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Bruno A. Neumann-Saavedra
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Dirk C. Mattfeld
Michael Römer

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Service Network Design of Bike Sharing Systems with Resource Constraints

Bruno A. Neumann-Saavedra[1], Teodor Gabriel Crainic[2,3,*], Bernard Gendron[2,4], Dirk C. Mattfeld[1], Michael Römer[5]

[1] Decision Support Group, University of Braunschweig, Mülehnpfordtstraβe 23, 38106 Braunschweig, Germany

[2] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[3] Department of Management and Technology, Université du Québec à Montréal, P.O. Box 8888, Station Centre-Ville, Montréal, Canada H3C 3P8

[4] Department of Computer Science and Operations Research, Université de Montréal, P.O. Box 6128, Station Centre-Ville, Montréal, Canada H3C 3J7

[5] Institute of Business Information Systems and Operations Research, Martin Luther University, Halle-Wittenberg, Universitätsring 3, 06108 Halle (Saale), Germany

Abstract. Station-based bike sharing systems provide an inexpensive and exible supplement to public transportation systems. However, due to spatial and temporal demand variation, stations tend to run full or empty over the course of a day. In order to establish a high service level, that is, a high percentage of users being able to perform their desired trips, it is therefore necessary to redistribute bikes among stations to ensure suitable time-of-day all levels. As available resources are scarce, the tactical planning level aims to determine efficient master tours periodically executed by redistribution vehicles. We present a service network design formulation for the bike sharing redistribution problem taking into account trip-based user demand and explicitly considering service times for bike pick-up and delivery. We solve the problem using a two-stage MILP-based heuristic and present computational results for small real-world instances. In addition, we evaluate the performance of the master tours for multiple demand scenarios.

Keywords: Bike sharing systems, bike redistribution, tactical planning, service network design, master tours.

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* Corresponding author: teodorgabriel.crainic@cirrelt.net

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1 Introduction

Station-based bike sharing systems (BSS) enhance the public transportation system in several cities by offering bike rentals. After registration, users can perform trips between any pair of stations scattered in the service area. Bike rentals are usually free of charge for the first (half) hour, additional driving time incurs fees. Thus, BSS have become a valid and inexpensive approach for the ”last mile” between the metro/bus station and the final destination. For a review of the history of BSS, see [1].

The percentage of users who can successfully perform desired trips, defining the service level, is an important measure for the reliability of a BSS. For a high reliability, a sufficient number of bikes and free bike racks need to be provided at stations within the day. Still, given spatial and temporal demand variation, together with different trip purposes such as commuting, leisure and tourism, ensuring a high service level is a challenging task [2]. For instance, stations near to working areas run full in the morning peak hour and empty in the afternoon peak hour. Full and empty stations may negatively affect the service level since users cannot return or rental bikes at them, respectively. To ensure availability of bikes and free bike racks when demanded, bikes need to be redistributed among stations. Resources such as vehicles, fuel and drivers are available to realize the necessary redistribution operations. Unlimited resources would fulfill user demand by setting up many redistribution operations with few bikes involved e.g., see [3]. However, given that due to the offered free-of-charge user trips, the revenues produced by BSS are limited, the resources available for redistribution operations are scarce. In fact, redistribution operations incur the most significant operational costs, putting on risk the bike sharing's profitability [4].

Information systems provide real-time status of BSS, including fill levels, user trips, and weather conditions. In addition, external information systems can be used for supporting the operation of vehicles, see e.g. [5, 6]. Although future user demand is unknown, it is possible to obtain estimates through the analysis of historical trip data, see e.g. [2]. Outputs of such analyses are used to anticipate at which time of day the rental or return rate is critical at particular stations. Although user demand varies between days, days with similar characteristics, e.g. commute activitiy during workdays in a summer season exhibit very similar demand patterns . Given these recurring patterns, it makes sense to think about a ”redistribution master plan” indicating how the redistribution vehicles should be regularly operated and forming the backbone for the operational redistribution planning.

In recent years, shared mobility systems have attracted a considerable amount of research regarding e.g. the location of stations, fill level at stations, user incentives, as well as the car/bike redistributions (for a review, see [7]). The BSS redistribution problem is related to the traditional inventory routing problem [8] since at stations, inventory decisions regarding the fill levels are made. A challenging feature of BSS is that bikes can be moved several times by both users and distribution vehicles. In the majority of related articles, however, it is assumed that repositioning only occurs during the night when user demand and traffic are considered as negligible; see for example [9, 10, 11].

Contrastingly, publications dealing with intra-day bike redistribution are still scarce. Most of these publications assume perfect knowledge of the user demand and consider the redistribution decisions in terms of bike flows through a time space network. They differ, however, in the way of handling user demand and in the considered objective functions. The approach presented in [12] assumes that nodes of the time space network can be partitioned into rental and return nodes, avoiding that both situations occur simultaneously. In other words, the user demand is not defined in terms of bike flows, but is associated at the nodes of the time space network as a rental or return request. Bikes are artificially added or removed when demand is not fulfilled, leading to an imbalance of the number of bikes in the system. Redistribution costs for operating vehicles are not considered in this approach. In [13], a multi-objective approach is proposed. User demand at stations is represented in terms of a expected accumulated demand over time. The expected unfulfilled demand is counted and penalized. A mismatch between initial and final fill levels for the given time horizon is also penalized. However, the initial fill level is not an output of the approach but selected arbitrarily. In [14], a cluster-first route-second approach is proposed, classifying stations according to user demand into pick-up or delivery stations. In [3], time-dependent origin-destination matrices are proposed for the resource allocation problem. The approach yields station-to-station redistribution decisions without considering the fact that these need to be performed by vehicles in a connected tour. In all papers described above, the service times for (un)loading bikes from the vehicle are neglected or assumed to be constant without regard to the number of (un)loaded bikes. To sum up, in the current literature, we identify a lack of properly representing critical issues such as time-dependent bike fill levels, service times incurred by redistribution decisions and user demand for the (intra-day) BSS redistribution problem in an optimization-based decision support system.

In this paper, we consider the intra-day BSS redistribution problem at a tactical planning level. At this planning level, the aim is to efficiently use the limited resources in order to yield a high expected service level for characteristic user demand patterns, e.g. for a working day in a given season. Redistribution operations are scheduled in master tours periodically operated by the redistribution vehicles. It is assumed that master tours are adjusted in an operational planning level based on the real-time BSS status by adapting the number of redistributed bicycles and/or by locally changing the route of the vehicle. The BSS redistribution problem can be addressed by service network design formulations [15] maximizing the service level while taking both vehicle fleet and monetary budget limitations into account. Outputs are the time-dependent fill level at stations and the necessary master tours to achieve these targeted fill levels.

We make the following contributions: First, we present a mixed-integer linear programming (MILP) formulation for the service network design of BSS. The MILP integrates the service level, the time-dependent fill levels, the master tours, the redistribution decisions, and the resources used for redistribution purpose. Second, based on a "small" real-world BSS, we conduct computational experiments to test different settings of available resources. Finally, we evaluate the quality of the master tours under different demand realizations and point to future research opportunities.

2 Problem Description and Model

The tactical BSS redistribution problem to be considered in this paper can be viewed as a special variant of a service network design problem. In this section, we first describe the key elements of this problem: The network underlying the BSS, the representation of fill levels, the tours conducted by the redistribution vehicle, the forecast of user demands as well as the service level and the costs incurred by the redistribution. This description, along with the introduced notation, forms the basis for the mathematical formulation of the problem as a mixed-integer linear program presented at the end of this section.

2.1 The network

The BSS infrastructure is defined on the set $\mathscr{N}'$ of physical nodes, i.e., the bike stations, and links connecting them, where the vehicles and users are allowed to drive. Each station $i \in \mathscr{N}'$ has a capacity of c_i bike racks. The vehicles are parked at the depot $\{0\} \in \mathscr{N}'$ considered as a station with big capacity and no bike demand. A total number of b' bikes are distributed among all stations. Theft of bikes, as well as damage of bikes or racks at stations, are neglected. It is supposed that the redistribution vehicles do not realize intermediate stops in order to simply represent them in terms of the corresponding vehicle paths. Figure 1 illustrates a small BSS infrastructure with three bike stations, two bikes allocated at each station, and the depot where a redistribution vehicle is parked. The solid line represents vehicle paths whereas the dashed lines are potential user trajectories between stations.

Let $\mathscr{T}$ be the target time horizon, e.g., a day, discretized into $\mathscr{T} = \{t\} = \{0, ..., T_{MAX}\}$ chronologically indexed time points; two adjacent time points represent one time period. We create a time space network represented by the graph $\mathscr{G} = (\mathscr{N}, \mathscr{A}_U \cup \mathscr{A}_V \cup \mathscr{A}_H)$. Each node $(i,t) \in \mathscr{N}$ represents a physical node $i \in \mathscr{N}'$ and a time point $t \in \mathscr{T}$. Each node (i,t) has a successor, i.e., the next time realization of the physical node, defined as $(i, t+1)$, if $t < T_{MAX}$.

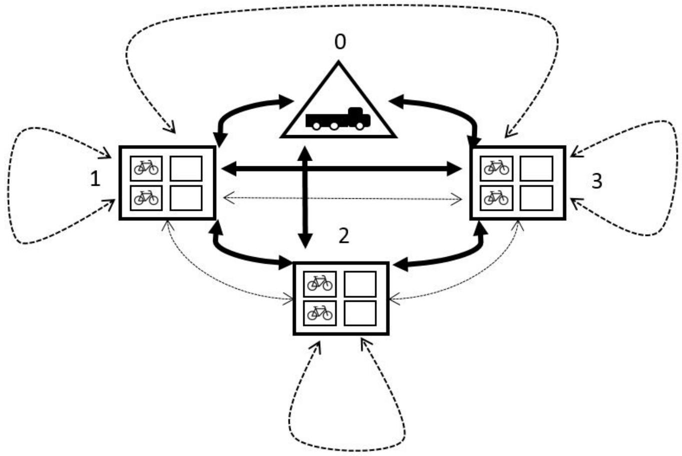

Figure 1: A small BSS infrastructure.

The arc set $\mathscr{A}_U$ contains the arcs $e_U = ((i,t),(j,\bar{t})), \bar{t} = t + \Delta^U_{ij}, \forall i,j \in \mathscr{N}' \setminus \{0\}, \forall t, \bar{t} \in \mathscr{T} \mid \bar{t} > t$, where Δ^U_{ij} is the number of periods a that a user requires to drive from station i to station j. Each arc e_U of the set $\mathscr{A}_U$ models the possibility that users realize trips, renting a bike from station i at time t and returning it at station j at time $\bar{t}$. The arc set $\mathscr{A}_V$ contains the arcs $e_V = ((i,t),(j,\bar{t})), \bar{t} = t + \Delta^V_{ij}, \forall i,j \in \mathscr{N}', \forall t, \bar{t} \in \mathscr{T} \mid \bar{t} > t$, where Δ^V_{ij} is the required number of periods that a vehicle needs to drive from station i to j. Each arc e_V of the set $\mathscr{A}_V$ models the possibility that a redistribution vehicle drives from physical node i at time t arriving at physical node j and, in the case that a bike station is located there, serving it until time $\bar{t}$. If the physical node j is the depot, the vehicle park there until time $\bar{t}$. Finally, the arc set $\mathscr{A}_H$ contains the arcs $e_H = ((i,t),(i,t+1)), \forall i \in \mathscr{N}', \forall t, \bar{t} \in \mathscr{T} \mid \bar{t} > t$. The arc set $\mathscr{A}_H$ models holding arcs, i.e., the possibility that a vehicle, loaded or not, stays in a physical node from time t to time $\bar{t}$. Holding arcs allow the vehicle to stay at a station for additional time in order to service the station with more bikes, if necessary. The union of both sets $\mathscr{A}_V \cup \mathscr{A}_H$ is referred to the set of vehicle arcs. The three type of arc sets allow bike movements through the time-dependent network. Thus, in the case that bikes are "moved" by one of these arcs, these bikes are not available for new purposes from the departure node (i,t), appearing instantaneously at the destination node $(j,\bar{t})$.

2.2 The time-dependent fill levels

Let I^t_i be the number of bikes at physical node i and time point t including both the bikes allocated at the station located in i and plus the load of the vehicles parked in the physical node i at time point t. Immediately after t, bikes can either be rented by users, transported by vehicles, or stay at the station. The number of bikes allocated at station

i between t and $t+1$ is denoted by β_i^t, whereas the number of free bike racks available at station i from time t until time $t+1$ is denoted by γ_i^t.

Dealing with the tactical planning level, we assume that the user demand exhibits similar patterns each day. That means we need to ensure a suitable fill level at the end of the time horizon, i.e., for the beginning of the new day. For that, we explicitly stipulate that the absolute value of the mismatch between the initial and final fill level is not bigger than a value Ψ, i.e., $|I_i^0 - I_i^{T_{MAX}}| \leq \Psi$. Clearly, the closer Ψ is to zero, the more redistribution effort is necessary to satisfy this condition.

2.3 The vehicle routing and bike redistribution decisions

The size of the vehicle fleet available during the time horizon is denoted by $v \in \mathbb{Z}^+$ bounded by a maximal vehicle fleet size V_{MAX}. Each vehicle can transport a maximal lot size of l bikes. Let $y_{ij}^{t\bar{t}} \in \mathbb{Z}^+$ be a variable capturing the number of vehicles which implement the corresponding vehicle arc in $\mathscr{A}_V \cup \mathscr{A}_H$. When a vehicle arc is implemented, the driver can handle, i.e., pick up or deliver, a maximal number of $\delta_{ij}^{t\bar{t}} \in \mathbb{Z}^+$ bikes at station j until time $\bar{t}$. Note that $\delta_{ij}^{t\bar{t}}$ depends on the time left after the driving time of the vehicle. The number of picked up or delivered bikes at station j until time $\bar{t}$ is denoted by $\rho_j^{\bar{t}} \in \mathbb{Z}^+$ or $\sigma_j^{\bar{t}} \in \mathbb{Z}^+$, respectively. Holding arcs allow the bike handling at stations during several time periods if it is required. $x_{ij}^{t\bar{t}} \in \mathbb{R}^+, \forall((i,t),(j,\bar{t})) \in \mathscr{A}_V \cup \mathscr{A}_H$ represents the total load of the vehicle implementing the corresponding vehicle arc e_V, i.e., the vehicle bike flows. In order to avoid symmetries in the optimization model, the presented formulation operates with a set of aggregated vehicles. Note that assumption is only suitable when the master tours are implemented by a homogeneous vehicle fleet.

A time-space diagram is showed in Figure 2 based on the BSS infrastructure presented above. The vertical axis represents the stations (and the depot), whereas the horizontal axis represents the time horizon, discretized into 6 time points. At each node (i,t) the the number of bikes on it, i.e., I_i^t, is illustrated. The solid lines represents all the vehicle arcs which describe the master tour operated by the vehicle. Let suppose that the driver can handle only one bike per time period. Thus, the vehicle starts from the depot at time point 0, arriving at station 2, and picking up one bike until time point 1. For loading one additional bike at the vehicle, the driver has to stay at the station one additional time period, i.e., a holding arc is implemented at station 2 between time points 1 and 2. Thus, at time point 2, two bikes are still on the physical node 2, but now the load of the vehicle, whereas the station is actually empty. At time point 2, the vehicle drives from station 2 to station 1, handling one bike from the load of the vehicle to the station. To deliver the second bike at the station, a holding arc is implemented again. Now, the loaded vehicle drives from station 2 to station 1, and the delivering process begins. Finally, the vehicle returns to the depot. Note that for the sake of clarity, Figure 2 does not illustrate user

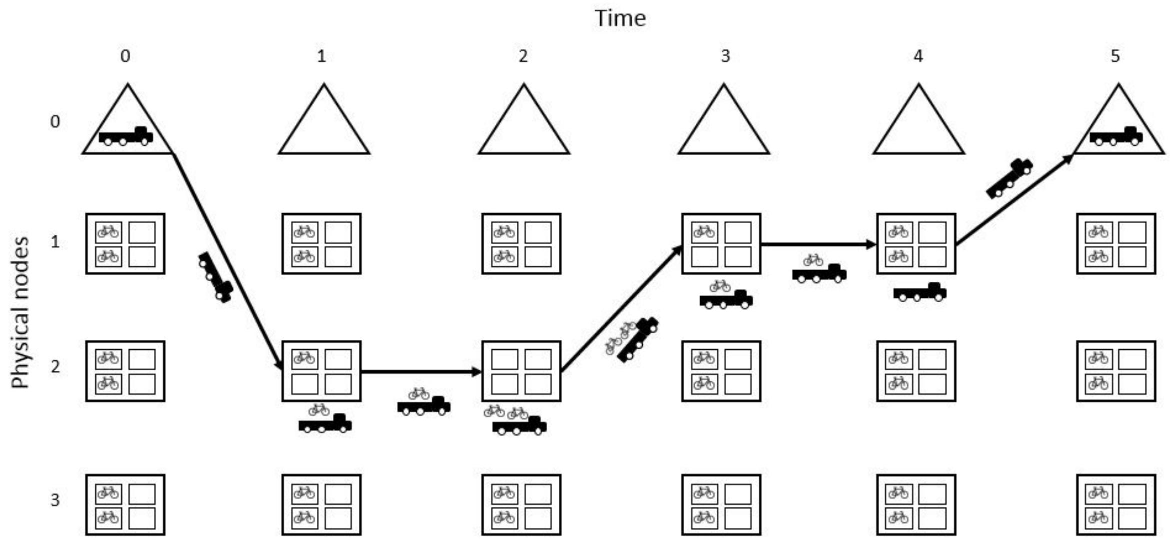

Figure 2: Time-space network. One redistribution vehicle operating.

trips through the network.

2.4 The representation of the user demand

We assume that the demand can be defined in terms of time-dependent origin-destination matrices representing expected user bike flows. Let $\zeta_{ij}^{t\bar{t}} \in \mathbb{Z}^+, \forall e_U \in \mathscr{A}_U$ be the number of expected user bike flows for the corresponding arc e_U. The decision variable $f_{ij}^{t\bar{t}} \in \mathbb{Z}^+, \forall e_U \in \mathscr{A}_U$. represents the user bike flows actually met. An expected user bike flow is only met if there is at least one bike at the departure station i and time t and at least one free bike rack at the destination station j and time point $\bar{t}$. Otherwise, the expected user bike flow is not realized.

Note that this demand representation has some limitations: It is assumed that users know the status at stations by means of information systems and do not realize a desired trip if they become aware that the trips cannot successfully be realized, even if there are stations close to the departure and destination stations with available bikes and racks. If a user bike flow is met, the bike is only available again when it is returned at time $\bar{t}$.

2.5 The service level

We defined the service level λ as the percentage of successfully realized demand trips during a time horizon. The service level is calculated as follows:

$$\lambda = \frac{\sum_{((i,t),(j,\bar{t}))\in\mathscr{A}_U} f_{ij}^{t\bar{t}}}{\sum_{((i,t),(j,\bar{t}))\in\mathscr{A}_U} \zeta_{ij}^{t\bar{t}}} \tag{1}$$

In order to address out approach with a MILP solver, we consider to maximize the successful user trips. The coefficient $\phi_{ij}^{t\bar{t}}$ may be considered in order to prioritize particular spatial and temporal demand (see Equation 2).

$$max \qquad \sum_{((i,t),(j,\bar{t}))\in\mathscr{A}_U} \phi_{ij}^{t\bar{t}} \cdot f_{ij}^{t\bar{t}} \tag{2}$$

2.6 The redistribution costs

Regarding the operational expenses ω, a cost F is associated with each redistribution vehicle used, a fixed cost $k_{ij}^{t\bar{t}}$ is incurred if each a vehicle implements an arc in $\mathscr{A}_V \cup \mathscr{A}_H$ (except the holding arcs when the vehicle stays in the depot) and a variable cost q_i^t is incurred per picked up or delivered bike at the node (i,t). Operational expenses are limited by a maximal budget L. The total operational expenses are calculated as Equation 3.

$$\omega = F \cdot v + \sum_{((i,t),(j,\bar{t}))\in\mathscr{A}_V\cup\mathscr{A}_H} k_{ij}^{t\bar{t}} \cdot y_{ij}^{t\bar{t}} + \sum_{(i,t)\in\mathscr{N}|i\neq\{0,T_{MAX}\}} q_i^t \cdot \left(\rho_i^t + \sigma_i^t\right) \tag{3}$$

2.7 The model

With the notation introduced above, the optimization model reads as follows:

$$\max \quad z = \sum_{((i,t),(j,\bar{t}))\in\mathscr{A}_U} \phi_{ij}^{t\bar{t}} \cdot f_{ij}^{t\bar{t}} \tag{4}$$

subject to

$$f_{ij}^{t\bar{t}} \leq \zeta_{ij}^{t\bar{t}}, \quad \forall((i,t),(j,\bar{t})) \in \mathscr{A}_U \tag{5}$$

$$\sum_{i\in\mathscr{N}'} I_i^0 = b' \tag{6}$$

$$I_i^{t+1} = I_i^t - \sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_U}} f_{ij}^{t\bar{t}} + \sum_{\substack{((j,\bar{t}),(i,t+1)) \\ \in \mathscr{A}_U}} f_{ji}^{\bar{t},t+1} - \sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{ij}^{t\bar{t}} + \sum_{\substack{((j,\bar{t}),(i,t+1)) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{ji}^{\bar{t},t+1}, \quad \forall i \in \mathscr{N}', t < T_{MAX} \tag{7}$$

$$I_i^t - \sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_U}} f_{ij}^{t\bar{t}} - \sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{ij}^{t\bar{t}} = \beta_i^t, \quad \forall i \in \mathscr{N}', t < T_{MAX} \tag{8}$$

$$c_i - \beta_i^t - \sum_{\substack{((j,\bar{t}),(i,t+1)) \\ \in \mathscr{A}_U}} f_{ji}^{\bar{t},t+1} - \sum_{\substack{((j,\bar{t}),(i,t+1)) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{ji}^{\bar{t},t+1} = \gamma_i^t, \quad \forall i \in \mathscr{N}', t < T_{MAX} \tag{9}$$

$$x_{ij}^{t\bar{t}} \leq l \cdot y_{ij}^{t\bar{t}}, \quad \forall ((i,t),(j,\bar{t})) \in \mathscr{A}_V \cup \mathscr{A}_H \tag{10}$$

$$\sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} y_{ij}^{t\bar{t}} = \sum_{\substack{((j,\bar{t}),(i,t)) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} y_{ji}^{\bar{t}t}, \quad \forall (i,t) \in \mathscr{N}, t \neq \{0, T_{MAX}\} \tag{11}$$

$$v \leq V_{MAX} \tag{12}$$

$$\sum_{\substack{((0,0),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} y_{0j}^{0\bar{t}} = \sum_{\substack{((j,\bar{t}),(0,T_{MAX})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} y_{j0}^{\bar{t}T_{MAX}} = v \tag{13}$$

$$\sum_{\substack{((0,0),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{0j}^{0\bar{t}} = \sum_{\substack{((j,\bar{t}),(0,T_{MAX})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{j0}^{\bar{t}T_{MAX}} = 0 \tag{14}$$

$$\rho_i^t - \sigma_i^t = \sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{ij}^{t\bar{t}} - \sum_{\substack{((j,\bar{t}),(i,t)) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} x_{ji}^{\bar{t}t}, \quad \forall (i,t) \in \mathscr{N}, t \neq \{0, T_{MAX}\} \tag{15}$$

$$\rho_i^t + \sigma_i^t \leq \sum_{((j,\bar{t}),(i,t)) \in \mathscr{A}_V \cup \mathscr{A}_H} \delta_{ji}^{\bar{t}t} \cdot y_{ji}^{\bar{t}t}, \quad \forall (i,t) \in \mathscr{N}, t \neq \{0, T_{MAX}\} \tag{16}$$

$$F \cdot v + \sum_{\substack{((i,t),(j,\bar{t})) \\ \in \mathscr{A}_V \cup \mathscr{A}_H}} k_{ij}^{t\bar{t}} \cdot y_{ij}^{t\bar{t}} + \sum_{\substack{(i,t) \in \mathscr{N} | \\ i \neq \{0, T_{MAX}\}}} q_i^t \cdot \left(\rho_i^t + \sigma_i^t\right) \leq L \tag{17}$$

$$I_i^{T_{MAX}} - \Psi \leq I_i^0 \leq I_i^{T_{MAX}} + \Psi, \quad \forall i \in \mathscr{N}' \tag{18}$$

$$I_i^t \in \mathbb{Z}^+, \forall (i,t) \in \mathscr{N}, \quad \beta_i^t, \gamma_i^t, \rho_i^t, \sigma_i^t \in \mathbb{Z}^+ \quad \forall (i,t) \in \mathscr{N}, t \neq \{0, T_{MAX}\} \tag{19}$$

$$y_{ij}^{t\bar{t}} \in \mathbb{Z}^+, x_{ij}^{t\bar{t}} \geq 0, \forall ((i,t),(j,\bar{t})) \in \mathscr{A}_V \cup \mathscr{A}_H \tag{20}$$

$$v \in \mathbb{Z}^+ \tag{21}$$

As explained above, the objective function (4) maximizes the service level. Constraints (5) ensure that the number of realized user trips are not higher than the expected trips from the demand data. At the beginning of the target time horizon, all bikes are allocated at the stations (6). Constraints (7) model the bike flow conservation taking into account the total number of bikes allocated at each station, user trips and bike relocation activities. The number of allocated bikes and available free bike racks at a station immediately after a time point is defined by equations (8) and (9). Equations (10) limit the load of the vehicles. The design-balanced constraints, that is, the vehicle flow constraints, are presented in (11). Equation (12) limits the size of the redistribution vehicle fleet. The master tours needs to start at end from the depot (13) with no bikes on the load (14). Equations (15) relate the number of picked-up and delivered bikes to the number of incoming and outgoing of bikes due relocation activities. Equations (16) restrict the handle time that a driver has to pick up or deliver bikes at a station until a time point. Constraint (17) models the limitation of the total relocation costs to the provided budget. Similar fill levels are expected at the beginning and end of the time horizon (18). Finally, all variables are non-negative, whereas the decision of implementing vehicle arcs, as well as the size of the vehicle fleet, are represented as integer variables (19, 20, 21).

3 Computational Experiments

This section presents computational experiments conducted to test our service network design formulation based on a "small" real-world BSS. Section 3.1 describe the input data, Section 3.2 presents the selected strategies to tackle our BSS instance, whereas results are reported on Section 3.3.

Table 1: San Francisco's Bay Area: instance description

Bike sharing system	San Francisco's Bay Area
Number of stations	35
Min - Max - Avg. bike racks per station	15 - 27 - 19
Year period	01 Mai - 31 Sep
Avg. trips per day	824

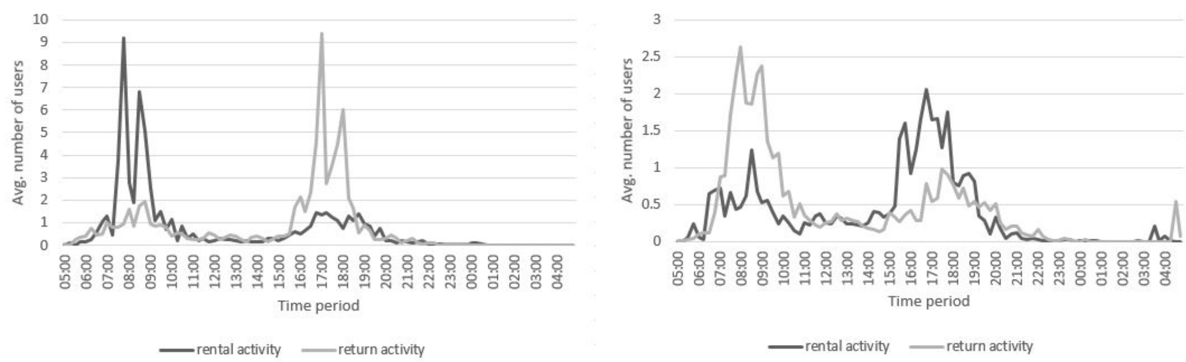

Figure 3: Rental and return activity of two Bay Area's bike stations. On the left, the San Francisco Caltrain 2 (330 Townsend). On the right, the Townsend at 7th.

3.1 Input data

We use the data of the San Francisco's BSS "Bay Area" to generate instances. Although Bay Area covers more cities, we only consider the data of San Francisco's service area. As we are interested in days with similar user demand patterns, we only consider the bike trips recorded during the summer season 2015, i.e., between May and September, excluding weekends. The bike sharing's infrastructure, as well as the recorded user trips, are presented and described on its website `http://www.bayareabikeshare.com/` and summarized in Table 1.

Analyzing the selected trip data, there are around 824 user trips per day out of which most happen in the morning and afternoon peak hours. In general, the user trips follow the activity patterns observed in [2]. To obtain a suitable user bike flow input for our service network design formulation, we aggregate the user trip data from multiple days to obtain the demand rate for each time-dependent origin-destination pair corresponding to the user bike flows utilized in the model. As the mean trip duration is around 12 minutes, we decided to split the time horizon into 15-minute time intervals. We assume that every user bike flow only takes one time period.

Figure 3 illustrates the rental and return activity at two Bay Area's bike stations. On the left, the San Francisco Caltrain 2 (330 Townsend), a station next to the train station, presents a high rental activity in the morning peak hour and a high return activity in the afternoon peak hour. On the right, the Townsend at 7th, a station located near to work places and shopping centers, presents the opposite behavior: A high return activity is observed in the morning peak hour and rental activity in the afternoon peak hour. Both stations exhibit typical commute activity observed in most BSS. Note, however, that the user activity at the Caltrain 2 station is clearly higher than at the Townsend at 7th.

After aggregating the data set, we obtain real-valued time-dependent user bike flows. For a suitable input for out service network design formulation, we need to generate integer user bike flows based on the real-valued ones. Assuming a Poisson distribution on the real-valued user bike flows, we generate 100 demand realizations with integer bike user flows. To obtain the master tours, we run our MILP with only one of the demand realizations. After that, we evaluate the quality of the master tours for all demand realizations by fixing the vehicle movements decisions of this solution and solve the resulting residual formulation once for each demand realization.

In addition, we use the following parameters: Regarding the redistribution vehicles, having a vehicle available during the time horizon costs 25 €/day. Based on the input data used in [15], each vehicle movement costs 0.5€/km, whereas the bike handling costs are 2€/bike between 8 and 17 hours, otherwise 3.5€/bike. The vehicle speed is 1m/s, and the service time is 1min/bike. 665 bikes are distributed among stations at the beginning of the time horizon. All user bike flows are weighted with the coefficient $\phi_{ij}^{t\bar{t}} = 1, \forall((i,t),(j,\bar{t}) \in \mathscr{A}_U$. Finally, $\Psi = 5$ is considered as the allowed mismatch between the initial and final fill levels.

3.2 Solution strategy

Even for small instances, solving the service network design formulation with standard MILP solvers is not possible within a reasonable amount of computation time. We propose the following approaches to speed-up the solution. First, we follow the "Two-phase solution method" proposed in [9]: In a first step, the integrality constraints for the fill level and bike flow variables are relaxed in order to obtain vehicle tours with fractional bike flows. In the second step, the vehicle tour decisions are fixed and the rest of the problem is solved again, now considering integer fill levels and bike flows.

We test with different number of vehicles by fixing v to 1, 2, or 3. In this first phase, the monetary budget is considered as unlimited. In the second phase, we aim at finding the minimal redistribution costs to obtain the optimal service level from the first phase. Note that it is possible that there exist solutions yielding the same service level with a fewer use of resources. To check that, we fix the optimal service level by introducing an additional constraint and select the left hand side of Equation 3 as objective function.

Table 2: Computational results after 10 h running time per objective

	objective		MIP gap	
v	λ	ω (€)	λ	ω
0	90.17%	0.00	-	-
1	97.90%	293.46	2.39%	15.73%
2	98.48%	306.23	1.54%	1.62%
3	99.65%	403.71	0.35%	3.04%

The service network design formulation is implemented in Java using the ILOG Concert Technology to access CPLEX 12.5. An Intel Xeon X7559 CPU at 2GHz processor with 80 RAM was used to run the experiments. All experiments were run for a maximal running-time of 10 hours per objective function.

3.3 Results

Table 2 shows the results for the Bay Area instance for different numbers of available vehicles. With one vehicle, a service level λ of 97.90% is obtained, meaning a 7.73% improvement comparing with a solution where no vehicles are used. Nevertheless, considering additional vehicles does not increase the service level significantly. For a second vehicle, only a 0.58% improvement is achieved in comparison to the one-vehicle solution. With three vehicles, the expected user bike flows are almost completely fulfilled. For the service level objective, the remaining MIP gap after reaching the time limit is always under 2.5 %.

The redistribution cost ω obtained after solving for the cost objective with a fixed service level from the first phase increases with the number of vehicles. Interestingly, the second vehicle incurs only 12.77€ more cost than the one-vehicle solution, whereas the redistribution cost increases considerably for the third vehicle. Note, however, that the results for the cost objective suffer from a high variation in the remaining MIP gap. In particular for the one-vehicle case, the solution quality may be improved considerably – the lower bound after the time limit is 247.31€.

Figure 4 illustrates the number of available bikes $\beta_{i,t}$ throughout the day for the two San Francisco stations discussed above, when one vehicle is used. The San Francisco Caltrain 2 station on the left almost runs full before the morning peak hour. As expected, a lot of bike rentals means that the stations is almost empty during the midday – this is not particularly critical since only few rentals are expected in that time. Moreover, a low fill level is desired before the afternoon peak hour to enable several bike returns. A significant decrease of the fill level is observed around 18:00 since bikes are removed from the station by a vehicle. This avoids that the station runs full, allowing more bike returns later on. Finally, low activity is observed during the night. The Townsend at 7th displayed on the right exhibits a completely different pattern. It begins with a low fill level, running almost full after the morning peak hour due to a large number of bike returns. In the afternoon peak hour, there is a high decrease of the fill level due to the large number of rentals. Around 17:00, bikes are delivered to the station to facilitate additional bike rentals during the evening and to maintain proper fill levels for the next day.

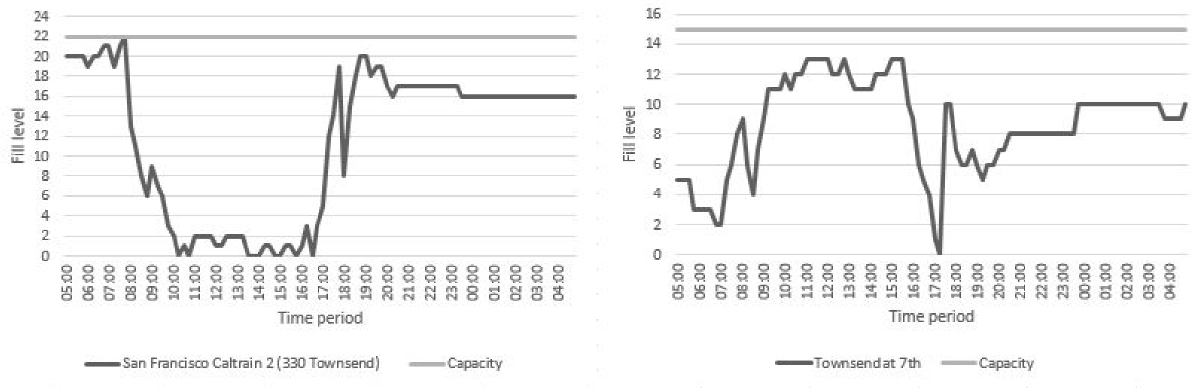

Figure 4: Available number of bikes within the day for the San Francisco Caltrain 2 (330 Townsend) (on the left) and the Townsend at 7th (on the right).

To evaluate the quality of the master tour obtained for a single vehicle and for a single of set of demand realizations in presence of different demand scenarios, we fix the master tour and solve the remaining bike flow problem for different demand scenarios. The results are depicted in Figure 5 by means of boxplots for the service level and redistribution costs obtained. The average service level is 95.23%, with a standard deviation of 1.53%. In fact, for most demand realizations, the service level lies between 94.00% and 96.00%. In fewer cases, the service level can range between 91% and 98%. A notch is used to show the 95.00% confidence interval. Regarding the redistribution costs, the mean is 218.33€, with a standard deviation of 44.78€. Most of the redistribution costs are between 175.00€ and 250.00€.

4 Discussion

For the small instances employed in our experiments, using a single redistribution vehicle already leads to a high service level. In fact, given a comparably small number of stations with comparably few bike rentals allows serving a high percentage of stations during the day with a single vehicle. Depending on the BSS infrastructure and daily rentals, however, more vehicles can have a more significant impact on achieving a higher service level. Moreover, considering the redistribution costs is critical to evaluate the quality of the solutions. For instance, the amount of redistribution costs associated with a given vehicle gives an indication on its utilization which may help to decide if the extra vehicle is actually necessary. We also observe that good fill level decisions consider the final fill level at stations, i.e., the initial fill level for the next time horizon. For instance, a station with several rental request each morning should dispose with a suitable number of bikes before the morning peak hours. This condition needs to be modeled explicitly in the service network design formulation.

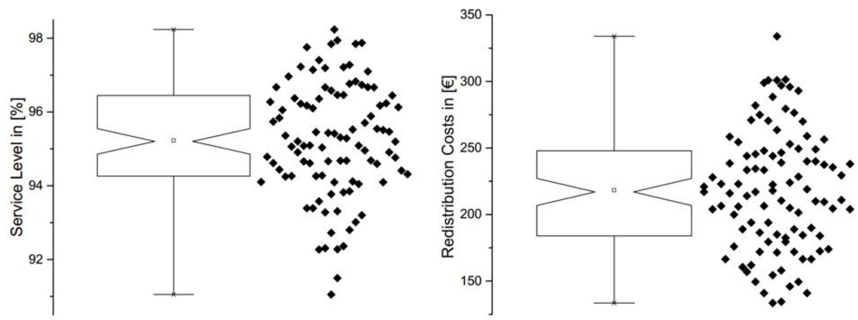

Figure 5: Box plots of service level and redistribution costs under different demand realizations with fixed vehicle movements.

Regarding the solution process, no optimal solution was found after the given running time. As for most service network design formulations, the design-balanced constraints necessary to set up the master tours are challenging for a standard MILP solver [16]. In fact, even solving the linear programming relaxation is very time-consuming and moreover only yields a weak lower bound for the optimal integer solution. Instances from bigger BSS with of hundreds of stations are not tractable using the MILP-based solution approach presented above. Alternative solution approaches are necessary to tackle bigger instances in an acceptable running-time. Heuristic search techniques should contribute to select a reduced but promising set of vehicle arcs to set up master tours.

Finally, our experiments show that the master tours are effective under different demand scenarios with similar characteristics for the selected BSS instance. Thus, master tours can support short-term operational redistribution decisions dealing with real-time fill levels and user demand as discussed by [5].

5 Conclusions

In this paper, we present a novel service network design formulation for the bike sharing redistribution problem. The model aims at obtaining master tours for the redistribution vehicles and bike redistribution operations in order to establish time-of-day-dependent station fill levels maximizing the service level. Our model uses a trip-based representation of user demand and explicitly considers the time needed for bike pick-up and delivery operations. The decision maker can evaluate the benefits of using different numbers of redistribution resources in order to make an informed trade-off between redistribution costs and service level. For example, our computational experiments show that for certain numbers of vehicles, an additional vehicle does not significantly improve the service level.

Taking a tactical planning perspective, we assume perfect knowledge of the user bike flows for the whole time horizon, i.e., a "deterministic" case. In our experiments presented in this paper, evaluated the performance of these master tours for multiple demand scenarios. In future work, we consider to explicitly model demand variations in our service network design formulation to obtain more robust master tour decisions. In addition, we aim at developing solution approaches to be able to tackle instances with a large number of stations.

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References

[1] DeMaio, P. (2009). Bike-sharing: History, impacts, models of provision, and future. Journal of Public Transportation, 12(4), 3.

[2] O'Brien, O., Cheshire, J., & Batty, M. (2014). Mining bicycle sharing data for generating insights into sustainable transport systems. Journal of Transport Geography, 34, 262-273.

[3] Vogel, P., Neumann-Saavedra, B. A., & Mattfeld, D. C. (2014). A hybrid metaheuristic to solve the resource allocation problem in bike sharing systems. In Hybrid Metaheuristics (pp. 16-29). Springer International Publishing.

[4] Büttner, J., & Petersen, T. (2011). Optimising Bike Sharing in European Cities-A Handbook.

[5] Brinkmann, J., Ulmer, M. W., & Mattfeld, D. C. (2015). Short-term Strategies for Stochastic Inventory Routing in Bike Sharing Systems. Transportation Research Procedia, 10, 364-373.

[6] Brinkmann, J., Ulmer, M. W., & Mattfeld, D. C. (2015). Inventory Routing for Bike Sharing Systems. Working Paper (2015-01-12).

[7] Laporte, G., Meunier, F., & Calvo, R. W. (2015). Shared mobility systems. 4OR, 13(4), 341-360.

[8] Coelho, L. C., Cordeau, J. F., & Laporte, G. (2013). Thirty years of inventory routing. Transportation Science, 48(1), 1-19.

[9] Raviv, T., Tzur, M., & Forma, I. A. (2013). Static repositioning in a bike-sharing system: models and solution approaches. EURO Journal on Transportation and Logistics, 2(3), 187-229.

[10] Rainer-Harbach, M., Papazek, P., Raidl, G. R., Hu, B., & Kloimüllner, C. (2015). PILOT, GRASP, and VNS approaches for the static balancing of bicycle sharing systems. Journal of Global Optimization, 63(3), 597-629.

[11] Dell'Amico, M., Hadjicostantinou, E., Iori, M., & Novellani, S. (2014). The bike sharing rebalancing problem: Mathematical formulations and benchmark instances. Omega, 45, 7-19.

[12] Contardo, C., Morency, C., & Rousseau, L. M. (2012). Balancing a dynamic public bike-sharing system (Vol. 4). CIRRELT.

[13] Kloimüllner, C., Papazek, P., Hu, B., & Raidl, G. R. (2014). Balancing bicycle sharing systems: an approach for the dynamic case. In Evolutionary Computation in Combinatorial Optimisation (pp. 73-84). Springer Berlin Heidelberg.

[14] Kloimüllner, C., Papazek, P., Hu, B., & Raidl, G. R. (2015). A Cluster-First Route-Second Approach for Balancing Bicycle Sharing Systems. In Computer Aided Systems Theory–EUROCAST 2015 (pp. 439-446). Springer International Publishing.

[15] Neumann-Saavedra, B. A., Vogel, P., & Mattfeld, D. C. (2015). Anticipatory Service Network Design of Bike Sharing Systems. Transportation Research Procedia, 10, 355-363.

[16] Andersen, J., Crainic, T. G., & Christiansen, M. (2009). Service network design with asset management: Formulations and comparative analyses. Transportation Research Part C: Emerging Technologies, 17(2), 197-207.