CIRRELT

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport

Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation

Integrating Production, Maintenance and Quality: a Multi-Period Multi-Product Profit-Maximization Model

Hossein Beheshti Fakher
Mustapha Nourelfath
Michel Gendreau

August 2017

CIRRELT-2017-59

Bureaux de Montréal :
Université de Montréal
Pavillon André-Aisenstadt
C.P. 6128, succursale Centre-ville
Montréal (Québec)
Canada H3C 3J7
Téléphone : 514 343-7575
Télécopie : 514 343-7121

Bureaux de Québec :
Université Laval
Pavillon Palasis-Prince
2325, de la Terrasse, bureau 2642
Québec (Québec)
Canada G1V 0A6
Téléphone : 418 656-2073
Télécopie : 418 656-2624

www.cirrelt.ca

Integrating Production, Maintenance and Quality: a Multi-Period Multi-Product Profit-Maximization Model

Hossein Beheshti Fakher[1,2], Mustapha Nourelfath[1,2,*], Michel Gendreau[1,3]

[1] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[2] Department of Mechanical Engineering, Pavillon Adiren-Pouliot, 1065, av. De la Médecine, bureau 1504, Laval University, Canada G1V 0A6

[3] Department of Mathematics and Industrial Engineering, Polytechnique Montréal, P.O. Box 6079, Station Centre-Ville, Montréal, Canada H3C 3A7

Abstract. We consider a set of items that must be produced in lots in a capacitated production system. Two types of failures are considered. At the occurrence of Type I failure, the system shifts to an out-of-control state where it produces a fraction of nonconforming items. After a shift, the system is restored to its initial in-control state. With Type II failures, the system stops and it is minimally repaired. During each period, the system is inspected and imperfect preventive maintenance (PM) activities can be performed to reduce its age, proportional to the PM level. At the end of each period a complete repair is performed. We develop an integrated optimization model where the objective is to maximize the expected profit. Computational experiments are performed to analyse the trade-offs between maintenance, quality and production. It is found that the increase in PM level leads to reductions in quality related impacts; but, if the cost of performing PM is high to the point where it is not compensated for by quality improvement, then performing PM is not beneficial. It is also found that using no-periodic inspections with the possibility of non-uniform PM levels may result in an improvement of the total profit.

Keywords: Production; maintenance; quality; integration; multiple periods and products; profit.

Acknowledgements. The authors would like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC) for supporting this project.

---

* Corresponding author: Mustapha.Nourelfath@cirrelt.ca

Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2017

Acronyms

PM	Preventive maintenance
EPQ	Economic production quantity
CLSP	Capacitated lot-sizing problem

Notation

H	Planning horizon
T	Number of periods
t	Index of periods
L	Length of a period (all periods have the same length)
P	Number of products
p	Index of products
Q	Number of PM levels
q	Index of PM levels
k	Index of PM/inspection intervals
I_t^k	Length of the *k*th PM/inspection interval in period *t*
d_t^p	Demand of product *p* in period *t*
π_t^p	Unitary cost of processing product *p* in period *t*
b_t^p	Backorder cost of product *p* in period *t*
h_t^p	Holding cost of product *p* in period *t*
s_t^p	Set-up cost for product *p* in period *t*
δ^p	Unitary cost of separation for product *p*
g^p	Production rate of product *p*
α^p	Fraction of nonconforming items *p* produced when the machine is out-of-control
PC_t^p	Selling price of a conforming item of product *p* in period *t*
PN_t^p	Selling price of a nonconforming item of product *p* in period *t*
CMR	Cost of minimal repair
$CPM(q)$	Cost of PM level *q*
$TPM(q)$	Time of a PM activity of level *q*
TMR	Time of a minimal repair
CCR	Cost of a complete repair
β	Cost of an inspection

η	PM imperfectness factor
ξ	Restoration cost parameter
$F(u)$	Cumulative distribution of the time to shift (Type I failure)
$f(u)$	Probability density function of the time to shift distribution (Type I failure)
$r(u)$	Hazard function of Type I failures
$G(u)$	Cumulative distribution of the time to Type II failure
$g(u)$	Probability density function of the time to Type II failure distribution
$\zeta(u)$	Hazard function of Type II failures
$\lambda,\ \varphi$	Parameters of Weibull distribution for Type I failures
θ, ρ	Parameters of Weibull distribution for Type II failures
NF_t^k	Number of Type II failures in interval *k* of period *t*
APT_t	Available production time in period *t*
PS_t^k	Probability of shift in interval *k* of period *t*
QC_t	Separation cost in period *t*
w_t^k	Age at the beginning of the k^{th} interval in period *t*
y_t^k	Age at the end of the k^{th} interval in period *t*
XN_t^p	Number of nonconforming items of product *p* in period *t*
XC_t^p	Number of conforming items of product *p* in period *t*
x_t^p	Lot-size of product *p* in period *t*
B_t^p	Backorder level of product *p* in period *t*
IC_t^p	Inventory of conforming product *p* at the end of period *t*
IN_t^p	Inventory of nonconforming product *p* at the end of period *t*
S_t^p	Set-up decision variable for product *p* in period *t*
m_t	Number of PM activities in period *t*
M_t^k	PM level at the end of interval *k* of period *t*
SC_t^p	Number of conforming items of product *p* sold in period *t*
SN_t^p	Number of nonconforming items of product *p* sold in period *t*

1. Introduction

1.1. Motivation and problem description

Production, maintenance and quality are key functions in manufacturing systems. Using common resources and due to mutual influences, these functions are all strongly linked to each other [1-3]. As a result, several papers have stressed the need and the economic importance of their integration, *e.g.* [4-6, 22]. However, models combining production, maintenance and quality are relatively scarce. There is a substantial amount of research dealing, in a separate way, with preventive maintenance (PM) planning, *e.g.* [7-9]. There exist also a lot of papers devoted to production planning without considering maintenance and quality issues, *e.g.* [10-12]. Finally, a vast literature separately deals with quality problems, *e.g.* [13-15]. Recently, the issue of integrating production planning and PM became an active area of research [16,17]. Recent literature also provides models that deal with the effect of quality and defective items produced by an imperfect process on economic production quantity (EPQ) [18,4]. Although recognizing the importance of integrated planning, the high majority of the existing literature is either limited to the integration of PM with production planning, or integrates production, maintenance, and quality by developing single machine EPQ models.

The present paper develops a profit maximization model which integrates PM, quality and production decisions; instead of dealing with the EPQ determination, we rather consider a multi-period multi-product capacitated lot-sizing problem (CLSP). The existing EPQ models apply only to single inventory items. Although we could certainly compute optimal order quantities separately for each of the different items, the existence of capacity constraints would make the resulting solution infeasible [18]. Furthermore, for time-varying demands, EPQ models do not give an optimal solution to the problem of production planning. By considering a multi-period lot-sizing model, we are able to find the policy that maximizes the total profit over the planning horizon under time-varying demands and costs. Classical multi-period multi-product lot-sizing models have usually ignored the possibility of machines deteriorations and the existence of nonconforming items in the production lots. Also, the use of machine inspection for maintenance and restoration purposes has not been considered in the context of profit maximization and CLSP. To the best of our knowledge, there is no existing model similar to our proposed profit-maximization model. The objective of this model is to determine the optimal values of the production plan, the sales and the PM times and levels, while taking into account quality related parameters.

Many challenging methodological difficulties arise when considering the above extensions. Our proposed approach is based on an evaluation method and an optimization algorithm. The evaluation method combines reliability, quality and production planning concepts to calculate the expected costs, the expected numbers of conforming and nonconforming items, and finally the expected profit. The optimization algorithm compares the results of several multi-product capacitated lot-sizing problems. Numerical examples are presented to validate the proposed model and to illustrate its important aspects. For example, we highlight the role of PM as a mean to reduce the cost of poor quality and to increase the total expected profit. In fact, the increase in PM level may lead to reductions in quality related impacts. However, when the cost of performing PM is high to the point where it is not compensated for by quality improvement, then performing PM is not beneficial in term of total profit maximization.

1.2. Prior literature

There are three separate main bodies of literature that are related to our research. The first is the literature on PM planning models. The second is the literature on production planning, and the third is related to quality control. Next, we briefly discuss each of these three research areas. Finally, we review the literature on the integration of maintenance, production and quality.

First, it is widely recognized that organizations are engaged increasingly in improvement of the systems availability and machines reliability, since they play a crucial role in performance, safety, organizational success, and economic efficiency. Therefore, PM planning are considered as key functions in manufacturing systems. There is a substantial amount of research dealing with PM planning. The advancement in this area is covered, for example, in [23] where the authors present an interesting classification, based on the modeling approach used for the problem formulation, such as Bayesian approach, mixed integer linear programming, fuzzy approach, simulation, Markovian probabilistic models and analytic hierarchy process. Generally, the objective of PM planning models is either to minimize the maintenance cost, or to maximize the availability, the production rate or the profit. These models are often solved by coupling optimization methods with analytical tools or simulation [23, 29]. Several papers have been also published on the problems of generating inspection instants of systems whose state of deterioration can be known only through inspection. In [38], Barlow and Proschan have worked on the determination of the inspection sequence which minimizes the total average cost per time unit. Other researchers, e.g. [39-41], proposed improvements and extensions to this basic model. An overview on existing inspection models for non-self-announcing failure single component and multi-components systems can be found in [42].

Second, there exist also a lot of papers dealing with production planning. For example, references [20,24] cover the majority of the production planning models and their related solution methods. Generally, production planning models are deterministic or stochastic optimization models designed either to minimize the total cost or to maximize the profit. In the capacitated lot-sizing problem, the optimal quantity of products to be processed on each machine and during each period should be determined subject to several constraints. Solution methodologies vary from traditional linear mixed integer programming, and associated branch and bound exact methods to heuristic methods [20,24].

Third, statistical process control (SPC) and quality sampling are conventional approaches to control and improve the production quality and the firm's productivity. Among SPC tools, the $\bar{x}$-chart is one of the frequently used tools to control the variations of process mean. The author of [25] first established a criterion that measures approximately the average net income of a process under surveillance of a control chart when the process is subject to random shifts in the process mean. He showed how to determine the sample size, the interval between samples, and the control limits that will yield approximately maximum average net income. According to this study, the PM-inspections should be equally spaced for an exponential failure time function. In [26], the authors suggested an acceptance sampling plan to control the quality in an unreliable imperfect system. Several other existing models dealt with the effect of defective items produced by an imperfect process on EPQ. Interestingly, the authors of [27] have found that, when the production process is subject to random process deterioration shifting the system from an in-control state to an out-of-control state, the resulting optimal EPQ is smaller than that of the classical model. Observing similar results, the author of [28] explored different options for investing in quality improvement.

Finally, we review the literature dealing with the integration of maintenance, production and quality. In [19], the authors incorporated maintenance by inspection with restoration cost dependent on the detection delay. In [13], the optimal design of a quality system was studied under Weibull shock models. In [30], the authors formulated a cost minimization model for the joint optimization of process control and perfect preventive maintenance for equipment with two quality states and general failure time distributions. The author of [31] studied the same quality decision variables in the context of imperfect maintenance using a discrete-time Markov chain approach. The author of [2] suggested a strategy for monitoring single-machine manufacturing process through the simultaneous implementation of an $\bar{x}$-chart and an age-based PM. The authors of [33] have developed an integrated production, inventory and preventive maintenance model for a multi-product production system and a single period. In [34], the athors developed a selective maintenance strategy for multi-state systems under imperfect maintenance. A joint modeling of preventive maintenance and quality improvement for deteriorating single-machine manufacturing systems was proposed in [35]. The author of [36] developed an optimal economic production quantity policy for randomly failing process with minimal repair, backorder and preventive maintenance. In [37], the author proposed a model to incorporate minimal repair and rework possibilities in order to determine the optimal number of inspections, the inspection interval, the EPQ, and the PM level. The effect of machine failure and corrective maintenance on lot-sizing problem was studied in [32]. In [16,17], the authors have shown that the integration of preventive maintenance and production planning is more economical than separate optimizations. The author of [5] developed an integrated profit model for imperfect production system incorporating an imperfect rework process, scrapped items, and PM errors. All the above-mentioned papers recognize the importance and the complexity of integrating PM, quality and/or production decisions. For further review of the integration of these three inter-dependent functions, the reader is referred to [21].

The existing papers are either limited to the integration of PM with production planning, or integrate production, maintenance, and quality by developing EPQ models. The present paper develops a profit maximization model integrating the three aspects (production, maintenance, and quality), while considering the multi-period multi-product capacitated lot-sizing problem (CLSP). The decisions involve determination of quantities of items (lot-sizes) to be produced in each period. In CLSP with limited demands, the answer obtained from maximizing the production or minimizing the maintenance cost can be different from the optimal solution of the joint model that maximizes the profit. Lot-sizing is one of the most important problems in production planning. Almost all manufacturing situations involving a product-line contain capacitated lot-sizing problems, especially in the context of batch production systems. The setting of lot sizes is in fact usually considered as a decision related to tactical planning, which is a medium-term activity. In aggregate planning, the lot sizing models are extended by including labor resource decisions. Tactical planning bridges the transition from the strategic planning level (long-term) to the operational planning level (short-term). Clearly, the time horizons may vary for each planning level depending on the industry. Typical values are one week (or less) for operational planning; one month (or more) for tactical planning; one year (or more) for strategic planning. In several modern production systems, the PM and quality decisions should be integrated at the tactical level. Although the literature recognizes the importance of integrated planning and the industrial relevance of tactical planning, there is no existing model dealing with profit maximization in the context of the CLSP. The present paper contributes to the existing literature by developing an integrated profit maximization model and a solution method based on reliability, quality and production planning concepts.

The paper remainder is organized as follows. Section 2 defines the problem and states the working assumptions. Sections 3 and 4 develop the evaluation method and the mathematical model, respectively. Section 5 develops a solution algorithm, and Section 6 presents numerical results. Finally, Section 7 concludes the paper.

2. Problem definition and assumptions

2.1. The production system

Consider a production system that is designed to produce a set of *P* products during a given planning horizon *H* subdivided into *T* discrete periods of a fixed length *L*. For each product *p*, a demand d_t^p should be satisfied at the end of period *t*. Figure 1 shows an example of a production plan with *H* = 24 months and *L* = 6 months. The system nominal capacity corresponds to its production rate g^p, measured in parts per time unit for each product *p*. At the beginning of each period *t*, the system is assumed to be in an *in-control* state producing conforming items, *i.e.* items of perfect or acceptable quality, with nominal capacity (g^p). Our model allows two types of failures, called Type I and Type II failures.

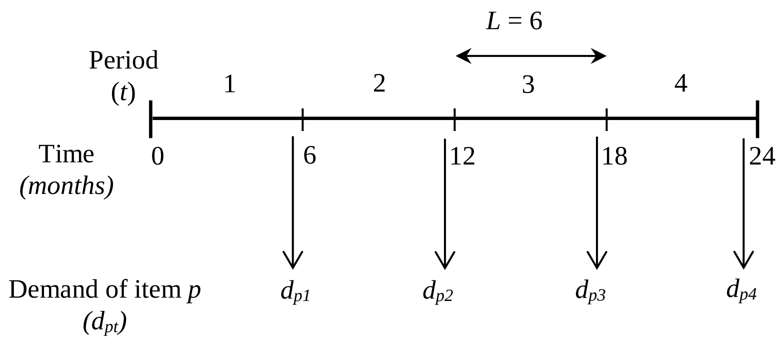

Figure 1. An example of a production plan.

Type I failures: As time progresses, the machine may shift to an *out-of-control* state where it produces a fraction α^p of nonconforming items, *i.e.* of substandard quality. Type I failures are detected only after inspection. If this inspection reveals that the system is in out-of-control state, it is possible to bring it back to its in-control state by some restorative work. Restoration refers to the actions required to identify and eliminate the causes of nonconformity. We consider minimal restoration, which means that the restored system condition is as good as it was immediately before the failure occurred. In other words, minimal restoration means that the age of the system is not disturbed by the restorative actions. The elapsed time for the machine to be in the in-control state before a Type I failure occurs is a random variable that follows a general distribution with increasing hazard rate. We qualify as operational the state where the system is producing items. That is the operational state can be either the in-control state or the out-of-control state.

Type II failures: These failures are self-announcing, *i.e.* detected without inspection. A blown fuse and an interruption of the delivery of consumable materials are examples of this kind of failure. At the occurrence of Type II failures, the system stops and it stays stopped until it is repaired. When the repair is completed, the system returns to the condition it was in when the failure occurred and operations resume. This means that we have a minimal repair as the system is repaired without affecting its age. The elapsed time for the machine to be in the *operational* state, before a Type II failure occurs, is a random variable that also follows a general distribution with increasing hazard rate.

Figure 2 summarizes the production system states described above. It is worth mentioning that the terms "in control" and "out of control" states are widely used in the literature of quality control and statistical process control charts and in the literature on the integration of maintenance and quality.

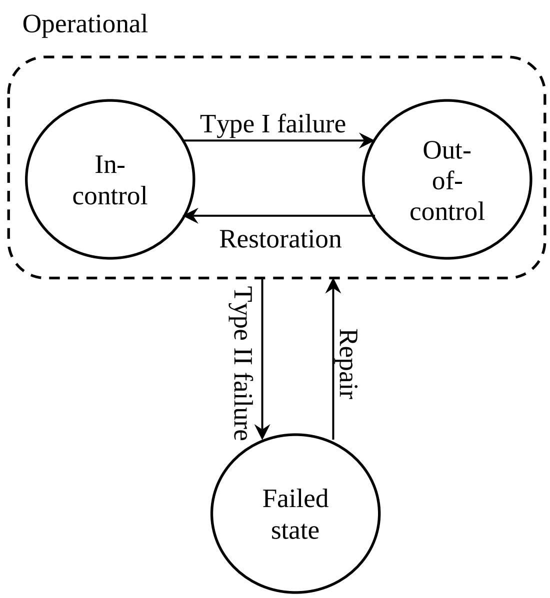

Figure 2. States of the production system.

2.2. Inspection and preventive maintenance

The used policy is illustrated in Figure 3, and it suggests that

- During each period t, the system is inspected at times t_k (k = 1, …, m_t+1) to assess its state and at the same time PM activities can be carried out.
- Each inspection can be followed by a PM activity which reduces the age of the machine. This age reduction depends on the level of the PM activity performed.
- Each time the machine is detected to be in an out-of-control state (Type I failures) conforming and nonconforming products are separated.
- Conforming items are used to satisfy the demand, whereas nonconforming items may be sold at the end of each period in a second market with reduced prices.
- At the end of each period, a complete repair is performed to renew the machine, *i.e.* to bring it back to its as good as new conditions. By introducing this assumption, the system can be analyzed for a

given period (t), and the evaluated performance measures and costs are summed over the number of periods (T). In general, such renewal is convenient in modeling the stochastic behavior of systems subjected to random failures [4,6,16].

The model is developed under the following additional assumptions:

- Inspections are error free.
- Inspection and restoration times are negligible. If this is not the case, the model can be easily modified to take restoration and inspection times into account.

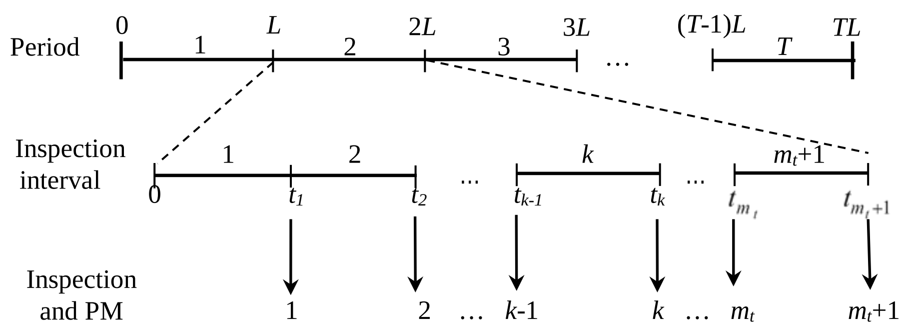

Figure 3. Inspection and PM for a given period; the k^{th} PM is performed at inspection time t_k.

In the considered model, the inspection is performed on the produced items to check their quality, while PM is performed on the process (i.e., the system producing these items). If the inspection reveals that the system has been producing non-conforming items (of bad or lower quality), production ceases and restoration work is carried out; otherwise, PM activities are carried out. The optimal solution can suggest that PM is not necessary in some cases (although inspection has been performed). That is, PM is triggered by quality inspection in some cases only, and the number of inspections can be different from the number of PM actions. The inspection and PM are then separately considered in our model, which is linking between maintenance, quality and production.

2.3. Integrated model

We develop a profit maximization model considering a multi-product, multi-period CLSP with time-varying demands and costs. A setup cost is charged whenever the processing of a product is planned. The usual assumptions of the CLSP model apply here. The trade-off between maintenance, quality and production is highly complex. Performing PM will yield reductions in quality related parameters, such as the restoration cost and the number of nonconforming items. However, if the cost of performing PM is high to the point where it is not compensated for by the reductions in nonconforming items and quality related costs, then performing PM will not be justifiable. Furthermore, the selling price of nonconforming items has an influence on the income. In each period, the production capacity is random and its expected value depends on the available production time, which is impacted by the PM and repair activities. The number of nonconforming items is also a random variable. Under such uncertainties, it is challenging to find the best trade-off between the various decisions on quality, PM and production.

3. Evaluation method

3.1. The profit as objective function

In the integrated model, the objective is to maximize the total profit which is calculated as

Profit = (Income from selling conforming items + Income from selling nonconforming items) − *(Production cost + Inspection cost + Preventive maintenance cost + Restoration cost + Separation cost + Repair cost).* (1)

In what follows, we detail each term in Equation (1).

1) Incomes

They are given by $\sum_{t=1}^{T}\sum_{p=1}^{P}\left(SC_t^p PC_t^p + SN_t^p PN_t^p\right)$, with SC_t^p is the number of conforming items sold, PC_t^p is the selling price of one conforming item, SN_t^p is the number of nonconforming items sold, and PN_t^p is the selling price of one nonconforming item (for each product *p* and period *t*).

2) Production cost

For a given product *p* and period *t*, this consists of

- A processing cost $\pi_t^p x_t^p$, with π_t^p is the cost of processing one unit, and x_t^p is the lot size;
- An inventory holding cost $h_t^p\left(IC_t^p + IN_t^p\right)$, with h_t^p is the inventory holding cost per unit, IC_t^p is the inventory level of conforming items, and IN_t^p is the inventory level of nonconforming items;
- A set-up cost $s_t^p S_t^p$, with s_t^p is the fixed set-up cost, and S_t^p is a binary variable which is equal to 1 if the set-up of product *p* occurs at the end of period *t*, and 0 otherwise; and
- A backorder cost $b_t^p B_t^p$, with b_t^p is the backorder cost per unit, and B_t^p is the backorder level.

Therefore, the total cost of production is

$$Production\ cost = \sum_{t=1}^{T}\sum_{p=1}^{P}\left(\pi_t^p x_t^p + h_t^p\left(IC_t^p + IN_t^p\right) + s_t^p S_t^p + b_t^p B_t^p\right). \quad (2)$$

3) Inspection cost

This is given by $\beta \sum_{t=1}^{T}(m_t + 1)$, with β is the cost of one inspection, and $(m_t + 1)$ is the number of inspections in period *t*.

4) Preventive maintenance cost

The concept of imperfect PM considers that, after PM, the failure rate of the system is somewhere between 'as good as new' and 'as bad as old' [7,9]. It can be assumed that the failure rate of the equipment is decreased after each PM. This amounts to a reduction in the age of the equipment. In this paper, we consider that the reduction in the age of the equipment is proportional to the cost of PM. Our model assumes the existence of a finite number of PM alternatives with various costs and impacts on the machine's reliability. Each PM alternative is called a PM level. In the proposed optimization model, the PM level is a decision variable, and the PM level that produces the highest total expected profit corresponds to the optimal PM level.

We denote by $CPM(q)$ the cost of the PM level *q*, such as *q* = 1, …, Q , with Q is the index of the lowest PM level, and $CPM(1)$ is the cost of the maximum PM level. The PM cost ($CPM\left(M_t^k\right)$) depends on the decision variable M_t^k, which specifies the level of the PM activity performed at the end of interval *k* in period *t*. As no PM action is performed at the last inspection, the number of PM actions in each period is m_t. Thus, the total PM cost is $\sum_{t=1}^{T}\sum_{k=1}^{m_t} CPM\left(M_t^k\right)$.

A PM strategy is indicated by T vectors, where each vector represents the PM levels for each period. Consider an example of 2 periods, 4 PM per period and 3 possible PM levels. Under PM strategy (1,1,1,1)-(1,1,1,1) all PM are of the highest level, whilst under PM strategy (3,3,3,3)-(3,3,3,3) all PM are of the lowest level.

5) Restoration cost

Each time the inspection reveals an out-of-control state, the machine is restored by identifying and eliminating assignable causes of variation. Using the idea in Reference [19], we consider that the machine restoration cost is a function of the detection delay. The detection delay is defined as the elapsed time since the production process has deteriorated until it is identified by inspection and repaired. In general, the sooner we realize that the machine is out-of-control, the less costly it would be to repair and restore it. Considering that the restoration cost during interval k in period t changes linearly with the detection delay τ_t^k, we have

$$Restoration\ cost = \sum_{t=1}^{T} \sum_{k=1}^{m_t+1} \left(\xi \tau_t^k\right), \tag{3}$$

where ξ is a given restoration parameter.

6) Separation cost

When detecting a shift, nonconforming and conforming items are separated. This separation cost is given by $\sum_{t=1}^{T} QC_t$, with QC_t is the separation cost in period t.

7) Repair cost

The model considers two repair activities.

- Complete repairs (at the end of production periods): the cost of one complete repair is given by CCR.
- Minimal repairs: taking into account the expected number of Type II failures, the average cost of minimal repair (MR) in period t is $CMR \times NF_t^k$, with CMR is the MR cost of one failure and NF_t^k is the expected number of failures in interval k and period t. For all the T periods and the $(m_t + 1)$ intervals, the total MR cost is $CMR \sum_{t=1}^{T} \sum_{k=1}^{m_t+1} NF_t^k$. Therefore, the total repair cost is

$$Repair\ cost = (T \times CCR) + CMR \sum_{t=1}^{T} \sum_{k=1}^{m_t+1} NF_t^k. \tag{4}$$

The profit is stochastic because many quantities in the above equations are stochastic. Not only the production capacity and the number of conforming (and nonconforming) items are random variables, but also other quantities, such as the costs of repair and restoration, are subjected to uncertainties. Our model maximizes the *expected* profit. Using the same notations for the variables and their expected values, we have

$$Expected\ profit = \sum_{t=1}^{T}\sum_{p=1}^{P}\left(PC_t^p SC_t^p + PN_t^p SN_t^p\right) - \sum_{t=1}^{T}\sum_{p=1}^{P}\left(\pi_t^p x_t^p + h_t^p\left(IC_t^p + IN_t^p\right) + s_t^p S_t^p + b_t^p B_t^p\right)$$
$$- \beta\sum_{t=1}^{T}(m_t+1) - \sum_{t=1}^{T}\sum_{k=1}^{m_t} CPM\left(M_t^k\right) - \xi\sum_{t=1}^{T}\sum_{k=1}^{m_t+1}\tau_t^k - \sum_{t=1}^{T} QC_t - (\,T \times CCR)$$
$$- CMR\sum_{t=1}^{T}\sum_{k=1}^{m_t+1} NF_t^k. \quad (5)$$

To estimate the expected profit, let us first evaluate the age of the system at important time points of the planning horizon.

3.2. Age evaluation

We use the concept of age-based imperfect PM [7,9]. After each PM, the age of the system is somewhere between as good as new and as bad as old depending on the level of PM activities. The reduction in the age of the system is a function of the cost of PM. Let denote by w_t^k and y_t^k the ages of the system at the beginning and at the end of the k^{th} interval in period t (respectively). That is, w_t^{k+1} is the actual age *right after* the *k*th PM; and y_t^k is the actual age *right before* the *k*th PM. Let

$$\gamma_t^k = (\eta)^{k-1}\frac{CPM(q)}{CPM(1)}, \quad (6)$$

where $q = 1, \dots, Q$, and η $(0 < \eta \leq 1)$ is the PM imperfectness factor.

Linear and nonlinear relationships between age reduction and PM cost can be considered. In the linear case, we have

$$w_t^{k+1} = \left(1-\gamma_t^k\right)y_t^k\ . \quad (7)$$

Since at the beginning of each period t the machine is in its perfect conditions with age zero, we have

$$w_t^1 = 0. \quad (8)$$

Considering that the decision variable M_t^k specifies the level of the PM activity, by using Equations (6) and (7) we have for $k = 1, \ldots, m_t + 1$:

$$w_t^{k+1} = y_t^k \left(1 - \frac{CPM\left(M_t^k\right)}{CPM(1)} \eta^{k-1}\right). \tag{9}$$

The parameter η implies that there is a degradation in the effect of PM on the age of the machine. A full PM brings the machine farther from the as good as new condition as more PM actions are performed. Let us denote by I_t^k the length of the k^{th} inspection interval in period t. During minimal repairs and PM, the machine age will not increase. Given the expected number of machine failures (NF_t^k), the time required for one minimal repair (TMR), the selected PM level (M_t^k) and its corresponding PM time ($TPM\left(M_t^k\right)$), the total maintenance time is given by ($TPM\left(M_t^k\right) + NF_t^k \cdot TMR$). So, for each period t ($t = 1, \ldots, T$), the machine age at the end of interval k ($k = 1, \ldots, m_t + 1$) is

$$y_t^k = w_t^k + I_t^k - TPM\left(M_t^k\right) - NF_t^k TMR. \tag{10}$$

Since no PM is performed at the last interval of each period, the ending age for the last interval is

$$y_t^{m_t+1} = w_t^{m_t+1} + I_t^k - NF_t^k TMR. \tag{11}$$

If $\zeta(u)$ is the Type II failure rate of the machine at age u, the expected number of machine failures in the k^{th} interval of period t is

$$NF_t^k = \int_{w_t^k}^{y_t^k} \zeta(u) du. \tag{12}$$

Knowing the probability distribution of Type II failure, Equations (8) to (12) are simultaneously solved to yield the age values in all of the intervals and periods.

Since the operational time is equivalent to the age variation ($y_t^k - w_t^k$), the expected available production time in period t is

$$APT_t = \sum_{k=1}^{m_t+1} \left(y_t^k - w_t^k \right). \tag{13}$$

3.3. *Expected numbers of conforming and nonconforming items*

Let $F(u)$ be the cumulative distribution of the time to shift (i.e. Type I failure). The conditional probability that the system shifts to the out-of-control state during interval k (*i.e.* from w_t^k to y_t^k), given that it was in control at the beginning (*i.e.* at w_t^k), is

$$PS_t^k = \frac{F\left(y_t^k\right) - F(w_t^k)}{1 - F(w_t^k)}. \tag{14}$$

Let $f\left(u\middle|\, w_t^k\right) = \frac{f(u)}{1-F\left(w_t^k\right)}$ be the conditional probability density function of the time to shift distribution given that the system was in control at time w_t^k.

If a shift occurs in interval k of period t, with a probability of PS_t^k, then the average time that the system remains in this out-of-control state before being fixed is $\int_{w_t^k}^{y_t^k}\left(y_t^k - u\right) f\left(u\middle|\, w_t^k\right)du$. Here, $\left(y_t^k - u\right)$ is the remaining allocated time $\left(w_t^k \le u \le y_t^k\right)$, and $f\left(u\middle|\, w_t^k\right)dt$ is the probability to shift during du (given that the machine was in control at w_t^k). So, the expected number of items processed in the shifted state (for product p in period t) is $x_t^p PS_t^k \int_{w_t^k}^{y_t^k}\left(y_t^k - u\right) f\left(u\middle|\, w_t^k\right)du \,/ APT_t$. Since the nonconformity rate in a shifted state is α^p, the expected number of nonconforming items produced in period t (for product p) is

$$XN_t^p = x_t^p \frac{\alpha^p}{APT_t} \sum_{k=1}^{m_t} \int_{w_t^k}^{y_t^k} PS_t^k \left(y_t^k - u\right) f\left(u\middle|\, w_t^k\right)du. \tag{15}$$

Knowing that the lot-size x_t^p represents the total items produced, the expected number of conforming items is $XC_t^p = x_t^p - XN_t^p$. The analytical derivation of Equation (15) is inspired from existing papers dealing with EPQ problems [4,6,27]. These papers (and many others) use the same approach to calculate the number of non-conforming items in other contexts.

3.4. *Expected restoration cost*

According to Equation (3), the restoration cost depends on detection delay $\tau_t^k = \left(y_t^k - u\right)$ which is a random variable. Its expected value is the expected duration of time that the system operates in an out-of-control state in period *t*. Therefore, the expected restoration cost is

$$Restoration\ cost = \xi \sum_{t=1}^{T} \sum_{k=1}^{m_t+1} \left(\int_{w_t^k}^{y_t^k} \left(y_t^k - u\right) f\left(u \middle| w_t^k\right) du \right) PS_t^k. \tag{16}$$

3.5. *Expected separation cost*

This cost is proportional to the number of items produced when the system is out-of-control. Therefore, if δ^p is the unitary cost to separate one product *p*, the total expected separation cost in period *t* is

$$QC_t = \sum_{p=1}^{P} \frac{\delta^p x_t^p}{APT_t} \sum\nolimits_{k=1}^{m_t} PS_t^k (y_t^k - w_t^k). \tag{17}$$

In Equation (17), the quantity $\left(\frac{\delta^p x_t^p}{APT_t} \sum_{k=1}^{m_t} PS_t^k (y_t^k - w_t^k)\right)$ represents the number of items produced when the system is in out-of-control state (PS_t^k being the probability of a quality shift).

4. The mathematical model

The integrated model is mathematically formulated as a maximization problem of the expected profit objective function. Specific constraints are also formulated. The mathematical model is

Maximize (5)
Subject to
(8) - (17)

$$IC_t^p = IC_{t-1}^p - SC_t^p + \left(x_t^p - XN_t^p\right), \quad p = 1, \ldots, P;\ t = 1, \ldots, T \tag{18}$$

$$IN_t^p = IN_{t-1}^p - SN_t^p + XN_t^p, \quad p = 1, \ldots, P;\ t = 1, \ldots, T \tag{19}$$

$$B_t^p = B_{t-1}^p + d_t^p - SC_t^p, \quad p = 1, \ldots, P;\ t = 1, \ldots, T \tag{20}$$

$$x_t^p \leq Lg^p S_t^p, \quad p = 1, \ldots, P;\ t = 1, \ldots, T \tag{21}$$

$$\sum_{p=1}^{P} \frac{x_t^p}{g^p} \leq APT_t, \quad t = 1, \ldots, T. \tag{22}$$

$$IC_0^p = 0, \quad p = 1, \ldots, P \tag{23}$$

$$IN_0^p = 0, \quad p = 1, \ldots, P \tag{24}$$

$$B_0^p = 0, \quad p = 1, \ldots, P \tag{25}$$

$$M_t^k \in \{1, \ldots, Q\}, \quad t = 1, \ldots, T;\ k = 1, \ldots, m_t \tag{26}$$

$$m_t \geq 0, \quad t = 1, \ldots, T \tag{27}$$

$$S_t^p \in \{0,1\}, \quad p = 1, \ldots, P;\ t = 1, \ldots, T \tag{28}$$

$$x_t^p \geq 0, SC_t^p \geq 0, SN_t^p \geq 0, IC_t^p \geq 0, IN_t^p \geq 0, B_t^p \geq 0, \quad p = 1, \ldots, P;\ t = 1, \ldots, T \tag{29}$$

The objective function and constraints (8) to (17) have been evaluated in the previous section. They allow for an explicit evaluation of the expected profit. The flow constraints (18) and (19) link the inventory, backorder and sales to the expected number of conforming and nonconforming items. Constraint (20) links the backorders to the demands and sales. Constraint (21) forces $x_t^p = 0$ if $S_t^p = 0$, and frees $x_t^p \geq 0$ if $S_t^p = 1$. In (21), the quantity $g^p L$ is an upper bound of x_t^p. Constraint (22) indicates the capacity limitation. It states for each period t that the produced quantity x_t^p (lot size) cannot be higher than the quantity produced at rate g^p during the available production time APT_t. Equations (23) to (25) state the initial inventory and backorder levels. Constraint (26) specifies the PM decisions to be selected by choosing the appropriate PM levels, while constraint (29) is related to the number of inspections undertaken during each period. Constraint (28) indicates that the set-up variable is binary, and constraint (29) states that the other variables are positive. The decision variables are:

- The production planning variables, *i.e.* the set-up decision (S_t^p), the lot-size (x_t^p) and its corresponding backorder and inventories (B_t^p, IC_t^p, IN_t^p);
- The sales decisions given by the numbers of conforming and nonconforming items which are sold in each period (SC_t^p, SN_t^p); and
- The variables m_t and M_t^k representing the selected inspection/PM frequency and PM levels, respectively.

The proposed integrated model allows the joint determination of production planning, sales decisions, inspection and PM times and levels, while taking into account quality aspects, such as the restoration costs, and the quantities and selling prices of nonconforming items.

The above mathematical model corresponds to a mixed-integer non-linear programming problem. Each evaluation in the solution space requires, not only the handling of several integrals and complex constraints, but also the solution of the usual capacitated lot-sizing problem (CLSP). For a given inspection/PM solution, the objective function and the constraints are evaluated; and knowing the values of the inspection times and the PM levels, the problem becomes a mixed integer linear production planning problem corresponding to the classical CLSP.

5. Solution algorithm

The proposed algorithm is based on the following steps:

- **Step 1**: Select the probability distributions for Types I and II failures.
- **Step 2**: Determine the number of inspections and PM actions to undertake in each period.
- **Step 3**: For each combination of the PM decision variables, evaluate the expected profit, and solve the resulting CLSP.
- **Step 4**: Compare the resulting optimal solutions of all the optimization models solved in Step 3, and select the final optimal solution.

This algorithm is applied considering two variants of the problem depending on the frequency of inspection. The first variant considers periodic inspection, while the second deals with the more general case of non-periodic inspection. The difference between these two variants lies in Step 2 which calculates the inspection/PM frequency.

Step 1: **Selection of the probability distributions**

On the one hand, we consider that the time during which the system remains in the in-control state (time to shift or Type I failures) follows a Weibull distribution. Resulting in an increasing hazard rate, this is in line with the features of the existing quality models, *e.g.* [4,5]. The corresponding probability density function is

$$f(u) = \lambda\varphi u^{\varphi-1} e^{-\lambda u^{\varphi}}, \quad u > 0, \quad \varphi \geq 1, \quad \lambda > 0, \tag{30}$$

where λ and φ are the parameters of Weibull distribution.

The corresponding cumulative probability function is

$$F(t) = 1 - e^{-\lambda u^{\varphi}}, \quad u > 0, \quad \varphi \geq 1, \quad \lambda > 0. \tag{31}$$

Its hazard rate is

$$r(u) = \lambda\varphi u^{\varphi-1}, \quad u > 0, \quad \varphi \geq 1, \quad \lambda > 0. \tag{32}$$

Type II failures are also characterized by a Weibull distribution with parameters θ and ρ

$$g(u) = \theta\rho u^{\rho-1} e^{-\theta u^{\rho}}, \quad u > 0, \quad \rho \geq 1, \quad \theta > 0. \tag{33}$$

The Type II cumulative probability function is then

$$G(u) = 1 - e^{-\theta u^{\rho}}, \quad u > 0, \quad \rho \geq 1, \quad \theta > 0. \tag{34}$$

Such Weibull distribution is widely used in the reliability analysis and it has the flexibility to cover different failure models. The corresponding hazard rate is increasing and it is derived as follows

$$\zeta(u) = \theta\rho u^{\rho-1}, \quad u > 0, \quad \rho \geq 1, \quad \theta > 0. \tag{35}$$

Step 2: Number of inspections

Periodic case

The lengths of the inspection/PM intervals are given by

$$I_t^k = \frac{L}{m_t + 1}, \quad t = 1, \dots, T; \; k = 1, \dots, m_t. \tag{36}$$

The decision variables m_t define the frequency of the periodic inspection and PM.

Non-periodic case

It is generally admitted that while periodic PM can be more convenient to schedule, sequential or non-periodic PM is more realistic when the system requires more frequent maintenance as it ages. By applying the idea in [13], we consider that the lengths of inspection intervals are defined such that the integrated hazard over each interval is the same for all intervals. Knowing that the system age is reduced at the end of each interval because of PM activities, the inspection interval is then selected by solving the following equation:

$$\int_{w_t^k}^{y_t^k} r(u)du = \int_0^{I_t^1} r(u)du \,, k = 2, \dots, m_t, \tag{37}$$

where $r(u)$ is the hazard function corresponding the time to shift distribution (Type I failure).

Equation (37) is general in the sense that it can be used for any probability distribution with increasing hazard rate. For the particular case of exponential distribution, it yields equally spaced intervals. For a Weibull distribution, using Equation (37), the length of the variable inspection/PM intervals can be determined recursively as follows

$$I_t^k = \left[\left(w_t^{k-1}\right)^{\varphi} + (I_t^1)^{\varphi}\right]^{\frac{1}{\varphi}} - w_t^{k-1}, k = 2, \dots, m_t. \tag{38}$$

It is worth mentioning that the obtained inspection intervals lead to more frequent PM as the system gets older.

We consider that the number of inspections for all periods is the same. The length of intervals is calculated such that: (1) the integrated hazard over all intervals is constant; and (2) the length of all intervals plus the time of maintenance (PM time and expected corrective maintenance time) equals the length of period. For a given number of inspections, an initial value (H_0) of the integrated hazard is considered. To determine the age of machine in intervals, the length of the first interval is calculated. Then, the ages at the end of the first interval and at the beginning of the second interval are evaluated. Knowing the latter, the length of the second interval can be calculated such that its integrated hazard is also equal to H_0. This procedure continues until the last interval. Then, if the evaluated time (L') of period (or the sum of the length of all intervals and maintenance time) is much different from the period length (L), the initial value of the integrated hazard (H_0) is modified and the process repeats until the absolute difference $|L - L'| < \varepsilon$ (ε is a small number).

Step 3: **Evaluation of the expected profit and solution of the CLSP**

Evaluation

Knowing the failure rates probability distributions, this step solves Equations (8) to (12) for each PM level. After some easy manipulations, we obtain

$$\left[\theta \times \left(y_t^k\right)^{\rho} \times TMR\right] + y_t^k = w_t^k + I_t^k - \left(TPM \times M_t^k\right) + \left[\theta \times \left(w_t^k\right)^{\rho} \times TMR\right]. \tag{39}$$

For each combination of the PM decision variables (M_t^k and m_t), Equation (39) is solved numerically to yield the age values for each interval and period. Once the ages are calculated, we can use the method detailed in the previous section to estimate the expected profit.

Solving the CLSP

Once the values of the PM decision variables are fixed, the remaining decision variables are related to production (x_t^p, B_t^p, IC_t^p, IN_t^p, S_t^p) and sales (SC_t^p, SN_t^p). As a result, for each possible combination of M_t^k and m_t, a pure production planning profit-maximization model must be solved. We consider that an algorithm for solving such usual capacitated lot-sizing problem is available. In fact, this kind of problems has been extensively studied in the literature [20], and it can be solved using any selected existing algorithm. This pure production planning problem can be also solved using the mixed integer solver of a commercial optimization package. In our case, we used the mixed integer solver of CPLEX.

Step 4: Comparing the resulting optimal solutions

This last step determines the optimal integrated PM and production plan that takes into account the quality aspects considered in our model.

6. Numerical results

All the algorithms and solution methods are implemented in VB.NET and the numerical tests are performed on an Intel Core i7 – 3.4 GHz with 16 GB of RAM.

6.1. Input data

The planning horizon corresponds to three periods of one month each (H = 3, L = 1, T = 3 and t = 1, 2, 3). The system has to produce three kinds of products in lots so that the demands are satisfied (P = 3 and p = 1, 2, 3). The Weibull parameters are λ = 40 and φ = 2.5 for Type I failure, and θ = 20 and ρ = 2.5 for Type II failure. The number of PM options is Q = 4 (q = 1, 2, 3, 4). The other input parameters are given in Table 1. For each product and each period, Table 2 provides the demands, the conforming items selling prices, and the unitary costs of processing, backorder, holding and set-up. The price of the nonconforming product is 25% the price of the conforming item.

6.2. Analyzing the trade-off

The number of inspections is a decision variable. In this subsection, this variable is fixed to 3 in order to facilitate the analysis of the variations of profit with the PM cost. We illustrate how the algorithm selects the best PM level by determining the best trade-off between production, maintenance and quality. Considering a periodic inspection/PM interval of 1/4 month, Figure 4 sketches the profit as a function of the PM cost. The latter is the sum of all PM costs incurred for a given solution. The optimal integrated plan is chosen after enumerating all PM strategies. For each selected PM strategy, the algorithm optimizes the rest of the planning (decision) variables to maximize the profit.

The graph in Figure 4 is a discrete function because there is only a finite set of possible PM strategies, and both the PM cost and the profit depends on the strategy chosen. On the one hand, the lowest cost PM strategy is *Strat1* = (4,4,4)-(4,4,4)-(4,4,4). In this case, no PM is performed and a slightly negative profit is observed. On the other hand, the highest cost PM strategy is *Strat2* = (1,1,1)-(1,1,1)-(1,1,1). In this case, perfect PM actions are performed at each inspection, and the observed profit is again slightly negative. The calculated optimal strategy is *StratOpt* = (4,1,4)-(4,1,4)-(4,4,4). Although the profit function has several non-monotonic changes, we observe a general increasing trend between *Strat1* and *StratOpt*, and a general decreasing trend between *StratOpt* and *Strat2*. Once the optimal strategy is reached, the cost of performing PM is high to the point where it is not compensated for by reductions in the quality related impacts. In this case, performing PM is not justifiable in comparison to the optimal strategy.

We also remark that the highest incremental profit increase occurs when the PM strategy changes from (3,4,4)-(4,4,1)-(4,4,4) to (3,1,4)-(4,4,4)-(4,3,4). This corresponds to a PM cost increase from $5200 to $5400. The resulting increase in profit is $8173.7. Interestingly, this means that investing just $200 in PM may lead to a profit increase of about $8174. This kind of information is very useful. In fact, if the planner is facing a situation where the PM investment is limited to $5200, this suggests that increasing the PM investment by a small amount may advantageously lead to a relatively high profit increase.

Table 1

Input parameters.

Parameter	Notation	Respective values
Cost of PM level q ($)	$CPM(q)$, $q = 1, 2, 3, 4$	5000, 1000, 200, 0
Time of PM level q (month)	$TPM(q)$	0.05, 0.003, 0.001, 0
Production rate of product p (items/month)	g^p, $p = 1, 2, 3$	450, 400, 350
Nonconformity rate of product p	α^p	0.7, 0.7, 0.7
Cost of separating one product p ($)	δ^p	4, 5, 6
Cost of minimal repair ($)	CMR	500
Time of minimal repair (month)	TMR	0.02
Inspection cost ($)	β	40
PM Imperfectness factor	η	0,9
Restoration cost parameter ($/month)	ξ	3000
Cost of a complete repair ($)	CCR	6000

Table 2

Data of products.

		Demand (items)			Processing cost ($)			Backorder cost ($)			Holding cost ($)			Set-up cost ($)			Price of conforming ($)		
	Product	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Period	1	45	50	100	30	50	70	110	130	180	3	5.5	2.2	500	800	450	170	320	65
	2	30	40	150	26	47	74	110	130	170	2.5	6.1	2.5	550	780	420	150	300	70
	3	60	70	50	33	49	68	120	130	170	3.2	6.5	2.4	530	830	400	180	340	68

6.3. Additional experiments

The integrated model is solved here to select the optimal values of the inspection frequency and all the other decision variables. For more details about these experiments, the reader is referred to the report in [21]. We summarize here our main additional findings.

Using the input data in Tables 1 and 2, Figure 5 presents the profit as a function of the inspection frequency for the periodic case with a uniform PM. It shows that the maximum profit is reached when $m_t = 15$.

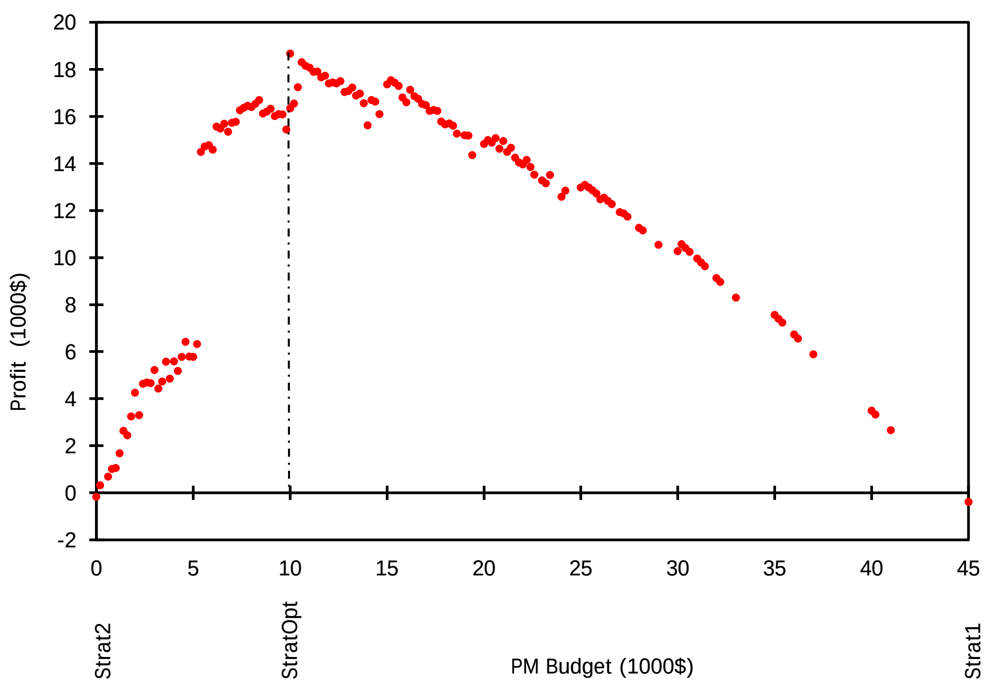

Figure 4. Variations of profit with the PM cost.

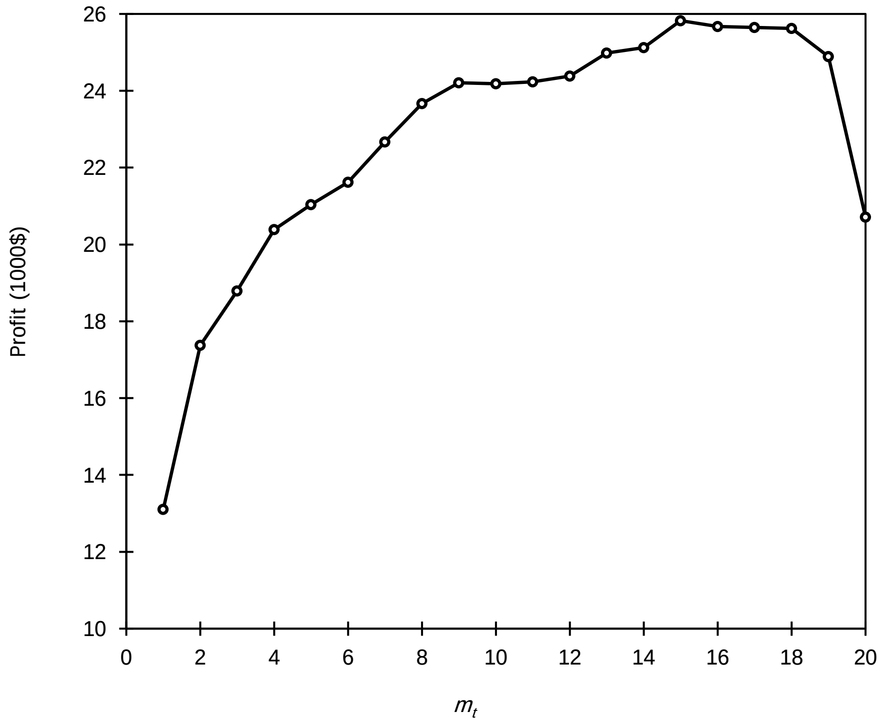

Figure 5. Effect of the number of inspections on the expected profit.

Comparing periodic and non-periodic inspection, we found that the non-periodic case is better in the sense that it leads to an expected profit which is higher than the periodic case.

We have also compared uniform and non-uniform PM. Uniform PM considers that the PM actions performed during each period are of the same level. This means that if a PM level is selected at the first inspection, the same PM level will be used for the remaining of the same period. Non-uniform PM rather considers different PM levels during the same period. As expected, we found that non-uniform PM actions are preferred to uniform PM, in the sense that its profit is higher.

7. Conclusion

The profit maximization model described here integrates maintenance, production and quality decisions. Using machine inspection for maintenance and restoration purposes, this model deals with a multi-period multi-product capacitated lot-sizing problem with time-varying demands and costs. Unlike the existing models, it takes into account the possibility of production system deteriorations and the existence of nonconforming items in the production lots. Ignoring these aspects may lead to a wrong determination of production and maintenance plans and overestimation of the service level. Our integrated model can help the manager make the right decision to maintain the system and to produce the required number of conforming products, in order to meet the customer demand. It was found that the increase in preventive maintenance investment leads to reductions in quality related impacts; but, if the cost of performing preventive maintenance is high to the point where it is not compensated for by quality improvement, then performing preventive maintenance is not justified. The proposed model can be used to determine the optimal values of the production plan, the sales and the preventive maintenance times and levels, while taking into account the quality of items produced by one machine. It can be extended to multi-machine systems. Furthermore, several critical equations related to cost and performance evaluations were analytically derived under some assumptions. To validate the proposed model and its related equations and assumptions, a simulation study could be conducted. Other useful extensions include the development of efficient heuristics to deal with large-sized instances of the problem. In the proposed algorithm, for each combination of the PM decision variables, the expected profit has been evaluated and the resulting CLSP solved. The optimal solutions of *all* the solved CLSP are compared to select the final optimal solution. This approach results in a worst case of a large number of linear mixed-integer programs to be solved. Because such a method may solve efficiently only small-size problems, heuristic algorithms are required to reduce the computation time burden, and to improve the ability to find the optimal solution when solving large-sized instances.

References

[1] Ben-Daya M, Duffuaa SO. Maintenance and quality: The missing link. J Qual Main Eng 1995; 1: 20–26.

[2] Cassady CR, Bowden RO, Liew L, Pohl EA. Combining preventive maintenance and statistical process control: A preliminary investigation. IIE Trans 2000; 32 (6): 471–478.

[3] Kenné JP, Gharbi A, Beit M. Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale. Eur J Oper Res 2007; 178: 408–420.

[4] Ben-Daya M, Rahim MA. Effect of maintenance on the economic design of x-control chart. Eur J Oper Res 2000; 120 (1): 131–143.

[5] Chen YC. An optimal production and inspection strategy with preventive maintenance error and rework. J Man Sys 2013; 32: 99–106.

[6] Nourelfath M, Nahas N, Ben-Daya M. Integrated preventive maintenance and production decisions for imperfect processes. Rel Eng Sys Saf 2016; 148: 21–31.

[7] Nakagawa T. A summary of imperfect preventive maintenance policies with minimal repair. Oper Res 1980; 14 (3): 249–255.

[8] Murthy DNP, Nguyen DG. Optimal age policy with imperfect preventive maintenance. IEEE Trans Rel 1981; 30: 80–81.

[9] Pham H, Wang H. Imperfect maintenance. Eur J Oper Res 1996; 94 (3): 425–438.

[10] Drexl A, Kimms A. Lot-sizing and scheduling - Survey and extensions. Eur J Oper Res 1997; 99: 221–235.

[11] Haugen KK, Olstad A, Pettersen BI. The profit maximizing capacitated lot-size (PCLSP) problem. Eur J Oper Res 2007; 176: 165–176.

[12] Salvietti L, Smith NR. A profit-maximizing economic lot scheduling problem with price optimization. Eur J Oper Res 2008; 184: 900–914.

[13] Banerjee PK, Rahim MA. Economic design of x-chart under Weibull shock models. Tech 1988; 30: 407–414.

[14] Chen YS, Yang YM. Economic design of x-control charts with Weibull in-control times when there are multiple assignable causes. Int J Prod Eco 2002; 77: 17–23.

[15] Lee PH, Torng CC, Liao LF. An economic design of combined double sampling and variable sampling interval x-control chart. Int J Prod Eco 2012; 138: 102–106.

[16] Aghezzaf EH, Jamali MA, Ait-Kadi D. An integrated production and preventive maintenance planning model. Eur J Oper Res 2007; 181: 679–685.

[17] Fitouhi MC, Nourelfath M. Integrating noncyclical preventive maintenance scheduling and production planning for multi-state systems. Rel Eng Sys Saf 2014; 121: 175–86.

[18] Nahmias S. Production and Operations Analysis. 5th ed. Boston: McGraw-Hill – Irwin; 2005.

[19] Lee HL, and Rosenblatt MJ. A production and maintenance planning model with restoration cost dependent on detection delay. IIE Trans 1989; 21 (4): 368–375.

[20] Sipper D, Bulfin R. Production: planning, control and integration. McGraw-Hill, 1997.

[21] Beheshti Fakher H. Integration of production, maintenance and quality: modelling and solution approaches. Ph.D. Thesis, Laval University, Canada; 210 pages, 2016.

[22] Beheshti Fakher H, Nourelfath M, Gendreau M. A cost minimisation model for joint production and maintenance planning under quality constraints. Int J Prod Res 2016, DOI: 10.1080/00207543.2016.1201605.

[23] Garg A, Deshmukh SG. Maintenance management: literature review and directions. Journal of Quality in Maintenance Engineering 2006; 12 (3): 205–238.

[24] Argoneto P, Perrone G, Paolo Renna P, Lo Nigro G, Bruccoleri M. Production Planning in Production Networks: Models for Medium and Short- term Planning. Springer-Verlag, London, UK, 2008.

[25] Duncan AJ. The Economic Design of *x* Charts Used to Maintain Current Control of a Process. Journal of the American Statistical Association 1956; 51: 228–242.

[26] Bouslah B, Gharbi A, Pellerin R. Joint optimal lot-scheduling and production control policy in an unreliable and imperfect manufacturing system. International Journal of Production Economics 2013; 144: 143–156.

[27] Rosenblatt MJ, Lee HL. Economic production cycles with imperfect production process. IIETrans 1986; 18: 48–55.

[28] Porteus EL. Optimal lot sizing, process quality improvement and setup cost reduction. Oper Res 1986; 34: 137–44.

[29] Dekker R. Application of maintenance optimization models: a review and analysis. Reliab Eng Syst Saf 1996; 51 (3): 229–40.

[30] Panagiotidou S, Tagaras G. Optimal preventive maintenance for equipment with two quality states and general failure time distributions. European Journal of Operational Research 2007; 180: 329–353.

[31] Xiang Y. Joint optimization of X control chart and preventive maintenance policies: A discrete-time Markov chain approach. European Journal Operations Research 2013; 229: 382–390.

[32] Groenevelt H, Pintelon L, Seidmann A. Production lot sizing with machine breakdowns. Management Science 1992; 38 (1): 104-123.

[33] Liu X, Wang W, Peng R. An integrated production, inventory and preventive maintenance model for a multi-product production system. Rel Eng Sys Saf 2015; 137: 76–86.

[34] Liu Y, Huang HZ. Optimal Selective Maintenance Strategy for Multi-State Systems Under Imperfect Maintenance. IEEE Trans Rel 2009; 59-2: 356–367.

[35] Lu B, Zhou X, Li Y. Joint modeling of preventive maintenance and quality improvement for deteriorating single-machine manufacturing systems. Comp Ind Eng 2016; 91: 188–196.

[36] Liao G. Optimal economic production quantity policy for randomly failing process with minimal repair, backorder and preventive maintenance. Int J Syst Sci 2012; 44(9): 1602–12.

[37] Wang WY. Optimum production and inspection modeling with minimal repair and rework considerations. App Math Model 2013; 37: 1618–1626.

[38] Barlow RE, Proschan F. Mathematical Theory of Reliability. Wiley and Sons, Inc., New York; 1965.

[39] Munford AG, Shahani AK. A nearly optimal inspection policy. Operational Research Quarterly, 1972; 23 (3): 373–379.

[40] Kaio N, Osaki S. Comparison of inspection policies. The Journal of the Operational Research Society 1989; 40 (5): 499–503.

[41] Keller JB. Optimum checking schedules for systems subject to random failure. Management Science 1974; 21 (3): 256–260.

[42] Chelbi A, Ait-Kadi D. Inspection strategies for randomly failing systems. Handbook of Maintenance Management and Engineering. Springer, London, pp. 303–335; 2009.