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An Integrated Routing-Scheduling Model for a Hybrid UAV-Based Delivery System

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Abstract. We study a delivery problem with heterogeneous fleets, in particular adopting UAVs and standard vehicles like trucks or boats. We present a novel multi-period mixed integer linear programming formulation which integrates not only the routing and the scheduling problems underlying the delivery, but also the charging cycles of drone batteries with the movements of the vehicle. We propose also some variants of the formulation by means of three main objective functions related to energy consumption minimization, maximum completion time minimization, and maximization of number of customers served. A scalarisation technique is adopted to partially explore the Pareto optimal frontier. These models are tested on two very different scenarios and different parameter settings to both validate the solutions and to assess model computational solvability. The model can be optimally solved in most of the cases and gets solutions very close to the optimum in the remaining cases. The analysis of the efficient points reveals that there are quite large differences in the system performances and in the routing and scheduling solutions depending on the weights of the objectives.

Keywords: Delivery system, routing, scheduling, hybrid fleets, mixed integer linear programming, multi-objective optimization, UAV.

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1 Introduction

Unmanned aerial vehicles (UAVs), or drones represent an emerging technology that provides new opportunities and have been successfully adopted in different contexts like for example in disaster management Chiaraviglio and et al. (2019), in the telecommunication sector for providing 5G coverage of rural zones Amorosi et al. (2019), for 3D mapping and video surveillance Nexand and Remondino (2014), in the agriculture for monitoring the crops growth Tokekaretal (2016) and many others (see Otto et al. (2018) for a complete survey of civil applications of UAVs and related optimization problems). The increasing interest in this technology is also proven by the growing number of research papers published on this topic. Delivery applications have recently received considerable attention from many of the companies operating in the commercial delivery sector, like Amazon, UPS, Google, FedEx, DPD mainly because of the prospect of door-to-door express deliveries at low cost. They include package deliveries to remote rural regions and first and last-mile deliveries in urban and suburban areas as well as express deliveries of, for example, medical supplies Scott and Scott (2017). Focusing on the use of this technology in the logistic of delivery systems, several advantages can be identified: they can be faster than trucks, have low cost per mile to operate, emit less CO2 than vehicles and they can reach delivery locations, like for example roofs and terraces, that are not possible for vehicles. Thus, they represent a greener and faster alternative to conventional delivery modes. However, micro, mini, and small drones, which are popular in civil applications, can spend only a very limited time airborne. On the other side, urban contexts can be characterized by pedestrian areas or old towns with narrow alleys. Therefore, combining drones with other means of transportation may lead to a significant increase of efficiency and effectiveness in performing package deliveries. In the combined operations of drones and vehicles for package deliveries, synchronization is needed and several constraints related to the UAVs motion and energy consumption have to be considered.

Many different delivery systems based on the use of UAVs can be defined depending on the characteristics of the considered scenario and a system applicable on a given context may not fit a different one. For this reason in the literature, a variety of possible configurations have been studied. Among delivery systems based on UAVs, we can distinguish those consisting only of a fleet of drones, configurations composed by one or multiple UAVs that cooperate with a vehicle, like for example a truck or a boat, where both drones and vehicle deliver packages to customers, and systems where a vehicle supports the operations of one or multiple UAVs. In this last case the drones serve all customers and the vehicle is a mobile launch base for the drones. It has to be noted that the UAVs are subjected to a limited energy autonomy and they must be recharged. To this end the truck can be used also as a mobile recharging station for the UAVs.

These different possible problem settings and assumptions originated several combinatorial optimization problems.

In this paper we focus on a problem that, at best of our knowledge, has not been formulated related to a hybrid delivery system based on a fleet of UAVs whose operations are supported by one vehicle (e.g. a boat or a truck) that is used as launch and recharging base for drones. This system has to serve, within a given delivery time window, a set of customers located in different points in an urban context starting from a depot. The UAVs, differently from most of the work in literature, are the only responsible for delivering packages to customers and their missions must start and end on the vehicle. Each drone mission serve a single package to one customer and can be recharged on the vehicle after each mission. The vehicle can dock, in the case of a boat, can park, in the case of truck, on a set of locations out of a predefined set of stop areas that, differently from previous configurations studied in literature, does not coincide with the set of customer locations. Thus, the problem consists in finding: (i) a tour on an unknown subset of stop locations that starts from the depot and end to the depot when the last customer has been served, (ii) the schedule of the UAVs missions such that each customer is served within the delivery time window and the autonomy of each drone is respected. Two different major criteria to be optimized are considered: (i) minimization of the system's energy consumption and (ii) minimization of the maximum completion time. Moreover, a further goal is taken into consideration, to deal with the case in which it is not possible to serve all customers within the time window (that is, their number is too big for the UAVs fleet size). Consequently, a third objective function, that consists in maximizing the number of served customers, is introduced.

We deal with this problem by means of a mathematical programming approach formulating it as a multi-period mixed integer linear programming model and we solved it on realistic scenarios showing that taking an exact solution approach is compatible with the use of the model as a management support tool at a planning level. Furthermore, adopting a scalarisation technique, we provide Pareto optimal solutions whose values reflect the significant changes of system performances. The paper is organized as follows: Section 2 contains the literature review, Section 3 describes the problem and reports its mathematical programming formulation, Section 5 illustrates the realistic scenarios on which the model has been validated and tested and discusses the obtained results. Eventually, Section 6 summarized the paper and conclusions.

2 State of the art

The literature on this area is quite recent, although it refers to well known studied problems in location, sizing and routing. As already mentioned, we can identify three main groups of UAVs based delivery configurations and related works in the literature: the ones considering only fleets of drones starting from hubs/depots and delivering packages to customers, the ones where drones and vehicles perform delivery services in a coordinated way (i.e. a subset of customers is served by drones and a subset by vehicles) and the group of works focusing on systems where UAVs are assisted by a vehicle to perform deliveries. In this latter configuration, drones are the only responsible for delivering packages to customers and the vehicle represents a launch and recharging station for the UAVs. As far the works belonging to the first group, in this section we give just a glimpse and then we mainly focus on the problems related to hybrid delivery systems based on UAVs. The articles belonging to the first group deal with location problems like in Chauhan et al. (2019) where the authors study the maximum coverage capacitated facility location problem with drones. The problem consists in determining the location of a given number of facilities and in assigning drones to the located facilities in order to maximize the coverage of a set of demand taking into account the drones energy consumption and autonomy. The problem has been mathematically formulated but, in order to obtain solutions in times compatible with fast global re-optimization, the authors proposed greedy and three-stage heuristics. A similar problem has been faced also in Hong et al. (2018) with a different heuristic approach to deal with larger scenarios. Sundar and Rathinam (2014) considers a single UAV routing problem where there are multiple depots where the drone can refuel. The problem consists in finding a path for the UAV such that each customer is visited at least once, the fuel constraint is respected and the total refuel is minimised. The authors adopt an approximation algorithm to solve the problem. Other works, considering fleets of drones for delivering services, study multi trip drone routing problems like Cheng et al. (2018) and Dorling et al. (2017). More in details, in Cheng et al. (2018), the authors model the drones' energy consumption as a nonlinear (convex) function of the payload and travel distance and propose logical and subgradient cuts to tackle with the non linearity of the problem and for developing two branch-and-cut algorithms. In Dorling et al. (2017) the authors propose two multi-trip vehicle routing problems (VRPs) for drone delivery considering delivery time limit or budget constraints and adopting a linear approximation for modeling the dependency of the drones' energy consumption and the payload and battery weight. Moreover, it is assumed that each drone can visit multiple customers per trip. The authors adopt a simulated annealing heuristic for finding sub-optimal solutions to practical scenarios. In the article Oh et al. (2017) it is presented a special case of the multi-trip VRP, that is, the cooperative parcel delivery problem with UAVs, where multiple drones must build a team to carry together parcels whose weight is greater than the payload limit of a single UAV. In Choi and Schonfeld (2017) the authors study the problem of sizing the fleet of drones to minimise the total cost of a delivery system where each drone can lift multiple packages within its maximum payload and serve customers in a service area of a given radius.

Focusing on works dealing with optimization problems arising from hybrid delivery systems, where one drone cooperates with one vehicle, we can mention Murray and Chu (2015) where a hybrid truck and drone model has been introduced for the first time and named Flying Sidekick Traveling Salesman Problem (FSTSP). In this delivery system the truck carries one drone to deliver parcels. As the truck is also used to perform deliveries, the UAV is launched from the truck, delivers a parcel to a customer and then the truck and the drone rendezvous at a new customer location. The authors proposed a heuristic method that solves a TSP for obtaining an initial truck routing and then, in a greedy way, select the package on the truck to be delivered by the drone preserving feasibility. In Ponza (2016) the authors use a simulated annealing heuristic to solve the same problem. In Ha et al. (2015) it is studied an extension of the FSTSP considering the time span, that is the maximum time that the truck or the drone can wait for each other at the rendezvous node. The authors propose both route first-cluster second and cluster first-route second methods. Another interesting extension of the FSTSP is the one introduced in Marinelli et al. (2017), where the rendezvous operations are allowed not only at a node but also along a route arc. In this case a greedy randomized adaptive search procedure has been developed for solving the problem. In Agatz et al. (2018) the authors study another extension of the FSTSP that the authors named Traveling Salesman Problem with a Drone (TSP-D), where the truck can wait at the start node for the drone to return. A route first-cluster second heuristic method is adopted. New formulations for the FSTSP and also an extensive survey on drone deliveries are presented in Dell'Amico et al. (2019). In Poikonen et al. (2019) it is presented a branch and bound approach where each node in the decision tree is associated with some sequence of package locations, derived by the one proposed in Coutinho and al. (2016) for the Close-Enough Traveling Salesman Problem (CETSP), which is a generalization of the TSP whereby a city is considered visited if the tour comes within a specified radius of the city. Moreover, the authors adopt a slightly modified version of the partitioning procedure proposed in Agatz et al. (2018) such that the truck may remain stationary while the drone makes a delivery.

As regards hybrid configurations, where a fleet of drones cooperates with a fleet of trucks, we can cite Wang et al. (2017) and Poikonen et al. (2017) where the Vehicle Routing Problem with one or multiple drones (VRP-D) have been formalised. In both cases the authors focus on the worst-case analysis, revealing the amount of time that can be saved by using truck-drone rather than trucks alone. An extension of the same problem, obtained considering time windows constraints, has been studied in Pugliese and Guerriero (2017). In Dayarian et al. (2017), a fleet of trucks makes deliveries to customers and the drones fly from a warehouse to resupply the trucks. The authors propose a heuristic algorithm for the case with one truck and one UAV. In a more recent paper Dell'Amico et al. (2020) the parallel drone scheduling traveling salesman problem is studied where the set of customers is divided between a truck and a fleet of drones. Matheuristics to minimize the total time required to service all the customers are proposed and evaluated on a set of benchmarks.

Other hybrid delivery systems, as previously mentioned, consist in one or multiple drones that are carried by a vehicle (e.g. a truck) and that are the only responsible for delivering parcels to customers. In Mathew et al. (2015) the UAV visit one customer for each trip and the truck can wait at the launching node for the drone to come back or move to a different rendezvous node. In Carlsson and Song (2017) it is studied a continuous approximation on the Horse Fly Problem, where the truck serves as a mobile depot for the drone, demonstrating that the improvement in efficiency is proportional to the square root of the speed ratio of the truck and the drone. In Campbell et al. (2017) the authors use a continuous approximation model to evaluate the economic impact of truck-and-drone hybrid models considering a variety of model parameters and customer densities. In Ferrandez et al. (2016) it is considered a delivery system where one truck supports the operations of multiple drones. The customers demand is clusterized by means of a K-means clustering algorithm to find truck stops that represent hubs for drone deliveries. Thus, the problem consists in determining a TSP of the truck among centroids of these clusters. The authors use a genetic algorithm to determine the truck TSP tour and they assumed that drones are not constrained by flight range. In Moshref-Javadi and Lee (2017) the authors minimize latency in a customer-oriented distribution system. In their problem, at each stop site, the truck waits until all drones come back. The truck then moves drones to the next site. The authors compare the benefits of using drones for a single trip versus multiple trips. In Poikonen and Golden (2020) the authors formalized the k-Multi-visit Drone Routing Problem (k-MVDRP) considering a tandem between a truck and k drones. The authors assumed that each drone is capable of launching from the truck with one or more packages to deliver to customers. Each drone may return to the truck to swap/recharge batteries, pick up a new set of packages, and launch again to customer locations. Moreover, the authors decouple the set of launch locations from the set of customer locations. Although the work presents a mathematical formulation including a drone energy drain function that takes into account each package weight, the problem is solved by means of a heuristic algorithm.

In this paper, we present an original multi-period mixed integer linear programming formulation for a routing-scheduling problem of a hybrid urban delivery system based on one vehicle and a fleet of UAVs. Differently from previous works in literature, with the exception of the most recent paper Poikonen and Golden (2020), we consider the set of nodes, representing possible vehicle stops, separated from the set of customer locations. Moreover, we included in the mathematical formulation, and in particular in the objective function to be maximized, the UAVs energy expressed as linear function of the distance travelled by the drones and we modeled the problem of determining a tour on an unknown subset of nodes for the vehicle and the scheduling of the UAVs missions. Differently from Poikonen and Golden (2020) we exactly solved the mathematical formulation, performing an experimental analysis on two realistic scenarios we generated and considering different combinations of the different objectives we examined, that is the minimization of the vehicle and UAVs energy consumption, the maximization of the UAVs energy level and the maximization of the served customers.

3 Problem description

In this work we consider a multi-period routing-scheduling problem of one vehicle and a set of UAVs for packages delivery in an urban area.

More in details, we assume that in a given working period T a set of customers C, distributed in an urban area, has to be served by a set D of UAVs which transport the packages from the vehicle to the customers. The vehicle can be parked in different points of the urban area and moved among them between two consecutive missions of the UAVs set. Each UAV is characterized by a maximum autonomy expressed in terms of energy and has to be recharged on the vehicle before or when it reaches the minimum battery level. The problem consists in determining the stop locations of the vehicle and the UAVs missions over the time window to minimize the total cost related to the vehicle and UAVs energy consumption and the maximum completion time subject to the constraints: (i) all customers have to be served within the working period, (ii) the UAV energy level must range between a minimum and a maximum level. We believe the above problem setting meets several practical requirements of shipping companies operating in urban contexts. Although there are a number of not structured constraints, the formulation we propose appears to be complete and relatively easy to handle requiring not too much data.

With respect to ordinary routing models with homogeneous fleets, the model combines requirements of vehicles with entirely different characteristics. In particular, the charging cycles of drone batteries with the truck movements. To seek an operating equilibrium among different business and environmental aspects, we simultaneously adopt three criteria to respect:

- **Sustainability** of deliveries, by minimizing the total energy consumption;
- **Service Quality**, by minimizing the maximum completion time;
- **Productivity**, by maximizing the number of served customers.

Hence, depending on the specific management objectives and the actual company operating conditions, the decision maker can choose among a set of Pareto optimal solutions to find the one more adequate to his/her vision.

These model capabilities have been tested on two realistic scenarios which we selected in the attempt to cover a wide spectrum of situations. The first one is taken from an Italian art city where the adoption of a heterogeneous fleet is almost mandatory, that is Venice, where the deliveries are performed by a boat and a set of drones. The second one, is taken instead from a Rome district, but not in the central area, where a truck has a number of possible locations to stop allowing drones missions. In this second case, the adoption of drones may not be mandatory, as there is an extensive street network, but can be preferable both for efficiency or to deliver medicine to infected or vulnerable people locked in their homes (see for example Mesar et al. (2018)).

4 Mathematical formulation

In this section we introduce notation and mathematical model formalization.

Input parameters
N set of admissible vehicle stops (node with label 1 is the depot)
C, set of customers
A_1, set of links between pairs of nodes in N, including self-loops
A_2, set of links between nodes in N and customers in C
D, set of UAVs
$d_{i,j}$ $\forall(i,j) \in A_1 \cup A_2$, distance between location i and location j (in Km)
c_1, vehicle energy consumption to travel 1 Km (in Wh)
c_2, UAV energy consumption to travel 1 Km (in Wh)
$[0,T]$, working period discretized in time slots $t = 0, 1, ..T$
E_{max}, maximum UAV energy level (in Wh)
E_{min}, minimum UAV energy level (in Wh)
$E_{(i,j)}$ $\forall(i,j) \in A_2$, UAV energy consumption for traversing arc (i,j) (in Wh)

Decision Variables

$$x^d_{i,j}(t) = \begin{cases} 1 & \text{if customer } j \text{ is served at time slot } t \text{ by UAV } d \text{ starting from } i \\ 0 & \text{otherwise} \end{cases}$$

$$y_i(t) = \begin{cases} 1 & \text{if the vehicle is parked in } i \text{ at time slot } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{i,j}(t) = \begin{cases} 1 & \text{if the vehicle moves from location } i \text{ to location } j \text{ at time slot } t \\ 0 & \text{otherwise} \end{cases}$$

$$w_d(t) = \begin{cases} 1 & \text{if drone } d \text{ is recharged at time slot } t \\ 0 & \text{otherwise} \end{cases}$$

$k(t) \geq 0$ (integer) $\forall t \in [1, T]$, number of served customers up to time slot t

$e_d(t) \geq 0 \, \forall d \in D, \forall t \in [0, T]$, energy of UAV d at time slot t

$f_{i,j}(t) \geq 0 \, \forall (i, j) \in A_1 \, \forall t \in [0, T]$, auxiliary flow variables for the vehicle routing

$T_{max} \geq 0$, maximum completion time (i.e. the time at which the last customer is served and the vehicle goes back to the depot)

Problem constraints

$$\sum_{t \in [1,T]} \sum_{d \in D} \sum_{i \in N:(i,j) \in A_2} x^d_{i,j}(t) = 1 \quad \forall j \in C \tag{1}$$

Constraints (1) impose that each customer must be served within the discretized working period $[1, T]$ by a drone.

$$\sum_{(i,j) \in A_2} x^d_{i,j}(t) \leq 1 \quad \forall d \in D \;\; \forall t \in [1, T] \tag{2}$$

Constraints (2) guarantee that each drone can serve at most one customer in each time slot. Each UAV can serve a customer, starting from a given vehicle location at a given time slot, only if the vehicle is placed in that location at that time slot, as stated by constraints (3).

$$x^d_{i,j}(t) \leq y_i(t) \quad \forall d \in D \;\; \forall i \in N \;\; \forall t \in [1, T] \;\; \forall j \in C : (i, j) \in A_2 \tag{3}$$

The initial location of the vehicle, at time slot 0, is the depot identified with label 1 as constraint (4) imposes.

$$y_1(0) = 1 \tag{4}$$

Constraints (5)-(7) ensure that the vehicle moves from the depot to one of the vehicle locations at time slot 0 and that if a given vehicle location is visited by the vehicle at a given time slot, an ingoing arc (starting from a different or the same location in the previous time slot) and an outgoing arc from that location are traversed.

$$y_i(t) = \sum_{j:(j,i) \in A_1} z_{j,i}(t-1) \quad \forall i \in N \;\; \forall t \in [1, T] \tag{5}$$

$$y_i(t) = \sum_{i:(i,j) \in A_1} z_{i,j}(t) \quad \forall i \in N \;\; \forall t \in [1, T] \tag{6}$$

$$\sum_{j \in N} z_{1,j}(0) = 1 \tag{7}$$

Moreover, constraints (8) and (9) impose that in each time slot t the vehicle must traverse exactly one arc of the multiperiod graph related to time slot t (not necessary changing its location) and each arc of the multiperiod graph can be traversed by the vehicle at most once during the time window.

$$\sum_{(i,j)\in A_1} z_{i,j}(t) = 1 \quad \forall t \in [1,T] \tag{8}$$

$$\sum_{t\in[1,T]} z_{i,j}(t) \leq 1 \quad \forall (i,j) \in A_1 : i \neq j \tag{9}$$

Constraints (10) and (11) are flow conservation constraints that permit to determine the vehicle's route including only visited nodes.

$$\sum_{j:(i,j)\in A_1} f_{i,j}(t) - \sum_{j:(j,i)\in A_1} f_{j,i}(t) = y_i(t) \quad \forall i \in N : i \neq 1 \;\; \forall t \in [1,T] \tag{10}$$

$$f_{i,j}(t) \leq (T-1) z_{i,j}(t) \quad \forall (i,j) \in A_1 \;\; \forall t \in [0,T] \tag{11}$$

The number of customers served up to each time slot t is counted by means of the constraints (12). Constraints (13) and (14) set the value of variable $z_{i,1}(t)$ equal to one when all customers have been served, that is the vehicle must go back to the depot.

$$k(t) = k(t-1) + \sum_{(i,j)\in A_2} \sum_{d\in D} x^d_{i,j}(t) \;\; \forall t \in [1,T] \tag{12}$$

$$\sum_{(i,j)\in A_1 : i\neq 1, j=1} z_{i,j}(t) \geq 1 + (k(t) - |C|) \;\; \forall t \in [1,T] \tag{13}$$

$$\sum_{(i,j)\in A_1 : i\neq 1, j=1} z_{i,j}(t) \leq 1 - (k(t) - |C|) \;\; \forall t \in [1,T] \tag{14}$$

The initial energy level of each drone is equal to the maximum one and it has to range between the minimum and maximum level in each time slot, as stated by constraints (15), (16) and (17).

$$e_d(0) = E_{max} \quad \forall d \in D \tag{15}$$

$$e_d(t) \leq E_{max} \quad \forall d \in D \;\; \forall t \in [0,T] \tag{16}$$

$$e_d(t) \geq E_{min} \quad \forall d \in D \;\; \forall t \in [0,T] \tag{17}$$

The drone battery level is set through constraints (18), by considering: (i) the drone energy at the previous time slot, (ii) the energy consumption serving a customer, (iii) the energy gained if the drone is recharging at the current time slot.

$$e_d(t) \leq e_d(t-1) - \sum_{(i,j)\in A_2} x^d_{i,j}(t) E_{i,j} + w_d(t) E_{max} \quad \forall d \in D \;\; \forall t \in [1,T] \tag{18}$$

$$w_d(t) + \sum_{(i,j)\in A2} x^d_{i,j}(t) <= 1 \quad \forall d \in D \ \forall t \in [0,T] \tag{19}$$

Constraints (19) impose that a drone can't make a delivery while recharging or being in quiet mode (neither delivering nor recharging). Each of these events is mutually exclusive.

$$\sum_{t\in[0,T]} \sum_{(i,j)\in A_1: i\neq 1, j=1} z_{i,j}(t) = 1 \tag{20}$$

Constraints (20) impose that the number of incoming edges in the depot must be exactly equal to 1.

Three different variants of the basic model are considered mainly based on the following objective functions.

1. Minimization of energy consumptions

$$\min\left(\sum_{(i,j)\in A_1}\sum_{t\in[1,T]} c_1 \cdot d_{i,j} \cdot z_{i,j}(t) + \sum_{(i,j)\in A_2}\sum_{t\in[1,T]}\sum_{d\in D} c_2 \cdot d_{i,j} \cdot x^d_{i,j}(t) - \sum_{d\in D}\sum_{t\in[1,T]} e_d(t)\right) \tag{21}$$

The objective function (21) is the minimization of the vehicle's energy consumption and the maximization of the UAVs energy.

2. Minimization of energy consumptions and maximum completion time

$$\min\left(\sum_{(i,j)\in A_1}\sum_{t\in[1,T]} c_1 \cdot d_{i,j} \cdot z_{i,j}(t) + \sum_{(i,j)\in A_2}\sum_{t\in[1,T]}\sum_{d\in D} c_2 \cdot d_{i,j} \cdot x^d_{i,j}(t) - \sum_{d\in D}\sum_{t\in[1,T]} e_d(t) + T_{max}\right) \tag{22}$$

The objective function (22) is the minimization of: (i) vehicle's energy consumption, (ii) maximum completion time and the maximization of the UAVs energy.
In this case, in order to correctly set the value of T_{max}, the maximum completion time, constraints (23) have to be added to formulation (1)-(20).

$$T_{max} \geq t - M(1 - x^d_{i,j}(t)) \quad \forall t \in [1,T] \ \forall (i,j) \in A_2 \ \forall d \in D \tag{23}$$

3. Minimization of energy consumptions and maximization of served customers

$$\min \left(\sum_{(i,j)\in A_1} \sum_{t\in[1,T]} c_1 \cdot d_{i,j} \cdot z_{i,j}(t) + \sum_{(i,j)\in A_2} \sum_{t\in[1,T]} \sum_{d\in D} c_2 \cdot d_{i,j} \cdot x^d_{i,j}(t) \right. \\ \left. - \sum_{d\in D} \sum_{t\in[1,T]} e_d(t) - \sum_{(i,j)\in A_2} \sum_{d\in D} \sum_{t\in T:t>0} x^d_{i,j}(t) \right) \tag{24}$$

The objective function (24) is the minimization of the vehicle's energy consumption and the maximization of : (i) the UAVs energy and (ii) the number of served customers.
In the third version of the model it is assumed that not all customers can be served within the time window under consideration and we want to maximize the number of covered clients. Thus, constraints (25) must substitute constraints (1).

$$\sum_{t\in[1,T]} \sum_{d\in D} \sum_{i\in N:(i,j)\in A_2} x^d_{i,j}(t) \leq 1 \quad \forall j \in C \tag{25}$$

The resulting formulation is a multi-period mixed integer linear programming model and in the next section we provide scenarios parameters and details of the obtained solutions. Please note that the above formulations in a practical context can be further tuned to meet particular aspects more or less relevant from the specific business perspective. In particular, the normalization of the different terms within each objective function and the way these terms are utilized within a scalarisation technique will be illustrated in section 5.

5 Model resolution and Experimental Results

In this section, we first present the scalarisation method adopted to perform a partial exploration of the Pareto frontier of the problem. Indeed, as mentioned at the end of section 4, each term within each objective function can have more or less importance for the decision maker and so there is the necessity to normalize and assign weights to each of them. Successively, we describe the system operating assumptions we adopted to generate realistic scenarios and conduct computational experiments of the three different variants of the model presented in Section 4.

5.1 Adopted scalarisation method

As each of the proposed variants of the model can be seen in the framework of multi-objective optimization, we adopted a scalarisation method to find efficient solutions. Note that the terms in the objective functions have different ranges and physical meanings. So we first normalized these terms dividing each one by an upper bound on its value. Some of these values can be computed in a straightforward way (e.g. the maximum number of time slots and the maximum number of customers which are known a priori). The others can be obtained considering one term of the objective function at a time and solving the so obtained maximization problem formulation with the same set of constraints. Clearly, the values of these normalization coefficients depend on the scenario parameters and must be computed for each different setting.

Successively, we applied the weighted sum scalarisation technique which requires the assignment of a weight λ_i to each normalized term i in the objective function, such that $\sum_i \lambda_i = 1$. Consequently, in order to generate efficient solutions, different combinations of these weights are considered: a base case having $\lambda_1 = ... = \lambda_i$ and then other $i\ (i-1)$ cases with different variations of weights. The latter are calculated by reducing with a constant step coefficient one of the weights λ_j having $j \in [1, i]$ at a time and distributing its residual on each of the other lambdas λ_k with $k \in [1, i]$ and $k \neq j$, at a time. In this way we get exactly $i\ (i-1)+1$ combinations which, multiplied by the objective function terms, will potentially yield different Pareto optimal solutions on the same case scenario.

5.2 Operational setting

Several assumptions are needed to simulate a realistic scenario. We assume to have 16 time slots, each one of 30 minutes, resulting in a time window of 8 hours. In a single time slot, a vehicle can move from one stop to another or stay in one of them to let a UAV perform a delivery. Customers' demand are generated with a spatial uniform distribution. Instances with more customers are obtained adding demand points in the same area with the same distribution function. The instances so generated have the characteristic that, in general, a customer can be served by more than one vehicle stop location. Once a vehicle is arrived at a certain stop, an on-board UAV can either be in three different states: charging (C), stationary (S) or working (W). The first two states mean that the drone cannot be used for deliveries. In the third state, instead, the UAV is busy on completing a single mission.

We proceed with a brief description of two scenarios and an analysis of the model outcomes.

5.3 Scenario I

The first realistic scenario that has been considered, in order to validate the model and its variants, is related to the city of Venice (Italy).

More in details, as shown in figure 1, we assumed that a boat can navigate through Canal Grande, one of the major water-traffic corridors in the city, where a set of 18 possible stops have been identified. As said before, we considered a time window related to a working day, that is 8 hours divided in 16 time slots of 30 minutes each. A set of UAVs has a cardinality ranging from 1 to 8. We assumed that the typology of UAVs is the DHL Parcel Copter 2.0, a drone developed by DHL for air deliveries (see DHL (2014) for details on its technological parameters). Moreover, the number of customers ranges from 10 to 70. We assumed that each

Figure 1: Venice Scenario and boat route (Google Maps)

UAV mission has the same duration equal to one time slot. Finally, we observe that with this setting from each boat stop only a subset of customers is reachable by a UAV mission and that each customer can be served by more than one boat stop. On this first scenario we run the three variants of the model in order to validate it.

5.4 Scenario II

A second more challenging realistic scenario on which the model has been tested is related to an urban area of the city of Rome (Italy), the EUR district, shown in figure 2.

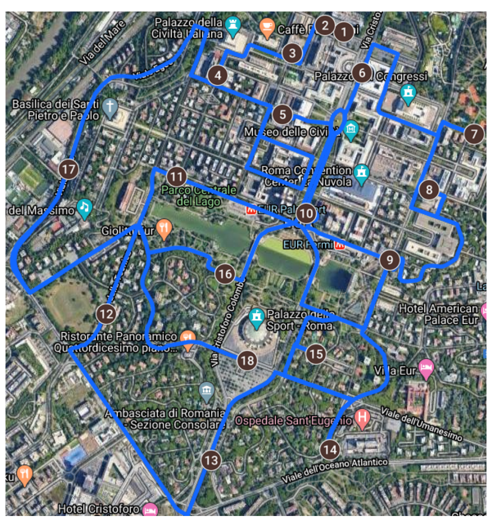


Figure 2: Rome Scenario (EUR district) and truck route (Google Maps)

Also in this case 18 possible stops have been identified corresponding to places where there is enough space for stopping a truck without congesting the traffic. In this context we assumed to adopt an electric truck with an autonomy of 100 Km. As in the Venice scenario, we considered a number of UAVs ranging between 1 and 8 and a set of customers uniformly distributed in the district, whose number varies between 10 and 70. As regards the others technological parameters related to the UAVs, we adopted the same of the Venice scenario.

5.5 Results discussion

All the results we present are obtained using one node of the cluster Terastat of the Department of Statistics in Sapienza and coding the model with Cplex 12.10. The model presented in section 4 is tested using its three different variants, respectively, on Venice and Rome scenarios and imposing a computational time limit of 4 hours for each combination of parameters (number of customers and available drones). Please note that for each variant and for each scenario we explore the Pareto frontier by changing the weights of the different terms of the objective function.

The study of model validation starts with the first scenario, Venice City, since its stops disposal does not provide for different routes choice and, therefore, lightens the calculation of the modified TSP[1]. This particular scenario has a fixed route for the boat, resulting in a strongly simplified problem which only has to manage the process of choosing stops for the mission while minimizing UAVs energy consumptions. In the second and third cases, instead, other terms are added to the objective function which results in the minimization of the maximum completion time and, next, the maximization of customers served, in case these latter are not satisfiable in a single boat mission.

More in details, we first considered the variant of the model given by the weighted sum of the drone energy consumption and the drone energy level and we run it with different weights combinations by means of the routine described in section 5. However, in this case, for each cardinality of the set of customers and UAVs, as the two objective functions are not really in conflict each other, it is possible to find a solution which simultaneously optimizes both functions instead of a Pareto frontier.

Figure 3 reports the UAVs energy level over time corresponding to the optimal solution, for the setting with 20 customers and 2 drones. We can observe that both UAVs start with the maximum energy level, then the energy of both decreases for serving customers and they are recharged when they reach the minimum energy level. One of them reaches it in 3 time slots while the other one in 5 time slots. After the first recharge the energy start again to decrease but only for one of the drones the second recharge is performed when the minimum energy level is reached. Indeed, in order to perform the second schedule of deliveries, the other UAV is recharged before to reach the minimum energy level. In this way, at time slot 10, both drones start again from the maximum energy level consuming the same amount of energy in the following time slots, recharging at time slot 13 after serving the last customer.

Figure 4 shows the solution configurations adopting respectively 2 and 4 drones for the same scenario and number of customers. The brown points identify possible stops for the boat,

[1]The Travelling Salesman Problem (TSP) in our case doesn't have to include all the stops, meaning that a vehicle can visit a subset of the scenario stops.

and the orange ones represent customer locations. We can notice that the increment in the number of UAVs from 2 to 4, as one can expect, permits to assign a bigger number of drone missions to the same boat stop. This produces a slightly different solution configuration and a reduction in the distance travelled by each drone in each mission. However, the number of used boat stops, for this first case, is independent from the number of available drones and it is always equal to 12. A similar behaviour can be observed also for the other cardinalities of the set of customers. The main difference is the minimum number of drones that is necessary for the feasibility. Indeed, increasing the number of customers to 30 at least 4 UAVs have to be available and for 50 or 70 customers at least 8.

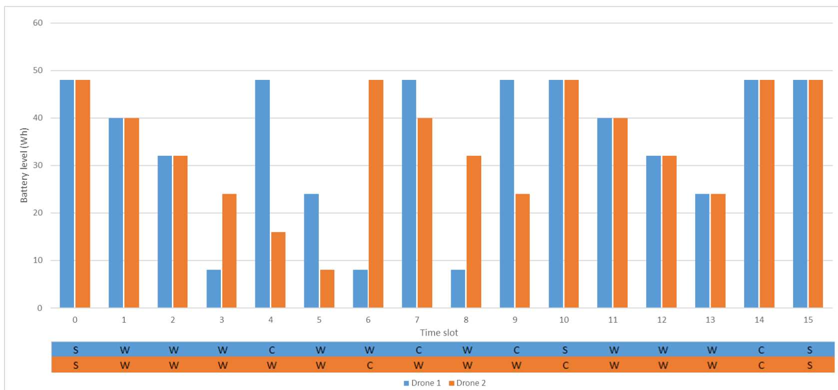


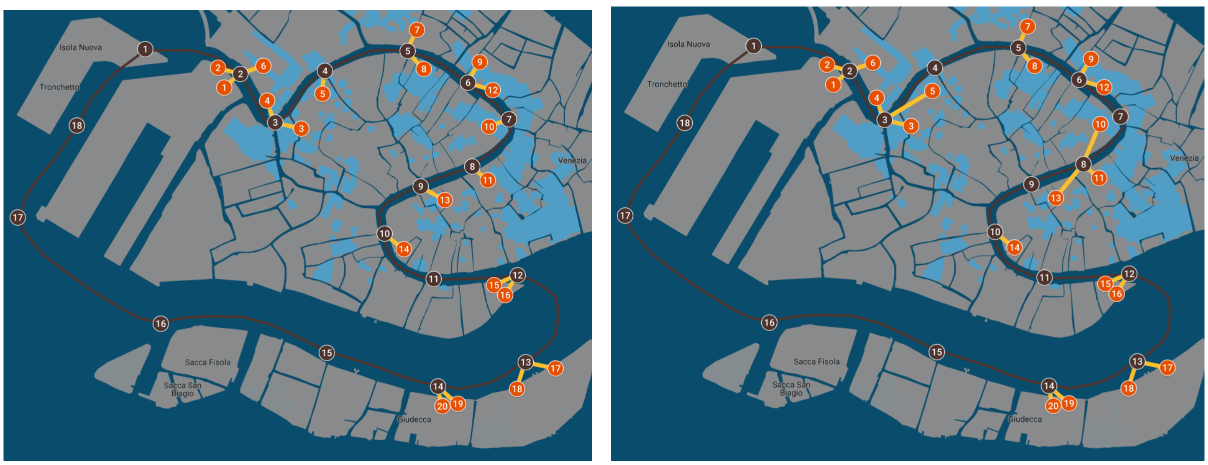
Figure 3: Venice Scenario case 1, 20 customers / 2 drones, energy levels of drones over time slots. A drone can be in 3 states: working (W), charging (C), stationary (S).

(a) 2 drones

(b) 4 drones

Figure 4: Venice Scenario case 1, 20 customers, $\lambda_1 = \lambda_2 = 0.5$

As regards the second model variant, in which we take into account also the minimization of the maximum completion time, we considered again, different combinations of weights assigned to the two objectives, for each cardinality of the customer set and of the available UAVs. Indeed, given the results obtained in the first variant, we considered as a unique term the sum of the drone energy consumption and the drone energy level, to which we assigned a weight. In this case, we generated different Pareto optimal solutions for each feasible instance solving the model at optimality in most of the cases or with a percentage relative gap very close to 0 within the time limit. With respect to the first variant, for this second case, for all the considered weight combinations and cardinality of the set of customers and drones, the solution strategy consists in concentrating the UAVs missions in less stops, as it can be observed, for example, comparing figure 4 and 5. Indeed, to minimize the maximum number of time slots for completing the deliveries, the model provides optimal solutions in which the customers are served by UAVs starting as soon as possible but not necessary from the closest possible boat stop. Thus, the distribution of the drone missions over the stops changes compared to the first case and the distance travelled increases.



(a) 2 drones (b) 4 drones

Figure 5: Venice Scenario case 2, 20 customers, $\lambda_1 = \lambda_2 = 0.5$

Table 2 reports the maximum completion time, in terms of number of time slots, for the different feasible instances and the different weight combinations considered. We can see that in general increasing the number of available drones the maximum completion time decreases. Moreover, making it the predominant term in the objective function ($\lambda_2 >> \lambda_1$), it is possible to save up to 66% of time to serve all customers.

Table 2: Venice case 2, 20 customers

#UAVs	#T slots	$\lambda_1 - \lambda_2$
2	13	0.5 - 0.5
	12	0.05 - 0.95
	15	0.95 - 0.05
4	9	0.5 - 0.5
	7	0.05 - 0.95
	12	0.95 - 0.05
8	7	0.5 - 0.5
	4	0.05 - 0.95
	12	0.95 - 0.05

In the last case analysed, we assumed it was not possible to complete all deliveries within the considered time window and thus we want to maximize the number of served customers.

Table 1: Venice case 2

#Cust.	#UAVs	#Pareto sol	Gap %		
			min	mean	max
10	1	3	0	0	0
	2	3	0	0	0
	4	3	0	0	0
	8	3	0	0	0
20	1	infeasible	-	-	-
	2	3	0	0	0
	4	3	0	0	0
	8	3	0	0	0
30	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	2	0	0	0
	8	3	0	0.012	0.037
40	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	3	0	0	0
	8	3	0.004	0.199	0.592
50	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	infeasible	-	-	-
	8	3	0.002	0.093	0.184
70	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	infeasible	-	-	-
	8	3	0.079	0.102	0.146

Table 3 reports the main results related to these latter tests always adopting the weighted sum method. In particular, with the exception of the last configuration consisting in 8 drones, the number of Pareto optimal solutions generated is always equal to 2. One of them, independently from the number of available drones, is the trivial one consisting in not serving customers at all, obtained making the drone energy consumption the predominant term in the objective function. On the contrary, when the number of served customers is the most important between the two criteria ($\lambda_2 >> \lambda_1$), we obtained solutions with a number of served customers which increases with the number of drones as shown in the last column of table 3. Assigning the same weight to each term in the objective function provides again the trivial solution with the exception of the configuration consisting in a fleet 8 drones. In this latter case it is possible to serve 57 customers. Consequently, in this case, given the customers distribution over the city, 8 appears to be the minimum size of the fleet of drones which makes convenient the adoption of such a delivery system, considering equally important energy saving and productivity.

Table 3: Venice case 3

#Cust.	#UAVs	#Pareto sol	Gap %			#Serv. cust.
			min	mean	max	$\lambda_1 = 0.05, \lambda_2 = 0.95$
140	1	2	0	0	0	12
	2	2	0	0	0	24
	4	2	0	0	0	48
	8	3	0	0	0	95

The results obtained on this first simplified scenario permitted us to validate the model, partially exploring the Pareto frontier of the problem, characterized by the presence of two distinct and conflicting criteria in its second and third variant. Indeed, we showed that, by varying the value of the weight assigned to each term in the objective function, it is possible to generate different Pareto optimal solutions which can reflect the different decision maker leanings.

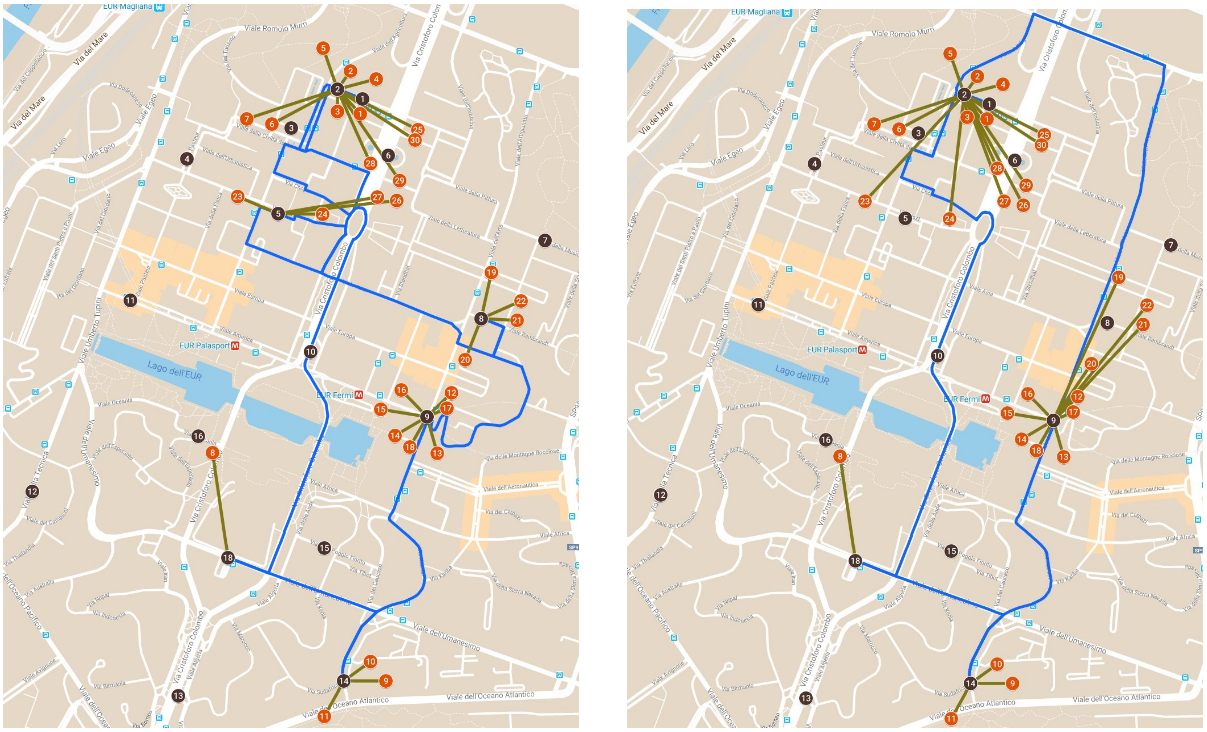
As already mentioned, we tested the model and its variants also on a second more challenging scenario related to the EUR district in the city of Rome, in order to verify its applicability in contexts where, differently from the particular configuration of the Venice city, the routing problem of the truck, supporting the operations of the fleet of UAVs, has to be solved. Table 4 reports the main results related to the first variant of the model on this scenario for which we again adopted the weighted sum method considering the three distinct terms representing respectively the truck energy consumption, the drone energy consumption and the drone energy level. We can notice that, differently from the Venice scenario, in this case we have conflicting criteria, and thus we generated different Pareto optimal solutions whose number increases with the number of customers and drones. However, also in this more complex scenario, the different weighted sum problems can be solved within the time limit at optimality or with a percentage relative gap that in most of the cases is very close to 0, as showed in the last column of table 4.

Table 4: Rome case 1

#Cust.	#UAVs	#Pareto sol	Gap %		
			min	mean	max
10	1	1	0	0	0
	2	1	0	0	0
	4	2	0	0	0
	8	2	0	0	0
20	1	infeasible	-	-	-
	2	2	0	0.016	0.114
	4	3	0	0.016	0.098
	8	3	0	0.071	0.060
30	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	3	0	0.168	0.113
	8	3	0	0.014	0.080
40	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	6	0	0.041	0.213
	8	5	0	0.017	0.070
50	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	4	0.005	0.281	1.233
	8	4	0.002	0.049	0.211
70	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	infeasible	-	-	-
	8	6	0.001	0.146	0.498

In particular, Figure 6 represents the Pareto optimal solutions, in term of truck route and customers assignment to the UAVs missions, for the instance with 30 customers and respectively 4 and 8 drones, obtained making the truck energy consumption the predominant term in the objective function especially with respect to the UAVs energy consumption ($\lambda_1 >> \lambda_2$). For this weight combination, increasing the number of available drones, it is possible to reduce the distance travelled by the truck concentrating in less stops the UAVs missions (6 with 4 drones and 4 with 8 drones).

From figure 7 we can visualize the Pareto frontier of the instance with 70 customers and 8 drones. This consists in 6 non-dominated points and it shows that it is possible to obtain different system configurations which differ each other up to around 31% for the drone energy consumption, up to around 36% for the truck energy consumption and up to around 7% for the drone energy level.



(a) 4 drone (b) 8 drones

Figure 6: Rome Scenario case 1, 30 customers, $\lambda_1 = 0.6\overline{3}, \lambda_2 = 0.0\overline{3}, \lambda_3 = 0.\overline{33}$

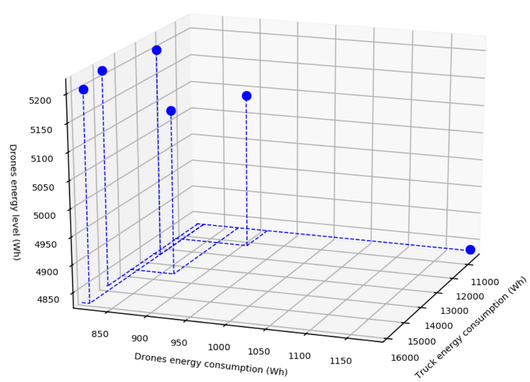


Figure 7: Rome Scenario, case 1, 70 customers, Pareto frontier

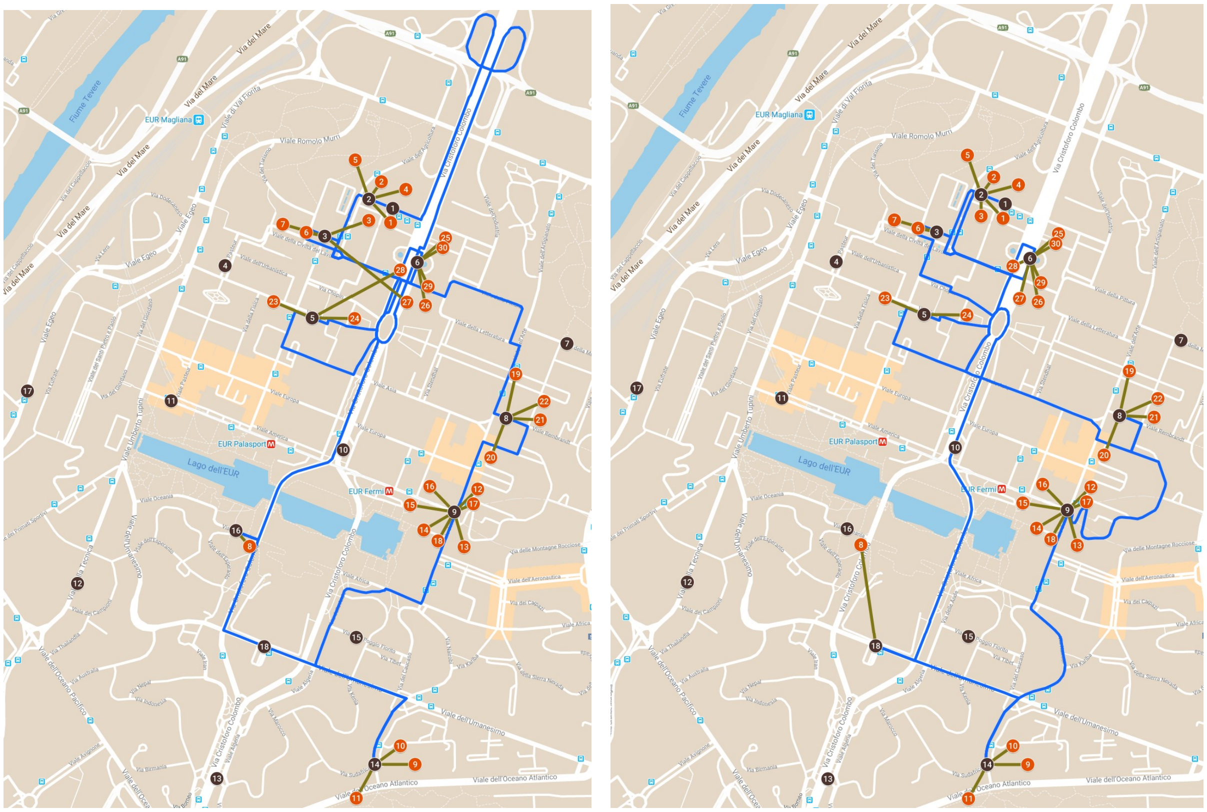
Similarly to the Venice scenario, adding the minimization of the maximum completion time, we incorporated in a unique goal all the energy terms (the drone energy consumption, the drone energy level and the truck energy consumption) assigning it a single weight. Then, we gave a different weight to the maximum completion time and varying their values we again performed a partial exploration of the Pareto frontier. Table 5 shows the main results obtained. We found up to 3 Pareto optimal solutions and the percentage relative gaps reported in the last column confirm that also for this second variant, the model can be solved within the time limit or provides a feasible solution of very good quality.

Table 5: Rome case 2

#Cust.	#UAVs	#Pareto sol	Gap %		
			min	mean	max
10	1	2	0	0	0
	2	2	0	0	0
	4	2	0	0	0
	8	2	0	0	0
20	1	infeasible	-	-	-
	2	2	0	0.016	0.114
	4	1	0	0	0
	8	2	0	0.004	0.012
30	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	3	0.110	0.707	1.257
	8	2	0	0.257	0.709
40	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	3	0.765	0.819	0.923
	8	3	0.284	0.660	1.209
50	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	2	0.191	0,402	0,819
	8	3	0.238	0.721	1.040
70	1	infeasible	-	-	-
	2	infeasible	-	-	-
	4	infeasible	-	-	-
	8	3	0.686	0.778	0.831

Figure 8 shows a comparison between the different Pareto optimal solutions generated making, respectively, the maximum completion time the predominant term (fig.8a) and viceversa, making the energy term the most important goal (fig.8b). We can observe that the truck route is different for the two solution configurations. Indeed, when the main goal is the minimization of the maximum completion time, the Pareto optimal solution consists in 9 time slots to complete the deliveries but in a bigger amount of energy consumption, especially for the drones. On the contrary, when the principal interest is the energy saving the last customer is served in 12 time slots and the total distance travelled by the UAVs is lower.

Table 6 reports the results related to the third variant of the model tested on this second scenario, again assigning a different weight to the energy term and the maximization of the number of served customers. As for the Venice scenario, when the energy saving is the main goal, the solution is trivial and it consists in not serving customers at all. On the contrary, making the number of deliveries the predominant term in the objective function, the number of served customers increases with the number of available UAVs, as shown in the last column of the table. However, also giving the same weight to each term, we obtained non-trivial solutions for a fleet of at least 4 drones, which can serve 29 customers. This number increases to 73 customers adopting 8 UAVs. These latter results provide a useful information on the minimum size of the fleet of drones which makes favourable the whole delivery system when both criteria are equally evaluated.



(a) $\lambda_1 = 0.05, \lambda_2 = 0.95$ (b) $\lambda_1 = 0.95, \lambda_2 = 0.05$

Figure 8: Rome Scenario case 2, 30 customers, 4 drones

Table 6: Rome case 3

#Cust.	#UAVs	#Pareto sol	Gap %			#Serv. cust.
			min	mean	max	$\lambda_1 = 0.05, \lambda_2 = 0.95$
140	1	2	0	0	0	13
	2	2	0	0	0	26
	4	3	0	0.004	0.012	52
	8	3	0	0.041	0.066	95

6 Conclusions

In this work we presented a new multi-period mixed integer linear programming model for the management of a hybrid delivery system consisting in a vehicle supporting delivery operations to customers, performed by a fleet of drones. In the problem formulation both the routing problem of the vehicle and the scheduling of the UAVs missions are incorporated. Different criteria are taken into account and simultaneously optimized, like the sustainability of the system, the service quality and the productivity. The model has been validated and tested on two realistic scenarios adopting a scalarisation technique to partially explore the Pareto frontier of the problem. The results appear encouraging. At the moment we do not know which will be the technological advances of UAVs and other vehicles, but we believe that the problem we described and faced in this paper could be of interest by shipping companies as it incorporates simple and robust operating routes: lunching and rendezvous of drones are not affected by network traffic conditions as the vehicle waits in safe locations; all deliveries are performed by drones releasing the vehicle driver from delivery tasks, furthermore the model is very adequate in all cases in which the use of drones is mandatory, like the absence of streets or when it is required an unmanned delivery like in the case of medicines shipping to infected people.

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