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CIRRELT-2020-21

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June 2020

Document de travail également publié par la Faculté des sciences de l'administration de l'Université Laval, sous le numéro FSA-2020-007.

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The Dynamic Routing Problem with Due Dates and Stochastic Release Dates

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Abstract. We study a variant of the multi-period routing problem in which deliveries may occur between release and due dates. The release date of each product is stochastic, and customer orders arrive dynamically over a planning horizon. The due date of an order is specified by the customer and no late delivery is allowed. The supplier reveals the delivery date to the customer in advance and any deviation from this date incurs a penalty. The probability of a product being available on any given day can be estimated. We introduce the notion of supplier risk aversion level to model the behavior of a supplier who must deal with disruptions in product supplies while trying to minimize the total cost of deliveries and of the penalties to be paid. Combining probabilities of product availability and the risk aversion level of the supplier, we formulate an a priori model for the problem in a deterministic fashion. This model, which is solved only once at the beginning of the planning horizon, allows the supplier to plan and schedule the upcoming deliveries. However, upon the occurrence of a supply disruption, delivery plans have to be modified and communicated to the customers. In this case, a recourse model is solved iteratively over the remaining days of the planning horizon, which can also handle dynamism in customer demand. The models are solved by branch-and-cut. Several sensitivity analyses are performed, and insights are developed to study the trade-offs between cost and stable plans. The results show how a more pessimistic attitude toward uncertainty results in more stability in planning but leads to higher costs, whereas lower risk aversion levels result in lower costs but come at the expense of more frequent changes in delivery plans.

Keywords: Routing problems, due dates, release dates, stochastic, disruption.

Acknowledgements. This project was partly funded by the Natural Sciences and Engineering Council of Canada (NSERC) under grants 2020-00401, 2019-00094 and 2015-06189. This support is greatly acknowledged. We thank Calcul Québec for providing high performance computing resources.

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Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2020

1. Introduction

Most of the research on routing problems assumes static and deterministic information, whereas in many real-life cases, the information is not fully known in advance (Azi et al., 2012; Coelho et al., 2016). Uncertainty can stem from different sources. Sanchez-Rodrigues et al. (2010) identify several uncertainty types with respect to shippers, customers, carriers, control systems, and the external environment. In stochastic routing problems, the main uncertain parameters relate to demand, travel time, service time, and the presence of customers (Gendreau et al., 2016; Hernandez et al., 2019).

There exist relatively few studies on supply uncertainty in routing problems. In distribution planning, products are usually assumed to be available at the time of shipment. For example, in inventory-routing (Coelho et al., 2014) and production-routing problems (Adulyasak et al., 2015; Díaz-Madroñero et al., 2015), products can be delivered if the inventory is sufficient, and these have a known and deterministic production rate. Supply disruptions caused by delays, breakdowns, quality failure, or any other factors are rarely considered in these contexts. In this paper, we investigate distribution planning in the presence of stochastic product availability.

Here we consider a routing problem with the following three characteristics: 1) customer orders for products arrive dynamically over a planning horizon, 2) they have a delivery due date specified by the customer, and 3) the time at which the products become available for delivery is uncertain, i.e., they have stochastic release dates. In this study, we consider as a first step the single-vehicle version of the problem. We call it the dynamic routing problem with due dates and stochastic release dates (DRPDSR).

In addition to e-commerce, applications of the DRPDSR arise in natural disasters emergency logistics planning where the demand for items such as medical supplies, personnel, food and water, clothing and bedding, is known for each location, and a deadline is given for their reception, but the availability of these products is not fully known in advance. The problem is also encountered in production systems dependent on the manufacturing or on the external supply of items to be shipped to customers.

1.1. Literature review

We now present a review of the literature on the vehicle routing problem (VRP) with release and due dates (VRPRDD). Shelbourne et al. (2017) and Archetti et al. (2018) introduced release and due dates to the vehicle routing literature. In this context, the challenge is to delay customer orders to be able to serve them together and benefit the most from the resulting reduction in delivery costs (Reyes et al., 2018). Shelbourne et al. (2017) defined the release date as the time at which the requests become available at the depot. They aimed to integrate production scheduling and vehicle routing decisions where penalties related to late deliveries also need to be minimized. These authors defined a setting with a fleet of homogeneous capacitated vehicles. To each customer, they associated a service time and an order characterized by its quantity, release and due dates, and a weight. They considered both the travel time and consistency of the sum of distance and total weighted tardiness. This problem is known to be NP-hard and shares similarities with the capacitated VRP and the VRP with time windows. The authors proposed a path-relinking algorithm to solve it. Archetti et al. (2015b) introduced the multi-period VRP with due dates in which the delivery decisions are more flexible than in the classical VRP, as every request from customers can be satisfied within a given deadline. Motivated by the integration of vehicle routing and production scheduling problems, the VRP with due dates was later extended to the VRPRDD (Cattaruzza et al., 2016). Shelbourne et al. (2017) provide a comprehensive review of the VRPRDD and other VRPs with time constraints.

Archetti et al. (2015a) solved two versions of the VRP with uncapacitated vehicles and release dates. In the first case, they considered a deadline for each order and minimized the total distance traveled, whereas in the second case, no deadlines were considered, but the total delivery completion time was minimized. They analyzed the complexity of these VRP variants and showed that the cases with either one uncapacitated vehicle performing several routes, or an unlimited fleet of vehicles performing only one route, could be solved in polynomial time for some graph structures. The authors developed a dynamic programming algorithm for the problem, which was also used by Reyes et al. (2018) who solved the same two variants, but in a different setting since they considered service guarantees. As an extension to the work of Archetti et al.

(2015a), Reyes et al. (2018) applied dynamic programming to minimize the completion time of the last route and the distance traveled while completing the last route before a specific deadline. Finally, focusing on the traveling salesman problem with release date and completion time minimization, Archetti et al. (2018) proposed a mathematical formulation and developed an iterated local search-based heuristic to solve it.

Another related variant of the VRPRDD is the VRP with release dates and time windows. Cattaruzza et al. (2016) introduced this problem with a fleet of identical capacitated vehicles serving customers within their preset time windows. Each order is associated with a release date. They developed a hybrid genetic algorithm to solve the problem. Zhen et al. (2020) extended this problem and studied a multi-depot multi-trip VRP with release dates and time windows. They proposed two metaheuristics: a hybrid particle swarm optimization algorithm and a hybrid genetic algorithm.

Although the recent VRP literature embraces the notion of stochasticity in which some elements of the problem are random, uncertainty has not yet been considered in the VRPRDD. The most studied stochastic elements in the VRP context are demand (Bertsimas, 1992) and stochastic travel times (Laporte et al., 1992). In this paper, we are particularly interested in uncertain supplies. In our problem setting, product availability is not deterministic, which forces the plans to change on a daily basis and with the release of information. Therefore, our problem is also related to disruption management in vehicle routing.

Eglese and Zambirinis (2018) review disruption management in vehicle routing. They consider four sources of disruption: vehicle breakdown, unavailable link in the road network, unknown supply, and unknown customer demand. The disrupted capacitated vehicle routing problem with order release delay (DCVRP-ORD) introduced in Mu and Eglese (2013) is the closest work to our problem as one needs to modify the routing plans when some products are unavailable at the start of the delivery period. The authors proposed two tabu search heuristics to deal with supply delays: to wait for the delayed products or to continue delivering the available ones. Despite the similarity of the DCVRP-ORD with the DRPDSR in the supply disruption aspect, the two problems differ in several important ways. In the single-period setting of DCVRP-ORD, some products become available later during the period, which requires certain vehicles to wait at the depot for the delayed products to become available. In the DRPDSR, given that each customer has a due date, one needs to decide on which day before the due date a delivery has to be made. In the DCVRP-ORD, a vehicle can wait at the depot for products to become available or multiple trips can also be assigned to vehicles, whereas in the DRPDSR, we consider an expedited shipping option (at a price), which encourages the supplier to make the best use of the vehicle capacity. Moreover, in the DCVRP-ORD, all delayed orders arrive at the same time at the depot, whereas in the DRPDSR the release day of products varies. Although in the DCVRP-ORD, a disruption occurs at the depot, the amount of the delayed orders and the length of the delay are known. In the DRPDSR, on the other hand, stochastic parameters are used to represent the release day of a product. While the DCVRP-ORD aims to minimize the delays, in the DRPDSR the goal is to reduce the deviations between the anticipated and the real delivery plans, and finally, no mathematical model is provided for the DCVRP-ORD.

1.2. Scientific contributions and organization of the paper

Although the VRPRDD literature extends the classical VRP by considering a release date for the orders, its underlying assumption is that the release dates are known and deterministic. In this paper, we extend the problems studied in Mu and Eglese (2013) and Archetti et al. (2020) to introduce due dates and stochastic release dates. We consider the availability of the product, i.e., the release date, to be a random variable. This paper differs from previous studies as information regarding demand and product availability is revealed on a daily basis after an a priori solution has been determined and communicated to the customers. When new information arrives, the a priori solution may need to be revised, and routes may have to be changed. Whenever the new plan results in a change in delivery schedules, the supplier may incur a penalty.

Our scientific contribution is manifold. We introduce, model, and solve the DRPDSR. We propose an a priori plan and several recourse actions whenever the a priori solution cannot be executed due to the non-availability of the products. We introduce the notion of supplier's risk aversion level and we combine it with the probabilities of product availability to yield a deterministic a priori model. We believe our study is the first to explicitly integrate this notion in the mathematical modeling of a stochastic vehicle routing problem (see, e.g., the recent survey of Gendreau et al. (2016)). We then iteratively solve a recourse model to handle stochastic realizations and demand dynamism.

The remainder of this paper is organized as follows. In Section 2, the DRPDSR is formally defined and modeled, with a detailed description, mathematical formulation, a priori solution, and recourse actions. The solution algorithm is explained in detail in Section 3. In Section 4, we present the results of our extensive computational experiments, along with sensitivity analyses and discussions on the results. Conclusions follow in Section 5.

2. Problem statement

We describe the DRPDSR, in general terms in Section 2.1 and through a mathematical model in Section 2.2, followed by disruption management procedures in Section 2.3.

2.1. General description

We consider a supplier delivering several products to its customers. The products are expected to become available at the supplier on a given date, with a given discrete probability distribution. According to the definition provided by Shelbourne et al. (2017), we call this the *release date*. Customers request products on different days of the planning horizon, and the day at which a product is requested is referred to as an *order date*. These requests can be met within a time interval, beginning at the order date and ending at their *due date*, which is known in advance and may vary for different requests. The supplier knows that all products will be available before the due date, but their availability on any given day between the order date and the due date is not guaranteed. The supplier must communicate the delivery plans in advance to the customers. Therefore, at the beginning of the planning horizon, the supplier makes an *a priori plan* that specifies delivery dates. If the products are unavailable on the planned distribution day or if the demand pattern changes, the supplier needs to change its a priori plan, *a recourse action* on the decisions is required. This entails iteratively solving a recourse model.

The supplier operates a single capacitated vehicle to serve the customers and has access to an express delivery service in order not to violate the due dates if the vehicle capacity is insufficient. When the vehicle makes a trip, it leaves the depot where the supplier is located, visits some customers, and returns to the depot. Two events may affect the routing plans of a day. First, a product planned for delivery turns out to be unavailable. In this case, the supplier has no choice but to postpone its delivery by bumping some customers from their planned route, which incurs a bumping cost. Due to this change of plan, extra space becomes available on the vehicle, and the supplier may decide to anticipate the delivery of other available products, in which case these delivery dates also need to be changed, and a bumping cost is again incurred. The other case occurs when a product that was considered unavailable in the initial plan becomes available on a given delivery day. In this case, the supplier may decide to change the delivery date of this product and incur a bumping cost. Given the uncertainties on the supply side, this decision is made to avoid future express delivery costs caused by the violation of the vehicle capacity, since some requests need to be sent urgently. The supplier always has the option of subcontracting deliveries to an express carrier, which yields more costly direct shipments to customers.

The cost minimization dilemma the supplier faces can be described as follows: is it better to be assured of the availability of products before announcing the delivery plans to the customers, or to assume the products will be available as early as possible and pay for the bumping costs if proved wrong? The choice depends on the risk aversion level of the supplier and therefore, the trade-off between the distribution costs and the bumping costs leads to different delivery plans. In summary, the a priori solution determines when to deliver to each customer and how to create vehicle routes, under the objective of minimizing the total cost of routing and expedited delivery. A recourse action aims at 1) recovering feasibility once a release date does not allow a product delivery, and its request must be postponed, 2) decreasing costs in case other deliveries can be anticipated, and 3) handling dynamism in customer demand.

2.2. Mathematical formulation of the a priori problem

The DRPDSR is defined on an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{0, \ldots, n\}$ is the node set and $\mathcal{E} = \{(i, j) : i, j \in \mathcal{N}, i < j\}$ is the edge set. Node 0 represents the depot and $\mathcal{N}_c = \mathcal{N} \setminus \{0\}$ is the set of customers. Let $\mathcal{P} = \{1, \ldots, P\}$ be the set of products that are delivered to the customers by a single vehicle of capacity Q. A routing cost c_{ij} is associated with each edge (i, j). The problem is defined on a finite planning horizon $\mathcal{T} = \{1, \ldots, T\}$ of T days or over an infinite horizon. In the latter case it can easily be solved through a rolling-horizon mechanism by iteratively solving the problem each day for the horizon $\{t, ..., t + T - 1\}$ and implementing the solution for day t . In what follows we model and solve the problem for the finite horizon case.

For each customer i the demand of product p on day t is known and denoted by d^t_{pi}. Multiple products may be requested on day t, and a maximum allowed lateness l^t_{pi} indicates within how many days the demand of customer i for product p must be fulfilled, defining the due date. A maximum allowed lateness of zero means same-day delivery.

The order fulfillment of product p depends on its availability, which is stochastic, but once a product becomes available on day t, it will remain so until the end of the planning horizon. Before its scheduled delivery on day t, the cumulative probability π^t_p of product p being available on day t is known. This means that if a customer i has a positive demand for product p on day t ($d^t_{ip} > 0$) with a maximum lateness l^t_{pi}, then we must have $\pi^{t'}_p = 1$, where $t' = t + l^t_{pi}$. If the product is available, delivery can occur; otherwise, it needs to be postponed. The subjective estimate of the supplier on the availability of product p on day t is denoted by e^t_p, which is a binary parameter incorporating the cumulative probabilities π^t_p and the risk aversion level of the supplier denoted by $\theta \in [0, 1]$. Specifically,

$$e^t_p = \begin{cases} 1 & \text{if } \pi^t_p \geq \theta \\ 0 & \text{otherwise.} \end{cases}$$

If $\theta = 0$, then the supplier is risk taking (i.e., an optimist) who assumes that product p will be available at the first available opportunity. If $\theta = 1$, then the supplier is risk averse (i.e., a pessimist) who assumes that product p will be available on the last possible day. Intermediate values of θ correspond to a continuum of risk aversion levels. Because the supplier does not know in advance whether the products will be available or not on a certain day and the delivery plan needs to be communicated to the customers beforehand, e_p^t is used to compute the delivery plan, which yields a deterministic-equivalent form of a stochastic model.

Knowing the availability probability of each product, the supplier schedules the deliveries in its a priori plan, but as time unfolds and stochastic release dates are realized, these schedules may need to be revised, which gives rise to a recourse action. A bumping cost f must be paid whenever a customer has one of its deliveries rescheduled. Expedited delivery to customer i costs c'_i, and we assume that c'_i is sufficiently large to encourage the shipment via routing. No split delivery is allowed for a given product, but different products ordered on the same day can be delivered separately.

By considering the subjective estimate of the supplier on the availability of the product, the deterministic counterpart of the DRPDSR can be formulated with the following decision variables: let x_{ij}^t be the number of times edge (i, j) is traversed on day t, $y_i^t = 1$ if and only if customer i is visited on day t via routing, $z_{pi}^{tt'} = 1$ if and only if product p ordered on day t is delivered to customer i on day $t' > t$ via routing, and $q_{pi}^{tt'} = 1$ if and only if product p ordered on day t is delivered to customer i on day $t' > t$ via an expedited delivery. The problem can then be modeled as follows:

$$\text{(P) minimize} \sum_{t' \in \mathcal{T}} \left(\sum_{(i,j) \in \mathcal{E}} c_{ij} x_{ij}^{t'} + \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{N}_c} \sum_{t \in \mathcal{T}} c'_i q_{pi}^{tt'} \right) \tag{1}$$

$$\text{subject to} \quad x_{ij}^t \leq y_i^t \quad (i, j) \in \mathcal{E}, t \in \mathcal{T} \tag{2}$$

$$x_{0i}^t \leq 2y_i^t \quad i \in \mathcal{N}_c, t \in \mathcal{T} \tag{3}$$

$$y_0^t \geq y_i^t \quad i \in \mathcal{N}_c, t \in \mathcal{T} \tag{4}$$

$$\sum_{j:(i,j)\in\mathcal{E}} x_{ij}^t + \sum_{j:(i,j)\in\mathcal{E}} x_{ji}^t = 2y_i^t \quad i \in \mathcal{N}_c, t \in \mathcal{T} \tag{5}$$

$$\sum_{\substack{(i,j)\in\mathcal{E}\\ i,j\in\mathcal{S}}} x_{ij}^t \leq \sum_{i\in\mathcal{S}} y_i^t - y_m^t \quad \mathcal{S} \subset \mathcal{N}_c, m \in \mathcal{S}, t \in \mathcal{T} \tag{6}$$

$$z_{pi}^{tt'} \leq y_i^{t'} \quad d_{pi}^t > 0, p \in \mathcal{P}, i \in \mathcal{N}_c, t, t' \geq t \in \mathcal{T} \tag{7}$$

$$\sum_{t\in\mathcal{T}, t\leq t'} z_{pi}^{tt'} \leq M^{t'} y_i^{t'} \quad p \in \mathcal{P}, i \in \mathcal{N}_c, t' \in \mathcal{T}, d_{pi}^t > 0 \tag{8}$$

$$\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}, t\leq t'} z_{pi}^{tt'} \geq y_i^{t'} \quad i \in \mathcal{N}_c, t' \in \mathcal{T}, d_{pi}^t > 0 \tag{9}$$

$$\sum_{p\in\mathcal{P}} \sum_{i\in\mathcal{N}_c} \sum_{t\in\mathcal{T}, t\leq t'} z_{pi}^{tt'} d_{pi}^t \leq Q \quad t' \in \mathcal{T} \tag{10}$$

$$\sum_{\substack{t'\in\mathcal{T},\\ t\leq t'\leq t+l_{pi}^t}} \left(z_{pi}^{tt'} + q_{pi}^{tt'}\right) = 1 \quad p \in \mathcal{P}, i \in \mathcal{N}_c, t \in \mathcal{T}, d_{pi}^t > 0 \tag{11}$$

$$z_{pi}^{tt'} + q_{pi}^{tt'} \leq e_p^{t'} \quad p \in \mathcal{P}, i \in \mathcal{N}_c, t \in \mathcal{T}, d_{pi}^t > 0, t' \in [t, t + l_{pi}^t] \tag{12}$$

$$x_{ij}^t \in \{0, 1, 2\} \quad (i,j) \in \mathcal{E}, \quad t \in \mathcal{T} \tag{13}$$

$$y_i^t, z_{pi}^{tt'}, q_{pi}^{tt'} \in \{0, 1\} \quad p \in \mathcal{P}, i \in \mathcal{N}_c, \quad t, t' \in \mathcal{T}. \tag{14}$$

The objective function (1) minimizes the total cost of routing and expedited deliveries. Constraints (2)–(4) link the routing and visiting variables. Constraints (5) define the node degrees, and constraints (6) eliminate subtours (see Gendreau et al. (1997)). Constraints (7)–(9) link the z variables to the y variables, where $M^{t'}$ in constraints (8) is a large number that can be set to the number of requests from customer i between time 1 and time t'. Constraints (10) ensure that the vehicle capacity for the routing shipments is respected. Constraints (11) state that each demand must be satisfied exactly once, and constraints (12) state that the delivery of a product can only be made after it has become available. These constraints also ensure that the due dates are respected. Finally, constraints (13) and (14) define the domains of the variables.

2.3. Managing disruptions: mathematical formulation of the recourse model

The problem at hand can be considered as a sequential decision problem since decisions have to be made at different points in time and are influenced by the previous ones (Hausman, 1969). Under this situation, adaptive optimization is the most natural approach a supplier can apply in which one adjusts the solutions when uncertain parts of the input are realized (Bertsimas et al., 2013). We apply this concept to manage disruptions in product availability and dynamism in customer demands.

In the a priori plan, we set the values of e_p^t according to the rules described before. We then solve model (1)–(14) to obtain a delivery plan, i.e., $x_{ij}^t = \bar{x}_{ij}^t$, $z_{pi}^{tt'} = \bar{z}_{pi}^{tt'}$, and $q_{pi}^{tt'} = \bar{q}_{pi}^{tt'}$. On each day t of the planning horizon, the true realization of the release dates for the products becomes known, and some of the previously made decisions may need to be adjusted. The same model can be used to handle dynamic customer demands.

In the recourse plan, we solve the problem iteratively, and at the beginning of each day t, we decide whether a modification to the a priori plan is required. Three situations may occur. First, the estimation e_p^t was correct, in which case no recourse action is needed. Second, if the product is not available when it was expected to be, at least one delivery will be bumped to a later date. One can also use the extra vehicle capacity to anticipate some deliveries. Third, a product that was not expected to be available on day t becomes available, meaning that some later deliveries can be anticipated, including to the day in question.

Considering the same risk estimate as before, if the due date for the product is reached or if this product has become available, then $e_p^t = 1$, otherwise, we need to recompute the values of e_p^t based on the remaining probabilities π_p^t and the value of θ, as follows. Considering that the decision is made on day t, we have

$$e_p^t = \begin{cases} 1 & \text{if } \left(\pi_p^t - \pi_p^{t-1}\right) \geq \theta \\ 0 & \text{otherwise.} \end{cases}$$

Specifically, on day t, the supplier either realizes that some products have been released or remain unavailable, effectively knowing the availability status of each product p. To determine the recourse action, a mathematical model optimizes the remaining decisions based on the same risk aversion level of the supplier. To this end, we define binary variables v_i^t, equal to 1 if and only if any product ordered by customer i is bumped from its delivery planned for day t. Also, let the values of variables $z_{pi}^{tt'}$ and $q_{pi}^{tt'}$ be $\bar{z}_{pi}^{tt'}$ and $\bar{q}_{pi}^{tt'}$, based on the solution of the a priori model. In order to calculate the bumping costs, we define parameters g_i^t based on the a priori plan and variables w_i^t for the recourse plan. The parameters g_i^t indicate whether a visit was planned for customer i in period t and are defined as follows:

$$g_i^{t'} = \begin{cases} 1 & \text{if } \sum_{p\in\mathcal{P}} \sum_{t\le t'} \left(\bar{z}_{pi}^{tt'} + \bar{q}_{pi}^{tt'}\right) \ge 1 \\ 0 & \text{otherwise.} \end{cases}$$

Variables w_i^t are equal to 1 if any delivery is scheduled for customer i on day t and 0 otherwise. The recourse model is then:

$$\text{(R) minimize } \sum_{t'\in\mathcal{T}} \left(\sum_{(i,j)\in\mathcal{E}} c_{ij} x_{ij}^{t'} + \sum_{p\in\mathcal{P}} \sum_{i\in\mathcal{N}_c} \sum_{t\in\mathcal{T}} c_i' q_{pi}^{tt'} + \sum_{i\in\mathcal{N}_c} f v_i^{t'} \right) \tag{15}$$

subject to (12)–(14) and to

$$w_i^{t'} \le \sum_{p\in\mathcal{P}} \sum_{t\le t'} \left(z_{pi}^{tt'} + q_{pi}^{tt'} \right) \quad i\in\mathcal{N}_c, t'\in\mathcal{T} \tag{16}$$

$$\sum_{p\in\mathcal{P}} \sum_{t\le t'} d_{pi}^t w_i^{t'} \ge \sum_{p\in\mathcal{P}} \sum_{t\le t'} \left(z_{pi}^{tt'} + q_{pi}^{tt'} \right) \quad i\in\mathcal{N}_c, t'\in\mathcal{T} \tag{17}$$

$$v_i^{t'} \ge g_i^{t'} - w_i^{t'} \quad i\in\mathcal{N}_c, t'\in\mathcal{T} \tag{18}$$

$$v_i^{t'} \ge w_i^{t'} - g_i^{t'} \quad i\in\mathcal{N}_c, t'\in\mathcal{T} \tag{19}$$

$$w_i^t, v_i^t \in \{0,1\} \quad i \in \mathcal{N}_c, \quad t \in \mathcal{T}. \tag{20}$$

The objective function (15) is similar to (1) with an extra term representing the bumping costs. Constraints (16) link delivery variables z (routing) and q (expedited delivery) with the visiting variables w. Quantities delivered are then ensured via constraints (17). If any visit was planned for customer i in period t, i.e., $g_i^t = 1$, but a new optimized plan does not include a visit to this customer in that period, i.e., $w_i^t = 0$, then variables v take value 1 as ensured by constraints (18) and (19). In this case, a bumping cost is then incurred as per the new objective function (15). Constraints (20) define the domains of the decision variables.

For any day $t \geq 2$, the values of $z_{pi}^{tt'}$ and $q_{pi}^{tt'}$ are stored in $\bar{z}_{pi}^{tt'}$ and $\bar{q}_{pi}^{tt'}$, respectively. Model R is then reoptimized by adding the following constraints to ensure that past decisions are no longer changed:

$$z_{pi}^{\tilde{t}t'} = \bar{z}_{pi}^{\tilde{t}t'} \quad \tilde{t} \in \{1, \ldots, t\} \tag{21}$$

$$q_{pi}^{\tilde{t}t'} = \bar{q}_{pi}^{\tilde{t}} \quad \tilde{t} \in \{1, \ldots, t\}. \tag{22}$$

3. Solution algorithm

To deal with the uncertainty from the supply side, we first make a priori decisions. On a given horizon, these decisions are updated, if necessary. To solve model (1)–(14), we use a general purpose mixed integer linear program solver, Gurobi in our case. However, using a heuristic, we first generate an initial solution as a warm start for a branch-and-cut algorithm.

3.1. Initial solution generation

An initial solution is given to the branch-and-cut algorithm to provide an upper bound and speed up the search. The heuristic process applied to obtain this initial solution is summarized as follows. As long as the capacity of the vehicle allows, the demand of each customer for each day t is scheduled for delivery on the earliest possible day, both with respect to the availability of the product and the delivery due date. It should be noted that all demands must be satisfied by their due dates. The pseudocode for this procedure is described in Algorithm 1.

Algorithm 1 Initial solution construction

```
for each demand of product p from customer i released on day t do
    if the demand is not assigned then
        for each day from t until the due date do
            if the product is available then
                if the capacity of the vehicle allows then
                    Assign the demand of customer i for product p to day t
                    Adjust the load of the vehicle
                end if
            end if
        end for
    else
        Go to the next demand to be assigned
    end if
end for
```

3.2. Branch-and-cut algorithm

To solve the DRPDSR, we have implemented the model defined by (1)–(14), excluding the subtour elimination constraints (6), relaxing integrality, and considering the supplier's risk aversion level. Once a solution is found, it is checked against these constraints using the separation procedure from the CVRPSEP library (Lysgaard et al., 2004). If the solution does not contain subtours, it is accepted; otherwise, it is discarded and the violated constraints are added to the model.

We observed in preliminary experiments that many small subtours arise early in the solution process. To speed up the search forbidding such subtours, and increasing the lower bounds from the root node, we apply a *lazy constraints* strategy. This means that these constraints are valid, written and ready, but not added to the model directly. Instead, they remain in a *lazy constraint pool* where they remain inactive until an integer feasible solution is found, at which point the solution is checked against the lazy constraint pool. If the solution violates any lazy constraints, they are added to the model. Within Gurobi, we used the *attribute* value of 1, which indicates that not all violated lazy constraints need to be added for a violated solution. This leads to a more parsimonious addition of constraints to the model, which helps prevent making the model too heavy for the later phases of the branch-and-cut search. This is valid as long as at least one violated constraint is added. Then the node is solved again, and if another integer feasible solution is obtained, it is checked against the lazy constraint pool again.

Our use of lazy constraints is as follows. First, all subtours of sizes two and three are forbidden by adding all constraints (6) with $|S| = 2$ and 3 as lazy constraints. Moreover, for some sets $|S| = 4$ and 5 the respective constraints are also enumerated in the lazy constraint pool. Since the number of possible subsets is very large, here for each node, we only write the constraints to forbid the 10 smallest subtours, i.e., we evaluate the size (routing cost) of all these subsets, and for each customer, we add to the lazy pool the constraints that would forbid the 10 smallest ones. Obviously, repeated constraints are discarded. Adding these constraints for subsets larger than five nodes would become too expensive to separate.

4. Computational results

The formulations presented in Sections 2.2 and 2.3 were coded in C++ and solved using Gurobi Optimization 8.1.0. The computational experiments are conducted on an Intel Core i7 processor running at 3.4 GHz with 64 GB of RAM installed with the Ubuntu Linux operating system. The a priori phase is run for five hours, and we limit the reoptimization time to one hour for each day in the planning horizon.

4.1. Instance generation

For our experiments, we have randomly generated several sets of instances, as described in Table 1. The instances differ in size defined by the number of products, customers, and days in the planning horizon. All the random selections follow a discrete uniform distribution. To ease the computational burden, we work with a fixed planning horizon and static demands varying between zero and five units. The value of T is selected from $\{5, 8, 10, 12\}$, and when five days are considered in the planning horizon, $|N_c|$ is selected from $\{10, 20, 50, 60, 80\}$. Each time, we move to a larger planning horizon, we remove the largest element of the set N_c. This means that for instances with 12 days, we consider 10 and 20 customers. As the products are customer specific, in all our instances we assume $P = |N_c| + 5$.

The coordinates for the customer nodes are randomly and uniformly generated as an integer number in the interval of $[0, 500]$. We located the depot at the center with coordinates $[250, 250]$. The distances between any two nodes are calculated using a Euclidean metric and rounded to the nearest integer, where transportation costs are proportional to the distances.

Table 1: Parameters of the instances

Parameter	Notation	Value
# of days	T	$\{5, 8, 10, 12\}$
# of customers	$\|N_c\|$	$\{10, 20, 50, 60, 80\}$
# of products	P	$P = \|N_c\| + 5$
Demand	d_{pi}^t	$[0, 5]$
Vehicle of capacity	Q	$1.5 \times \max_t \sum_{i \in \mathcal{C}} d_i^t$
Coordinates of node i	X_i, Y_i	$[0, 500]$
Routing cost	c_{ij}	$\lfloor \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} + 0.5 \rfloor$
Expedited delivery cost	c_j'	$5 \times c_{0j}$
Bumping cost	f	$\{500, 2000\}$
Maximum allowed lateness	l_{pi}^t	$[1, 1 + \frac{T+1}{2})$

To generate the release day of products, we randomly select a day between the day at which the product is demanded for the first time and the latest day at which the product can become available. The product will become available after this randomly selected day. It should be noted again that once a product becomes available, it remains so until the end of the planning horizon, and no product can become available before any demand is realized for it.

We consider four scenarios to generate the cumulative probability π_p^t for product p being available on day t. These scenarios are generated as follows. First, we select a random number X to have control over the probability increase. For the first two scenarios, this number is selected from the interval of $[0.05, 0.10]$ and for the last two scenarios from $[0.4, 0.5]$. The probability for the first day in this interval is then set to $\pi_p^1 = X$. For the next days, we proceed as follows:

Scenario LG (Low starting point- Gradual increase): We generate a random number Y in the interval $[0, 1 - \pi_p^{t-1}]$. If $Y > X$ then Y is set equal to X. The accumulated probability on day t is then calculated as $\pi_p^t = \pi_p^{t-1} + Y$. We repeat this procedure for every day of the interval.

Scenario LS (Low starting point- Skewed): We randomly select a day t' between the order day and the due date. The probability of the product being available on any day before t' is randomly selected from $Y = \min\{0.01, \text{rand}[0, 1 - \pi_p^{t-1}]\}$. If $Y > X$ then Y is set equal to X. For every day after day t' until the due date, $Y = \min\{0.4, \text{rand}[0, 1 - \pi_p^{t'-1}]\}$.

Scenario HA (High starting point- Abrupt increase): This scenario is similar to the first one but with different starting values for X.

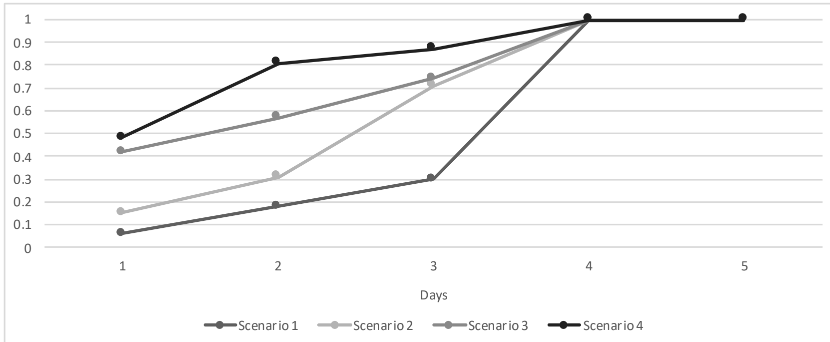
Scenario HS (High starting point- Skewed): We randomly select a day t' between the order day and the due date. The probability of the product being available on any day before t' is randomly selected as $Y = \text{rand}[0, 1 - \pi_p^{t-1}]$. If $Y > X$ then Y is set equal to X. For every day after day t' until the due date, $Y = \text{rand}[0, 1 - \pi_p^{t'-1}]$.

Basically, under scenarios LG and HA, we gradually increase the probability of the products becoming available. The slope of this increase differs in the two scenarios. In scenarios LS and HS, we use a random point within the first day at which the product is ordered and the day at which it becomes available. We then generate skewed probabilities before and after this random point. This is depicted in Figure 1, where an example with five days in the planning horizon and the cumulative probability for each scenario is provided.

4.2. Late deliveries strategy

The defined mathematical models are oblivious to the risk aversion levels. Given enough capacity, two solutions may differ only on the day at which a route is executed, but their cost is the same. However, a solution for which a delivery is planned for day $t + 1$ instead of day t has a lower associated risk, and should be preferred. In order to provide this information to the model, and to avoid such symmetries, we apply the following strategy to encourage delivering later than sooner, if capacity allows. We slightly modify the cost matrix c_{ij} structure to become time-

Figure 1: Comparing different cumulative probability scenarios for a five-day planning horizon.



dependent where $c_{ij}^t = c_{ij} - t/100$, implying that the cost of an arc traversed one day later would be reduced by 0.01. This value is small enough not to change the shape of the solution but it is large enough to provide an advantage to the model to plan for later deliveries. Once a solution is obtained, it is evaluated using the true c_{ij} matrix, not the time-dependent discounted one.

4.3. Results and analyses

This section presents the results of our computational experiments. First, we assess the effectiveness of the proposed solution algorithm for the a priori phase, and we then conduct a cost analysis for each category of risk threshold and product availability scenario in the recourse phase.

4.3.1. A priori phase

We now provide detailed information on the results obtained for each instance on the a priori phase. Table 2 shows the average gap obtained considering each product availability scenario and risk aversion level θ over all instances with the same number of customers $|N_c|$. The average results obtained for the random instance sets are distinguished. The lighter the color of a cell, the lower the gap. For both sets of instances, we observe that the combination of the product availability scenario and θ can have different behaviors depending on the number of customers.

Table 2: Average gaps (in %) obtained in the a priori phase

Instance		**Instance set I**				**Instance set II**			
$\|N_c\|$	Scenario\ θ	0.0	0.4	0.8	1.0	0.0	0.4	0.8	1.0
10	LG	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	LS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	HA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	HS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	LG	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	LS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	HA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	HS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	LG	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	LS	2.80	0.22	0.00	0.03	1.64	0.96	0.01	0.00
	HA	1.43	0.90	0.65	0.00	0.00	0.03	0.44	0.00
	HS	1.48	0.56	0.19	0.00	2.53	0.57	0.88	0.00
60	LG	1.57	0.00	0.00	0.00	1.96	0.66	0.66	0.67
	LS	1.81	0.00	0.00	0.00	1.39	0.48	0.05	0.66
	HA	2.18	0.52	0.00	0.00	1.67	2.89	0.76	0.66
	HS	1.85	1.80	0.41	0.00	2.02	0.71	1.11	4.65
80	LG	0.00	0.00	0.00	0.00	4.61	0.01	0.01	0.01
	LS	0.50	0.00	0.00	0.00	14.49	0.24	0.00	0.01
	HA	0.31	0.00	0.00	0.00	7.44	5.51	0.53	0.00
	HS	0.31	0.00	0.00	0.00	5.75	6.03	0.01	0.01

This table highlights that when a supplier is more optimistic, i.e., has lower values of θ, the problem becomes more difficult as the average gap after five hours of optimizing time remains high. This can be explained by the increased flexibility for the delivery plan as a more optimistic setting allows products to be delivered in more days, whereas a more pessimistic setting (corresponding to a higher value of θ) yields a more constrained problem in which there are fewer delivery options for each product. These results also show that the availability scenarios HA and HS exhibit the same behavior, being more difficult. This is also explained by the fact that products become available sooner, hence increasing flexibility.

Table 3 provides more details about the impact of the θ on the difficulty of the a priori problem. The results shown in this table are aggregated over all scenarios and all planning horizons. For each set of instances identified by the number of products, the number of customers, and θ, we provide the upper bound (UB), the lower bound (LB), the optimality gap (%), and the execution time in seconds. We observe that for all instances with optimal solutions, an increase in the value of θ results in a cost increase. In other words, the more pessimistic the supplier is, the higher is the estimated cost. In order to prevent further bumping costs, a pessimist supplier is willing to consider a distribution plan with a higher a priori cost.

Table 3: Average a priori results over all scenarios and all planning horizons

Instance		**Instance set I**				**Instance set II**			
$\|N_c\|$	θ	UB	LB	Gap (%)	Time (s)	UB	LB	Gap (%)	Time (s)
10		5,899.09	5,899.08	0.00	3	5,663.20	5,663.19	0.00	4
20		7,668.25	7,668.25	0.00	86	6,929.67	6,929.66	0.00	68
50	0.0	9,138.16	8,951.15	1.72	12,244	9,787.23	9,670.35	1.04	9,331
60		9,037.69	8,846.88	1.85	17,466	9,249.41	9,053.22	1.76	10,918
80		8,167.79	8,135.00	0.40	17,378	9,112.15	8,361.53	8.07	18,002
10		6,415.70	6,415.70	0.00	2	5,946.66	5,946.66	0.00	2
20		8,192.11	8,192.11	0.00	30	7,421.07	7,421.04	0.00	39
50	0.4	10,113.57	10,063.35	0.42	9,495	11,054.12	11,008.31	0.39	5,965
60		10,001.02	9,938.68	0.58	9,480	11,276.80	11,135.88	1.18	10,322
80		9,175.01	9,175.00	0.00	2,686	9,774.39	9,503.67	2.96	13,538
10		6,675.81	6,675.81	0.00	2	6,123.92	6,123.92	0.00	1
20		8,508.09	8,508.09	0.00	28	7,752.52	7,752.49	0.00	11
50	0.8	10,713.28	10,687.90	0.21	6,433	12,169.65	12,128.92	0.33	4,934
60		10,606.47	10,593.59	0.11	5,917	12,528.87	12,446.54	0.64	10,914
80		9,629.30	9,629.30	0.00	82	10,229.79	10,208.01	0.22	9,073
10		7,283.24	7,283.24	0.00	2	6,214.57	6,214.57	0.00	1
20		8,745.59	8,745.58	0.00	25	7,971.69	7,971.69	0.00	10
50	1.0	11,352.35	11,347.26	0.04	6,010	13,816.07	13,816.07	0.00	1,508
60		11,453.80	11,453.30	0.00	2,767	15,869.19	15,632.76	1.66	10,890
80		10,905.28	10,905.28	0.00	27	11,092.13	11,091.23	0.01	143

Table 4 isolates the effect of different scenarios by aggregating the data over all risk thresholds θ and the number of customers $|N_c|$. Once again, we present results for two sets of instances. For all the cases when an optimal solution or very small gaps are obtained, we observe that scenarios with high starting points (i.e., HA and HS) yield lower costs.

Table 4: Average a priori results over all θ and all planning horizons

Instance		Instance set I				Instance set II			
$\lvert N_c \rvert$	Scenario	UB	LB	Gap (%)	Time (s)	UB	LB	Gap (%)	Time (s)
10	LG	6,924.33	6,924.33	0.00	2	6,077.44	6,077.43	0.00	2
20		7,833.30	7,833.30	0.00	7	7,375.52	7,375.52	0.00	9
50		10,787.38	10,758.60	0.27	6,854	12,747.57	12,747.47	0.00	3,052
60		10,829.26	10,792.19	0.36	6,268	14,106.38	13,986.00	0.98	11,302
80		10,291.74	10,291.74	0.00	3,895	10,517.39	10,415.24	1.16	4,581
10	LS	6,568.70	6,568.70	0.00	2	6,050.90	6,050.90	0.00	1
20		8,401.77	8,401.77	0.00	32	7,581.94	7,581.91	0.00	20
50		10,524.25	10,484.81	0.36	7,534	11,941.61	11,867.02	0.65	5,517
60		10,351.24	10,310.38	0.38	7,141	12,343.61	12,266.12	0.64	11,111
80		9,530.07	9,519.90	0.12	5,130	10,253.23	9,893.20	3.68	9,061
10	HA	6,422.29	6,422.29	0.00	2	5,912.29	5,912.29	0.00	2
20		8,124.70	8,124.69	0.00	49	7,412.26	7,412.22	0.00	33
50		9,953.52	9,876.15	0.71	8,974	11,034.79	11,020.34	0.12	6,621
60		9,870.11	9,815.28	0.50	6,629.54	10,609.81	10,433.98	1.49	9,247.28
80		8,206.19	8,180.92	0.35	6,766.15	10,196	10,195.30	0.00	2,192.82
10	HS	6,358.52	6,358.52	0.00	2	5,907.71	5,907.71	0.00	3
20		8,118.02	8,118.02	0.00	60	7,369.57	7,369.57	0.00	51
50		10,003.53	9,944.05	0.55	8,878	11,103.10	10,988.81	1.00	6,832
60		9,964.82	9,862.26	0.96	12,281	11,309.95	11,028.92	2.12	10,534
80		8,224.38	8,224.42	0.24	9,738	10,213.33	10,212.86	0.00	3,063

Our computational results indicate that all the small size instances (with 10 and 20 customers) are solved to optimality in the a priori phase. With respect to the risk threshold, a higher value of θ generally yields an easier instance. This is quite expected, given that a higher value of θ, fewer routing decisions must be taken. In this situation, the supplier takes a pessimistic approach and only plans deliveries as late as possible; if no capacity is left on the vehicle then a more expensive distribution method must be used. This also explains the high costs of these solutions.

Looking at Table 2, it may appear that instances with 80 customers are easier than those with 50 customers since the gaps are lower. The execution times in Tables 3 and 4 also gives the same impression. In fact, as previously explained, this is because large instances with 12 days do not include many customers. Therefore, the instances of these tables are not made up of the same number of days. Detailed results of the a priori phase are provided in Appendix A.

4.3.2. Disruption management analysis

Tables 5–8 show how in an iterative approach, a supplier optimizes the costs based on a given risk aversion level. We present a table for each length of the planning horizon, T= 5, 8, 10, 12. All columns containing only zero costs are omitted from the tables. The results are averaged over all scenarios and two sets of instances. These tables show the current day (column) and a future day (row): for example, the routing cost of current day 1 for the future day 5 indicates the expected routing cost to be paid on day 5, considering the available information on day 1. Therefore, an optimistic supplier estimates the cost to be paid on day 5 when the decisions are made on day 1 as $2,438$ for the routing cost and no expedited deliveries. As time unfolds, these decisions may need to be revised, and finally, the supplier ends up paying $2,437$ for the routing and a total of 66 for the expedited delivery cost.

These tables highlight how a recourse phase can significantly change the cost and take advantage of new information. They also show how a too optimistic supplier ($\theta = 0$) relies on the routing capacities to avoid paying the expedited delivery fees. The situation is different for a too pessimistic supplier ($\theta = 1$). Initially expensive expedited delivery is planned, but to avoid bumping costs, the supplier sticks to its a priori delivery plans.

Table 5: Averages of cost components over demand and bumping cost scenarios for instances with five days

		Future days						
		2	3	4	5	3	4	5
θ	Current day	Routing				Expedited delivery		
0.0	1	1,781	787	1,381	2,438	0	0	0
	2		787	1,400	2,437	0	0	66
	3			1,400	2,437		0	66
	4				2,437			66
0.4	1	1,148	1,475	1,311	2,061	19	191	519
	2		1,475	1,358	2,060	19	191	541
	3			1,358	2,059		191	548
	4				2,059			548
0.8	1	1,322	1,708	1,708	2,396	0	278	758
	2		1,708	1,708	2,395	0	253	815
	3			1,708	2,396		257	806
	4				2,396			806
1.0	1	1,312	1,701	1,945	2,349	0	731	1,795
	2		1,701	1,945	2,349	0	731	1,795
	3			1,945	2,349		731	1,795
	4				2,349			1,795

Table 6: Averages of cost components over demand and bumping cost scenarios for instances with eight days

		Future days								
θ	Current day	3	4	5	6	7	8	6	7	8
		Routing						Expedited delivery		
0.0	1	675	1,186	503	1,364	1,157	2,265	0	0	0
	2	777	1,188	503	1,364	1,157	2,265	0	0	0
	3		1,231	503	1,364	1,157	2,265	0	0	0
	4			503	1,364	1,157	2,265	0	0	0
	5				1,364	1,157	2,265	0	0	0
	6					1,157	2,265		0	0
	7						2,265			0
0.4	1	1,271	1,059	1,074	875	1,181	1,720	0	0	6
	2	1,276	1,107	1,111	875	1,181	1,720	0	0	21
	3		1,107	1,111	881	1,181	1,720	0	0	21
	4			1,111	881	1,181	1,720	6	0	21
	5				881	1,181	1,720	0	0	21
	6					1,181	1,720		0	21
	7						1,720			21
0.8	1	1,398	1,321	1,275	879	1,310	2,232	0	0	1
	2	1,398	1,316	1,306	879	1,310	2,232	0	0	1
	3		1,316	1,306	879	1,310	2,232	0	0	1
	4			1,306	879	1,310	2,232	0	0	1
	5				879	1,310	2,232	0	0	1
	6					1,310	2,232		0	1
	7						2,232			1
1.0	1	1,396	1,291	1,482	803	1,229	1,838	0	0	2
	2	1,396	1,291	1,482	803	1,229	1,838	0	0	2
	3		1,291	1,482	803	1,229	1,838	0	0	2
	4			1,482	803	1,229	1,838	0	0	2
	5				803	1,229	1,838	0	0	2
	6					1,229	1,838		0	2
	7						1,838			2

Table 7: Averages of cost components over demand and bumping cost scenarios for instances with 10 days

θ	Current day	Future days											
		3	4	5	6	7	8	9	10	5	6	9	10
										Expedited delivery			
0.0	**1**	945	570	1,046	1,082	234	1,013	1,021	2,052	0	0	0	0
	2	952	570	1,046	1,082	234	1,013	1,021	2,052	0	0	0	0
	3		570	1,046	1,082	234	1,013	1,021	2,052	0	0	0	0
	4			1,046	1,082	234	1,013	1,021	2,052	0	0	0	0
	5				1,082	234	1,013	1,021	2,052		0	0	0
	6					234	1,013	1,021	2,052			0	0
	7						1,013	1,021	2,052			0	0
	8							1,021	2,052			0	0
	9								2,052				0
0.4	**1**	1,447	1,028	1,078	1,012	850	1,604	1,015	2,051	0	16	12	3
	2	1,452	1,032	1,096	1,011	850	1,604	1,015	2,051	12	19	12	0
	3		1,032	1,096	1,011	850	1,604	1,015	2,051	12	19	12	0
	4			1,096	1,011	850	1,604	1,015	2,051	12	19	12	0
	5				1,011	850	1,604	1,015	2,051		19	12	0
	6					850	1,604	1,015	2,051			12	0
	7						1,604	1,015	2,051			12	0
	8							1,015	2,051			12	0
	9								2,051				0
0.8	**1**	1,451	1,134	891	1,124	433	920	1,074	2,041	0	0	0	0
	2	1,451	1,152	891	1,124	433	920	1,074	2,041	0	0	0	0
	3		1,152	891	1,124	433	920	1,074	2,041	0	0	0	0
	4			891	1,124	433	920	1,074	2,041	0	0	0	0
	5				1,124	433	920	1,074	2,041		0	0	0
	6					433	920	1,074	2,041			0	0
	7						920	1,074	2,041			0	0
	8							1,074	2,041			0	0
	9								2,041				0
1.0	**1**	1,451	1,142	1,137	923	845	780	1,053	2,059	0	0	0	0
	2	1,451	1,142	1,137	923	845	780	1,053	2,059	0	0	0	0
	3		1,142	1,137	923	845	780	1,053	2,059	0	0	0	0
	4			1,137	923	845	780	1,053	2,059	0	0	0	0
	5				923	845	780	1,053	2,059		0	0	0
	6					845	780	1,053	2,059			0	0
	7						780	1,053	2,059			0	0
	8							1,053	2,059			0	0
	9								2,059				0

Table 8: Averages of cost components over demand and bumping cost scenarios for instances with 12 days

		Future days														
θ	**Current day**	2	3	4	5	6	7	8	9	10	11	12	4	5	8	12
		Routing											Expedited delivery			
0.0	1	900	320	636	566	530	783	394	559	665	658	1,545	0	0	0	0
	2		320	681	566	530	783	394	559	665	658	1,545	0	0	0	0
	3			681	566	530	783	394	559	665	658	1,545	0	0	0	0
	4				566	647	783	423	559	665	658	1,545		0	0	0
	5					647	783	423	559	665	658	1,545			0	0
	6						783	423	559	665	658	1,545			0	0
	7							423	559	665	658	1,545			0	0
	8								559	665	658	1,545			0	0
	9									665	658	1,545				0
	10										658	1,545				0
	11											1,545				0
0.4	1	591	923	775	776	455	809	437	622	620	669	1,535	24	0	12	25
	2		923	870	806	455	809	437	622	620	669	1,536	24	0	0	0
	3			869	833	467	809	447	622	620	669	1,536	0	0	0	0
	4				839	467	809	447	622	620	669	1,536		0	0	0
	5					467	809	447	622	620	669	1,536			0	0
	6						809	447	622	620	669	1,536			0	0
	7						0	447	622	620	669	1,536			0	0
	8								622	620	669	1,536				0
	9									620	669	1,536				0
	10										669	1,536				0
	11											1,536				0
0.8	1	571	878	818	916	586	717	402	700	586	684	1,544	0	0	0	0
	2		878	818	916	586	717	402	700	586	684	1,544	0	0	0	0
	3			818	949	614	717	402	700	586	684	1,544	0	0	0	21
	4				949	614	717	402	700	586	684	1,544		10	0	10
	5					614	717	402	700	586	684	1,544			0	10
	6						717	402	700	586	684	1,544			0	10
	7							402	700	586	684	1,544			0	10
	8								700	586	684	1,544				10
	9									586	684	1,544				10
	10										684	1,544				10
	11											1,544				10
1.0	1	570	840	868	1,085	710	623	481	648	677	642	1,545	0	0	0	0
	2		840	868	1,085	710	623	481	648	677	642	1,545	0	0	0	0
	3			868	1,085	710	623	481	648	677	642	1,545	0	0	0	0
	4				1,085	710	623	481	648	677	642	1,545		0	0	0
	5					710	623	481	648	677	642	1,545			0	0
	6						623	481	648	677	642	1,545			0	0
	7							481	648	677	642	1,545			0	0
	8								648	677	642	1,545			0	0
	9									677	642	1,545				0
	10										642	1,545				0
	11											1,545				0

4.3.3. Cost components analysis

Table 9 shows the average deviation of the realized cost (RC) with respect to the a priori cost (AC) obtained in the a priori phase. The deviations are calculated as $100 \times \frac{(\mathrm{RC} - \mathrm{AC})}{\mathrm{AC}}$ where RC is the total cost, including routing, bumping cost, and the expedited delivery costs obtained at the end of the planning horizon. Table 9 provides averages over all instances and for each length T of the planning horizon. The averages are presented for the four availability scenarios and four θ values. The shade of the cell colors indicates the degree of deviation between the a priori and the realized costs: the darker the shade, the higher the percentage of this deviation. As can be observed from the table, irrespective of the bumping cost, a pessimistic approach leads to close to zero deviations. Basically, under this approach, the total cost paid is always the same as the estimated cost in the a priori phase. This stability, of course, comes at a cost. It also should be noted that, interestingly, the results show that the highest deviations in the initial plans are observed for the cases with $\theta = 0.4$; in these plans the main source of the deviation is the bumping costs paid for moving deliveries to other days.

Table 9: Average percentage increase of the realized cost value to the a priori cost

		Low bumping				**High bumping**			
T	**Scenario\\θ**	**0.0**	**0.4**	**0.8**	**1.0**	**0.0**	**0.4**	**0.8**	**1.0**
5	LG	3.34	0.04	0.04	0.04	5.82	0.04	0.04	0.04
	LS	3.73	0.27	0.48	0.04	6.57	0.27	1.60	0.04
	HA	49.83	57.75	0.56	0.04	5.94	200.10	0.56	0.04
	HS	2.57	64.73	1.78	0.04	5.00	230.12	3.97	0.04
8	LG	5.45	0.06	0.06	0.06	108.78	0.25	0.06	0.06
	LS	7.63	0.71	0.06	0.06	25.83	2.60	0.06	0.06
	HA	7.65	57.75	2.26	0.06	28.78	200.10	7.76	0.06
	HS	7.64	38.80	4.34	0.43	25.78	139.76	15.20	0.03
10	LG	0.83	0.06	0.06	0.06	2.96	0.06	0.06	0.06
	LS	0.83	0.06	0.06	0.06	3.28	0.09	0.06	0.06
	HA	0.78	20.21	0.07	0.06	2.89	69.54	0.07	0.07
	HS	0.83	33.45	2.56	0.06	2.94	113.49	6.53	0.06
12	LG	7.49	0.06	0.06	0.06	22.33	0.06	0.06	0.06
	LS	7.49	0.06	0.06	0.06	22.33	0.06	0.06	0.06
	HA	7.49	19.74	5.36	0.06	22.33	55.63	15.65	0.06
	HS	7.49	59.20	4.53	0.06	22.33	206.55	9.78	0.06

For each scenario and each value of θ, Tables 10 and 11 show the distribution of the routing, the bumping cost, and the expedited delivery cost. As before, the table is organized for each planning horizon, with respect to the four availability scenarios and four values for θ. Generally, the main contributor to the total cost is the routing cost. However, depending on the scenario and the risk threshold, the proportions change. When the supplier is an optimist, the routing, bumping, and expedited delivery costs are the main contributors. The highest proportion of the bumping cost is paid for $\theta = 0.4$ and scenario HS. These tables confirm that a pessimistic supplier ($\theta = 1.0$) obtains more stable solutions which come at the expense of more costly expedited deliveries.

Table 10: Final cost components distribution (averages over all instances with low bumping cost)

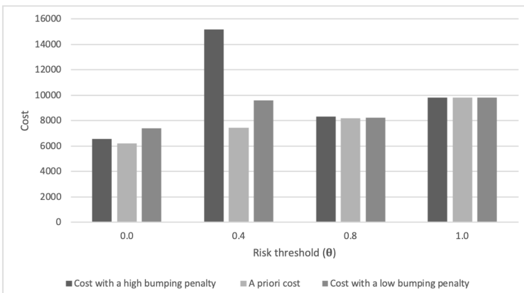
		Routing	Bumping	Expedited	Routing	Bumping	Expedited	Routing	Bumping	Expedited	Routing	Bumping	Expedited
T	Scenario\\θ		0.0			0.4			0.8			1.0	
5	LG	98.23%	0.85%	0.92%	80.91%	0.00%	19.09%	80.91%	0.00%	19.09%	80.91%	0.00%	19.09%
	LS	98.23%	0.85%	0.92%	99.79%	0.00%	0.21%	90.82%	0.36%	8.82%	80.91%	0.00%	19.09%
	HA	91.11%	7.97%	0.92%	71.38%	27.55%	1.07%	98.08%	0.00%	1.92%	80.91%	0.00%	19.09%
	HS	98.23%	0.85%	0.92%	67.96%	31.01%	1.03%	98.61%	1.13%	0.26%	81.38%	0.00%	18.62%
8	LG	91.19%	8.81%	0.00%	100.00%	0.00%	0.00%	99.99%	0.00%	0.01%	99.99%	0.00%	0.01%
	LS	90.74%	9.26%	0.00%	99.55%	0.45%	0.00%	100.00%	0.00%	0.00%	99.99%	0.00%	0.01%
	HA	90.22%	9.78%	0.00%	70.70%	28.85%	0.45%	98.76%	1.24%	0.00%	99.99%	0.00%	0.01%
	HS	90.25%	9.75%	0.00%	73.96%	26.02%	0.02%	97.19%	2.80%	0.01%	99.99%	0.00%	0.01%
10	LG	83.75%	16.25%	0.00%	99.95%	0.00%	0.05%	99.95%	0.00%	0.05%	99.95%	0.00%	0.05%
	LS	87.86%	12.14%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%	99.95%	0.00%	0.05%
	HA	88.07%	11.93%	0.00%	80.84%	18.89%	0.27%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	HS	88.01%	11.99%	0.00%	76.63%	23.21%	0.15%	99.14%	0.86%	0.00%	100.00%	0.00%	0.00%
12	LG	92.14%	7.86%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	LS	92.10%	7.90%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	HA	91.91%	8.09%	0.00%	83.54%	16.46%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	HS	96.35%	3.65%	0.00%	67.62%	32.38%	0.00%	84.88%	15.12%	0.00%	100.00%	0.00%	0.00%
Average		**91.77%**	**8.00%**	**0.23%**	**85.80%**	**12.80%**	**1.40%**	**96.77%**	**1.34%**	**1.89%**	**95.25%**	**0.00%**	**4.75%**

Table 11: Realized cost components distribution (averages over all instances with high bumping cost)

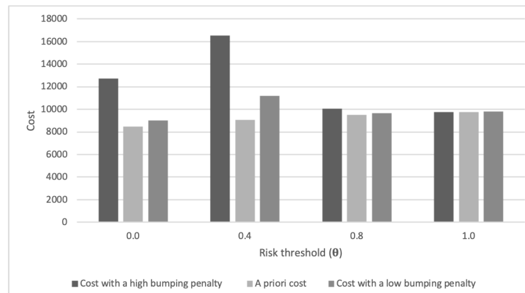
		Routing	Bumping	Expedited	Routing	Bumping	Expedited	Routing	Bumping	Expedited	Routing	Bumping	Expedited
T	Scenario\\θ		0.0			0.4			0.8			1.0	
5	LG	96.37%	2.71%	0.92%	80.91%	0.00%	19.09%	80.91%	0.00%	19.09%	80.91%	0.00%	19.09%
	LS	98.11%	0.00%	1.89%	99.79%	0.00%	0.21%	86.80%	1.28%	11.92%	80.91%	0.00%	19.09%
	HA	96.37%	2.71%	0.92%	44.02%	53.65%	2.33%	98.08%	0.00%	1.92%	80.91%	0.00%	19.09%
	HS	96.37%	2.71%	0.92%	41.79%	56.41%	1.80%	97.14%	1.91%	0.95%	81.38%	0.00%	18.62%
8	LG	75.84%	24.16%	0.00%	99.99%	0.00%	0.01%	99.99%	0.00%	0.01%	99.99%	0.00%	0.01%
	LS	81.17%	18.83%	0.00%	98.40%	1.60%	0.00%	99.83%	0.00%	0.17%	99.99%	0.00%	0.01%
	HA	81.01%	18.99%	0.00%	44.48%	54.79%	0.73%	95.01%	4.98%	0.01%	99.99%	0.00%	0.01%
	HS	80.81%	19.19%	0.00%	46.97%	53.01%	0.01%	91.38%	8.61%	0.01%	99.82%	0.00%	0.18%
10	LG	82.71%	17.29%	0.00%	100.00%	0.00%	0.00%	99.94%	0.00%	0.06%	99.94%	0.00%	0.06%
	LS	82.67%	17.33%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%	99.94%	0.00%	0.06%
	HA	72.22%	27.78%	0.00%	60.74%	39.07%	0.19%	99.34%	0.66%	0.00%	100.00%	0.00%	0.00%
	HS	80.38%	19.62%	0.00%	70.92%	28.45%	0.63%	96.70%	3.30%	0.00%	100.00%	0.00%	0.00%
12	LG	76.17%	23.83%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	LS	82.02%	17.98%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%	100.00%	0.00%	0.00%
	HA	75.53%	24.47%	0.00%	66.47%	33.53%	0.00%	91.24%	8.45%	0.31%	100.00%	0.00%	0.00%
	HS	83.26%	16.74%	0.00%	40.50%	59.50%	0.00%	94.12%	5.04%	0.84%	100.00%	0.00%	0.00%
Average		**83.81%**	**15.90%**	**0.29%**	**74.69%**	**23.75%**	**1.56%**	**95.65%**	**2.14%**	**2.21%**	**95.24%**	**0.00%**	**4.76%**

4.4. Analysis of the bumping costs

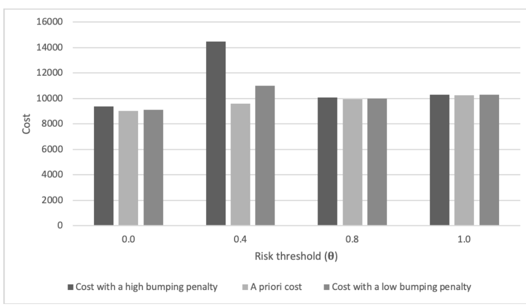
Finally, we provide a general overview of the impact of the bumping cost. In Figure 2, we present four figures, each representing the length of the planning horizon. Using average values over all instances and for all scenarios, we compare the cost obtained in the a priori phase and reoptimizations with low and high bumping costs. As shown in this figure, a pessimistic approach is more costly, but as there is almost no need to change the plans by the end of the planning horizon, these decisions always yield the same cost. However, a risk seeking approach yields a lower a priori cost but due to more deviations from the initial plans, the final cost is higher.



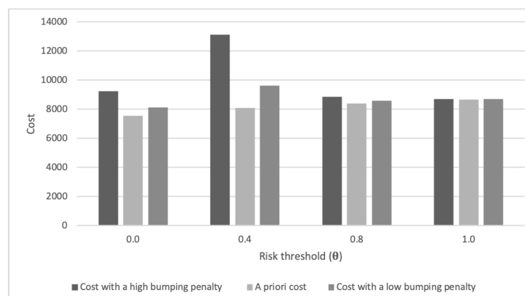
(a) Five days



(b) Eight days



(c) 10 days



(d) 12 days

Figure 2: Comparing the average costs from a priori, high and low bumping cases

5. Conclusions

We have introduced and modeled a dynamic routing problem with due dates and stochastic release dates (DRPDSR) in which deliveries may occur between release and due dates. The dynamism comes from the fact that customers place their orders over time. The stochasticity in the DRPDSR stems from product supply availability which is not known in advance.

To deal with uncertainty, we have introduced a binary parameter that combines stochastic information about the products' availability with the supplier's risk aversion level. To our knowledge, this notion is new in the context of stochastic vehicle routing. It enabled us to formulate the DRPDRS as a deterministic-equivalent a priori mathematical program which is solved on the first day of the planning horizon and its solution is iteratively updated every remaining day of the planning horizon by solving a recourse model. Since it is probable that not all expectations and plans for the future are realized, the goal of the recourse is to adapt the a priori plans to the changes so as to yield the least cost solution. We have proposed an exact branch-and-cut algorithm for this problem.

In extensive computational experiments, we have isolated the effect of bumping costs, risk aversion levels, and several shapes of the realization curves of the unknown product availability. The results show how different combinations of availability curve scenarios and risk aversion levels result in a different total cost. We have shown how a pessimistic supplier obtains more stable a priori solutions but more costly plans. Moreover, different risk aversion levels can lead to less costly operations, which come at the expense of more changes as new information becomes available.

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Appendix A. Detailed results from the a priori phase

First, we categorize in Table A.12 the instances and the results based on the thresholds, and then the results categorized based on scenarios are provided in Table A.13. The results for two sets of instances are shown in separate columns, and for each of them, the upper bound (UB),

Table A.12: Average a priori results over all scenarios

	Products	Customers	θ	Instance set I UB	Instance set I LB	Instance set I Gap (%)	Instance set I Time (s)	Instance set II UB	Instance set II LB	Instance set II Gap (%)	Instance set II Time (s)
Five planning days	15	10	0.0	4,162.05	4,162.05	0.00	0	3,498.01	3,498.01	0.00	0
	25	20		4,541.29	4,541.29	0.00	1	5,242.17	5,242.17	0.00	2
	55	50		7,018.69	7,018.69	0.00	728	6,568.70	6,568.70	0.00	559
	65	60		7,243.87	7,204.17	0.55	16,929	7,354.96	7,354.86	0.00	3,836
	85	80		8,167.79	8,135.00	0.40	17,378	11,356.35	8,332.49	21.55	18,002
	15	10	0.4	5,127.04	5,127.04	0.00	0	3,780.24	3,780.24	0.00	0
	25	20		4,889.27	4,889.27	0.00	1	5,693.15	5,693.02	0.00	1
	55	50		8,108.66	8,108.48	0.00	298	9,139.75	9,139.72	0.00	166
	65	60		8,280.65	8,269.66	0.15	6,756	10,337.76	10,337.19	0.00	2,642
	85	80		9,175.01	9,175.00	0.00	2,686	9,774.39	9,503.67	2.96	13,538
	15	10	0.8	5,740.04	5,740.04	0.00	0	4,015.23	4,015.23	0.00	0
	25	20		4,944.07	4,944.07	0.00	0	5,919.89	5,919.89	0.00	1
	55	50		9,078.18	9,078.10	0.00	215	11,015.52	11,015.32	0.00	329
	65	60		8,843.38	8,843.38	0.00	833	12,241.61	12,238.11	0.02	5,901
	85	80		9,629.30	9,629.30	0.00	82	10,229.79	10,208.01	0.22	9,073
	15	10	1.0	7,480.10	7,480.10	0.00	0	4,028.98	4,028.98	0.00	0
	25	20		5,048.33	5,048.33	0.00	1	6,043.10	6,043.10	0.00	0
	55	50		10,066.60	10,066.60	0.00	4	15,404.90	15,404.90	0.00	8
	65	60		10,188.90	10,188.90	0.00	7	18,032.30	18,030.60	0.01	3,549
	85	80		10,905.28	10,905.28	0.00	27	11,092.13	11,091.23	0.01	143
Eight planning days	15	10	0.0	5,879.06	5,879.06	0.00	3	6,400.41	6,400.41	0.00	1
	25	20		7,580.09	7,580.09	0.00	25	5,950.35	5,950.35	0.00	17
	55	50		9,236.29	9,145.59	0.98	18,001	10,875.40	10,524.75	3.13	15,971
	65	60		10,831.50	10,489.60	3.16	18,003	11,143.85	10,751.58	3.52	18,001
	15	10	0.4	6,120.95	6,120.95	0.00	1	6,855.87	6,855.87	0.00	1
	25	20		7,872.35	7,872.35	0.00	8	6,262.96	6,262.96	0.00	9
	55	50		10,096.20	10,081.41	0.16	10,209	11,506.15	11,368.75	1.17	14,350
	65	60		11,721.40	11,607.70	1.02	12,204	12,215.85	11,934.58	2.36	18,002
	15	10	0.8	6,315.15	6,315.15	0.00	1	7,064.11	7,064.11	0.00	1
	25	20		8,152.34	8,152.34	0.00	3	6,505.95	6,505.95	0.00	4
	55	50		10,515.60	10,515.58	0.00	2,389	12,363.78	12,242.23	0.99	11,18
	65	60		12,369.55	12,343.80	0.21	11,001	12,816.13	12,654.98	1.27	15,927
	15	10	1.0	6,411.88	6,411.88	0.00	1	7,118.85	7,118.85	0.00	0
	25	20		8,289.32	8,289.29	0.00	2	6,691.55	6,691.55	0.00	1
	55	50		10,943.10	10,943.10	0.00	541	12,532.90	12,532.90	0.00	1,381
	65	60		12,718.70	12,717.70	0.01	5,526	13,706.08	13,234.93	3.31	18,002
10 planning days	15	10	0.0	6,640.21	6,640.21	0.00	3	6,300.08	6,300.05	0.00	4
	25	20		9,685.58	9,685.58	0.00	81	8,561.02	8,561.02	0.00	78
	55	50		11,159.50	10,689.18	4.18	18,002	11,917.60	11,917.60	0.00	11,461
	15	10	0.4	7,031.60	7,031.60	0.00	1	6,488.42	6,488.42	0.00	2
	25	20		10,441.13	10,441.13	0.00	18	8,994.19	8,994.19	0.00	32
	55	50		12,135.85	12,000.15	1.10	17,977	12,516.45	12,516.45	0.00	3,380
	15	10	0.8	7,177.16	7,177.16	0.00	1	6,549.94	6,549.94	0.00	1
	25	20		11,000.73	11,000.73	0.00	19	9,276.72	9,276.61	0.00	14
	55	50		12,546.05	12,470.03	0.63	16,694	13,129.65	13,129.20	0.00	3,294
	15	10	1.0	7,518.07	7,518.07	0.00	1	6,633.23	6,633.23	0.00	2
	25	20		11,152.70	11,152.70	0.00	12	9,719.83	9,719.83	0.00	9
	55	50		13,047.35	13,032.08	0.12	17,484	13,510.40	13,510.40	0.00	3,134
12 planning days	15	10	0.0	6,915.02	6,915.02	0.00	4	6,454.30	6,454.30	0.00	11
	25	20		8,866.04	8,866.04	0.00	238	7,965.15	7,965.09	0.00	176
	15	10	0.4	7,383.22	7,383.22	0.00	5	6,662.10	6,662.10	0.00	5
	25	20		9,565.69	9,565.69	0.00	94	8,733.98	8,733.98	0.00	115
	15	10	0.8	7,470.90	7,470.90	0.00	4	6,866.38	6,866.38	0.00	4
	25	20		9,935.22	9,935.22	0.00	90	9,307.53	9,307.53	0.00	26
	15	10	1.0	7,722.92	7,722.92	0.00	6	7,077.21	7,077.21	0.00	1
	25	20		10,492.00	10,492.00	0.00	87	9,432.27	9,432.27	0.00	30

lower bound (LB), the percentage of the gap (Gap), and the execution time (Time in seconds) are presented.

The results for scenarios (averaged over different values of θ), presented in Table A.13, indicate that scenarios LS and HS are the most difficult ones for most instances, followed by scenario HA. Recall that in these two scenarios, the product availability starts with a higher probability (interval $[0.40, 0.50]$ for scenarios HA and HS, as opposed to $[0.05, 0.10]$ for scenarios LG and LS), meaning that more routing decisions can be made earlier in the planning horizon. In scenarios LG and LS, more deliveries are planned for the later days, when distribution capacity becomes limited and recourse to expedited delivery is more frequently used.

Table A.13: Average a priori results over all thresholds

				Instance set I				Instance set II			
	Products	Customers	Scenario	UB	LB	Gap (%)	Time (s)	UB	LB	Gap (%)	Time (s)
Five planning days	15	10	LG	6,650.59	6,650.59	0.00	0	3,896.24	3,896.24	0.00	0
			LS	5,467.04	5,467.04	0.00	0	3,896.23	3,896.23	0.00	0
			HA	5,355.79	5,355.79	0.00	0	3,767.24	3,767.24	0.00	0
			HS	5,035.80	5,035.80	0.00	0	3,762.75	3,762.75	0.00	0
	25	20	LG	4,921.57	4,921.57	0.00	1	5,842.87	5,842.87	0.00	1
			LS	4,921.57	4,921.57	0.00	1	5,780.38	5,780.25	0.00	1
			HA	4,784.28	4,784.28	0.00	1	5,634.66	5,634.66	0.00	2
			HS	4,795.54	4,795.54	0.00	1	5,640.41	5,640.41	0.00	1
	55	50	LG	9,304.62	9,304.62	0.00	110	13,195.85	13,195.85	0.00	118
			LS	8,752.39	8,752.21	0.00	317	10,813.51	10,813.42	0.00	259
			HA	8,056.66	8,056.66	0.00	197	9,087.50	9,087.34	0.00	492
			HS	8,158.46	8,158.38	0.00	622	9,032.03	9,032.03	0.00	193
	65	60	LG	9,452.64	9,445.18	0.10	4,507	15,362.97	15,361.62	0.01	4,603
			LS	8,655.59	8,655.59	0.00	3,513	12,194.56	12,190.99	0.02	6,298
			HA	8,206.19	8,180.92	0.35	6,766	10,195.76	10,195.30	0.00	2,193
			HS	8,242.38	8,224.42	0.24	9,738	10,213.33	10,212.86	0.00	3,063
	85	80	LG	10,291.74	10,291.74	0.00	3,895	10,517.39	10,415.24	1.16	4,581
			LS	9,530.07	9,519.90	0.12	5,130	10,253.23	9,893.20	3.68	9,061
			HA	9,071.28	9,064.93	0.08	5,249	9,731.90	9,425.90	3.37	13,524
			HS	8,984.04	8,973.17	0.13	5,878	9,718.90	9,433.55	3.15	11,864
Eight planning days	15	10	LG	6,278.68	6,278.68	0.00	1	6,963.61	6,963.61	0.00	0
			LS	6,181.94	6,181.94	0.00	1	6,931.12	6,931.12	0.00	0
			HA	6,070.48	6,070.48	0.00	2	6,739.88	6,739.88	0.00	1
			HS	6,195.94	6,195.94	0.00	2	6,804.63	6,804.63	0.00	1
	25	20	LG	8,113.33	8,113.33	0.00	7	6,506.25	6,506.25	0.00	5
			LS	7,983.31	7,983.31	0.00	8	6,373.45	6,373.45	0.00	8
			HA	7,847.60	7,847.57	0.00	11	6,232.19	6,232.19	0.00	6
			HS	7,949.86	7,949.86	0.00	12	6,298.93	6,298.93	0.00	11
	55	50	LG	10,515.90	10,504.72	0.12	4,945	11,997.93	11,997.63	0.00	4,698
			LS	10,335.16	10,317.64	0.19	6,609	12,059.35	11,835.85	1.95	11,102
			HA	9,962.45	9,921.47	0.44	10,150	11,488.93	11,445.73	0.36	13,267
			HS	9,977.68	9,941.86	0.39	9,437	11,732.03	11,389.43	2.99	13,814
	65	60	LG	12,205.88	12,139.20	0.61	8,029	12,849.80	12,610.38	1.95	18,001
			LS	12,046.90	11,965.18	0.75	10,768	12,492.65	12,341.25	1.26	15,924
			HA	11,683.55	11,573.90	1.00	13,062	12,132.13	11,780.63	2.99	18,002
			HS	11,687.25	11,500.10	1.68	14,823	12,406.58	11,844.98	4.24	18,004
10 planning days	15	10	LG	7,247.12	7,247.12	0.00	2	6,549.94	6,549.92	0.00	3
			LS	7,137.63	7,137.63	0.00	2	6,540.94	6,540.94	0.00	2
			HA	7,003.65	7,003.65	0.00	2	6,497.43	6,497.43	0.00	2
			HS	6,978.64	6,978.64	0.00	2	6,383.37	6,383.37	0.00	3
	25	20	LG	10,812.02	10,812.02	0.00	19	9,430.13	9,430.13	0.00	22
			LS	10,736.27	10,736.27	0.00	21	9,180.20	9,180.20	0.00	20
			HA	10,401.49	10,401.49	0.00	41	9,026.96	9,026.85	0.00	35
			HS	10,330.35	10,330.35	0.00	56	8,914.47	8,914.47	0.00	61
	55	50	LG	12,541.63	12,466.45	0.68	15,507	13,048.93	13,048.93	0.00	4,341
			LS	12,485.20	12,384.60	0.89	15,678	12,951.98	12,951.80	0.00	5,191
			HA	11,841.45	11,650.33	1.68	16,575	12,527.95	12,527.95	0.00	6,102
			HS	11,874.45	11,731.90	1.27	16,574	12,545.25	12,544.98	0.00	6,488
12 planning days	15	10	LG	7520.95	7520.95	0.00	5	6899.96	6899.96	0.00	3
			LS	7488.20	7488.20	0.00	5	6835.31	6835.31	0.00	3
			HA	7259.22	7259.22	0.00	4	6644.61	6644.61	0.00	6
			HS	7223.69	7223.69	0.00	4	6680.11	6680.11	0.00	7
	25	20	LG	10031.26	10031.26	0.00	82	9065.47	9065.47	0.00	67
			LS	9965.94	9965.94	0.00	95	8993.74	8993.74	0.00	49
			HA	9465.44	9465.44	0.00	143	8755.23	8755.17	0.00	88
			HS	9396.31	9396.31	0.00	140	8624.49	8624.49	0.00	118