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Synchronization in Two-Echelon Distribution Systems: Models, Algorithms, and Sensitivity Analyses

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Abstract. We address the problem of modeling a two-echelon location-routing system under tight synchronization constraints, in addition to several other interacting attributes. Prompted in particular by city-logistics applications, the problem settings we address include time-dependent multicommodity demand, time windows, limited storage capacity at intermediate facilities, and synchronization at these facilities of the fleets operating on different echelons. The problem requires the selection of facilities at both levels, the allocation of suppliers to platforms and of customers to satellites, and the routing and scheduling of vehicles at each echelon to deliver the freight from platforms to customers, through satellite facilities. Moreover, the limited storage capacity of the shared facilities, the satellites, requires the scheduling of the vehicle and demand routes, i.e., departure times from the platforms and satellites, and the synchronization of vehicle routes at satellites for efficient transshipment operations. We introduce the problem setting, present and compare two mixed-integer programming formulations as well as a dynamic time discretization scheme for the problem, and perform thorough analyses to assess the impact of the different attributes and of the synchronization itself on the system operation and the algorithm performance.

Keywords: Time-dependent two-echelon location-routing; synchronization; dynamic discretization discovery; city logistics

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1 Introduction

Freight distribution and logistics systems are essential to support the economic and social development of cities. At the same time, freight transportation activities also negatively impact urban life in terms of congestion, noise, and pollution. Multi-tier, especially *two-tier city logistics* concepts have become a mean to promote efficient and sustainable freight transportation in terms of economic, social, and environmental viability. In this context, the *two-echelon location routing problem (*2E-LRP*)* is one of the methodologies of choice to model and plan such two-tier systems through the integration of facility location and vehicle routing decisions. With urbanization expanding, operational models being restructured and e-commerce accelerating immediate delivery expectations, the interest of academics and practitioners has been shifting toward more realistic problem settings, mostly driven by the major opportunity that these attributes represent for the logistics industry in the upcoming years (Crainic and Montreuil, 2016). Nevertheless, the literature on more realistic 2E-LRPs characterized by several interacting attributes is still very limited (Dellaert et al., 2019). Particular developments are required in the 2E-LRP field, especially in relation to time-dependent components of operations, multicommody non-substituable demand, synchronization of the carriers involved, and the modeling and algorithmic challenges these considerations imply.

We address a two-echelon location-routing problem considering multiple interacting attributes. Prompted in particular by *city logistics* applications, the problem settings we address include time-dependent multicommodity demand, time windows, limited storage capacity (if at all) at intermediate facilities, and synchronization at these intermediary facilities of the fleets operating on different echelons. Our study aims to deepen the understanding of the effects of the integrated treatment of diverse attributes on both location and routing decisions, in particular under the presence of tight synchronization constraints and timing features. Incorporating temporal considerations, however, is challenging in multi-attribute problem settings, due to the level of detail that these attributes require (Crainic et al., 2009). From a modeling perspective, time-space networks is a widely-known modeling technique to efficiently capture and handle temporal information, in which physical locations are duplicated at discrete points in time defined by a given time interval (Ford and Fulkerson, 1962). Yet, while a time-space network yields to a static flow representation of the time-dependent problem, it suffers from dimensionality issues of the underlying network, which can become prohibitively large due to the time expansion.

Our goal with with paper is to provide methodology to respond to these modeling and algorithmic challenges and, thus, to contribute toward filling the gaps in the literature. The paper introduces the *Two-Echelon Multi-Attribute Location-Routing Problem with fleet Synchronization at intermediate facilities* (*2E-MALRPS*) proposing a unified view on the attributes considered, most notably, time-dependent multicommodity demands, fleet synchronization, and customers' time windows. We present and compare two mixed-integer programming (MIP) formulations, notably a compact and a time-space formulation for the 2E-MALRPS. An exact solution framework for the 2E-MALRPS based on a dynamic discretization scheme is also proposed to address the scalability issues provoked by the time-space formulation. In the computational study, we analyze the cost sensitivity, the infrastructure usage and the importance of fleet synchronization under time-sensitive distribution systems to derive managerial insights. It is worthwhile to mention that while we describe and develop our study under city-logistics guidelines, this research can also be of particular relevance for other freight distribution systems arising in contexts beyond urban distribution (Crainic et al., 2009).

The remaining part of the paper is organized as follows. The problem definition is given in Section 2 and an overview of related literature in Section 3. Section 4 is dedicated to the modeling of the two-echelon system considered, while Section 5 discusses the time-dependency characteristics of the problem and the modeling of time used in our formulations. Two mathematical formulations are proposed and described in Sections 6 and 7. Section 8 describes the solution framework we developed. Computational results are presented and analyzed in Section 9. We summarize our work and propose future research directions in Section 10.

2 Problem Setting

We develop our study on a two-echelon location-routing problem characterized by several interacting attributes. The system is composed of sets of suppliers (demand origins), platforms (primary facilities), satellites (the intermediate facilities), and customers (demand destinations), as well as two garages, each holding the fleet of capacitated vehicles operating at a specific echelon. *Demand* is defined between suppliers and customers, each individual demand being characterized by origin, destination, volume, availability time at each platform facility, and due time window at destination. As depicted in Figure (1), each origin-destination (OD) demand has to be assigned to an existing open platform, where it will be possibly consolidated with other commodities and moved by a first-echelon vehicle to a sequence of exiting satellites where demand flows are transferred. Loads delivered at satellites are transshipped and consolidated into second-echelon vehicles, which will perform the deliveries to the final destinations. Routes from each echelon are assumed to start and end at garages. Yet, for simplifying the presentation of the system, garage nodes are not displayed in Figure (1) or in any of the following illustrations of the system throughout this paper.

Platforms are large-sized infrastructures responsible for the storage, sorting and consolidation of the inbound freight provided by supply points through various modes of transportation. *Satellites*, on the other hand, are medium- to small-sized facilities located within the city limits, and are responsible for the second and last leg of transportation to the customers. From a physical point of view, satellites are multimodal trans-dock infrastructures with reduced or null storage capacity (for instance, cross-docking stations, parking lots) to enable transshipment operations. Freight delivery is performed by two independent fleets of homogeneous and limited-capacity vehicles, capable to transport any kind of demand indistinguishably. Vehicles are assumed to be available at strategically located vehicle garages for each echelon, where vehicles start and end their routes.

The problem requires the selection of facilities at both levels, the allocation of suppliers to platforms and of customers to satellites, as well as the routing and scheduling of vehicles at each echelon to deliver the freight from platforms to customers, through satellite facilities. Vehicles exchanging freight must be synchronized at a given satellite to enable efficient transshipment operations, considering the time dependency on demand and the null storage capacity at intermediate facilities. Vehicle routes and the demand freight itineraries according to which they arrive at, wait, and depart from each location in the system must be determined within

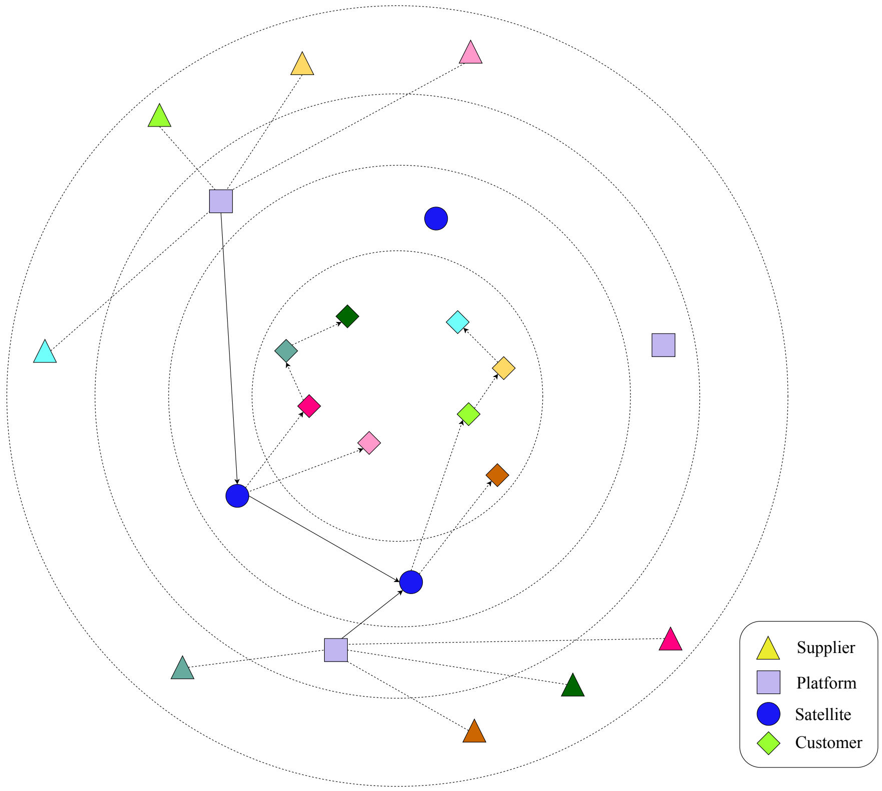


Figure 1: Two-echelon distribution system topology

the time restrictions imposed by each OD demand at platforms and customers as well as by the synchronization at satellites. The costs of transportation are assumed to be equal to the travel time to reach locations in the system, while, to simplify the presentation but without loss of generality, waiting times at locations do not yield additional costs. The main objective of the resulting 2E-MALRPS is to minimize the total cost of the system, composed of the cost of selecting/opening facilities at both levels and the transportation costs, while satisfying the demand and the capacities of the system elements.

3 Literature Review

The 2E-MALRPS is a two-echelon location routing problem, which thus belongs to the *Location-Routing Problem* class (*LRP*). LRPs concern the selection of the locations of urban freight infrastructure (receiving, handling, storing, distributing and dispatching terminals), and the design of vehicle routes to support the associated transportation operations. *Two-echelon* LRPs (*2E-LRP*) are LRPs involving location decisions of two types, “two levels” of facilities.

This section aims to situate the 2E-MALRPS within the relevant literature on the 2E-LRP and LRP, pointing out the gaps in knowledge with respect to time dependencies, time windows, origin-destination demand, and fleet synchronization. A brief discussion on time-space formulations is also provided, focusing on dynamic discretization schemes used as solution frameworks. For an overview of the different problems in two-echelon distribution systems and location routing problems considering attributes that are out of the scope of this work, we refer the interested reader to recent surveys by Prodhon and Prins (2014), Lopes et al. (2013), Cuda et al. (2015); Albareda-Sambola and Rodríguez-Pereira (2019); Drexl and Schneider (2015), and Schiffer et al. (2019).

The LRP has been the object of numerous studies since Maranzana (1964). From a methodological perspective, the complex structure of the problem made authors focus on metaheuristics (Prodhon and Prins, 2014). On the other hand, the study of richer problem settings, involving multiple attributes, is becoming more and more center stage providing opportunities for more realistic logistics-system modeling and handling (Cuda et al., 2015).

The literature on multi-attribute LRPs is rather sparse, due to the diverse variety of features that can be addressed within the problem setting. Gianessi and Alfandari (2015) address a multicommodity-ring LRP with a matheuristic that decomposes the problem into several subproblems (location, allocation, network design, and routing) and sequentially solves each subproblem, using its output as input to the subsequent problem. Boccia et al. (2018) tackle a multicommodity LRP by introducing the flow-intercepting facility location-routing problem inspired by city logistics applications, where the authors present a branch-and-cut algorithm strengthened by valid inequalities and a heuristic procedure. Ponboon et al. (2016) address the LRP with time windows (LRPTW) using a branch-and-price method. Farham et al. (2018) extend the method of Ponboon et al. (2016) by adding new valid inequalities and other acceleration features. Capelle et al. (2019) introduce the LRPTW with pick-up and deliveries and address it via column generation. Overall, contributions to branch-and-price applied to the LRPTW rely on the set-partitioning formulation of the problem. The main strategy consists in decomposing the problem in such a way that the pricing problem aims at finding feasible vehicle routes for each candidate depot location (platforms and satellites), whereas the master problem assures the location, demand satisfaction, and the respect of the depot capacities. Koç et al. (2016) address the fleet size and mix LRP with time windows. The work contributes to the structuring of different mathematical formulations for the problem. For this, the authors consider the addition of valid inequalities derived from variants of the LRP to either reduce the size of the formulation through the aggregation of variables or to tighten the linear relaxation bounds by disaggregating some of the constraints. In general, the literature is notably scarce, in particular in studies addressing time-dependent, non-substitutable demands and fleet synchronization.

The literature on multi-attribute 2E-LRP is very limited as well. Govindan et al. (2014) introduced a bi-objective 2E-LRP with time-windows, for the simultaneous minimization of distribution costs and greenhouses gas emissions, for perishable food supply chain distribution. Bala et al. (2017) address the 2E-LRP with synchronized production schedules and time windows. Wang et al. (2018) introduce a bi-objective model for the 2E-LRP with time windows through a clustering-based algorithm hinged on locations and purchase behavior. Lu et al. (2019) address the 2E-LRP heightening multimodal freight consolidation. Li et al. (2019) propose a 2E-LRP considering real-time trans-shipment capacity, varying with transshipment and consolidation operations. Darvish et al. (2019) address the 2E-LRP incorporating multiple periods and maximal due date on customer demands incorporating flexible decisions in terms of location of intermediate facilities on each time period. Mirhedayatian et al. (2019) propose a MIP formulation and a decomposition-based heuristic for a 2E-LRP with fleet synchronization and pick-up and delivery. In a recent work, Abbassi et al. (2020) propose two versions of the multi-objective particle swarm optimisation algorithm for a multi-objective 2E-LRP to minimize both distribution cost and makespan objectives. It is worthwhile to mention that, contributions in the literature for closely related problems as the two-echelon vehicle routing problem, have further explored variants with time-dependency (Soysal et al., 2015) and synchronization constraints (see, (e.g., Li et al., 2020; Dellaert et al., 2019; Anderluh et al., 2019, 2017; Grangier et al., 2016). However, the literature for both the 2E-LRP and 2E-VRP remains scarce, in particular for studies with time-dependent OD demands. Despite the fact that fleet synchronization is the most studied attribute in the related literature, to the best of our knowledge, its interaction with time-dependencies and non-substitutable demands is yet to be studied.

Concerning solution methods, it is noteworthy that, due to the complexity of 2E-LRP, exact methods have been limited to small- and medium-sized instances even when multiples attributes were not considered (Contardo et al., 2012). The effectiveness of these methods strongly depends on the quality of the lower bounds provided by the linear relaxation of the models. Large-scale and industrial applications are usually handled by metaheuristics (Boccia et al., 2010; Winkenbach et al., 2016; Bala et al., 2017; Mirhedayatian et al., 2019; Grangier et al., 2016; Anderluh et al., 2017, 2019; Lu et al., 2019). Contardo et al. (2012) introduce a new compact modeling framework and an exact method for the 2E-LRP that decomposes the problem into two capacitated location routing problems (CLRPs), linked via the flows at satellites. The exact method is strengthened by the addition of valid inequalities derived from the CLRP. On the other hand, Darvish et al. (2019) present an exact method combining the interaction of a pure branch-and-bound algorithm and a hybrid branch-and-bound with local search. The proposed method relies on the parallel execution of both procedures, where the best known incumbent is shared. Optimality is proven by a branch-and-bound algorithm. In general, exact methods have been combined with decomposition strategies and refined mathematical formulations to derive strong lower bounds. Although the literature covers numerous time-sensitive problem settings, time dependencies and fleet synchronization are scarcely addressed. In contrast, non-substituable demand and its interaction with time-dependent attributes have not been studied from an optimization point of view.

Problems with tight or complex time considerations have often been modeled in the scientific literature by means of time-space networks. When discretizing time, a time-space network is a well-known approach used to model time-sensitive networks as static networks, as introduced by Ford and Fulkerson (1962). Although some recent works address time-space networks for modeling purposes in some vehicle routing problems (Fink et al., 2018; Anders et al., 2011), the potential of discretization methods for obtaining lower bounds or feasible solutions has been limited to service network design studies (Wang and Regan, 2009; Boland et al., 2017; Scherr et al., 2020). Vu et al. (2019) proposed a solution framework, inspired by the dynamic discretization discovery of Boland et al. (2017), for the time-dependent traveling salesman problem with time windows (TDTSPTW). The algorithm iteratively solves a time relaxed version of the TDTSPTW defined on a partial time-space network followed by a refinement procedure without creating a fully time-space network. Lagos et al. (2020) address the impact of time discretization for a continuous-time inventory-routing problem. The dynamic discretization techniques have yet to be addressed on further problem settings involving both location and routing decisions, in particular regarding the implications of multiple vehicle routes on iterative refinement schemes.

4 2E-MALRPS System Modeling

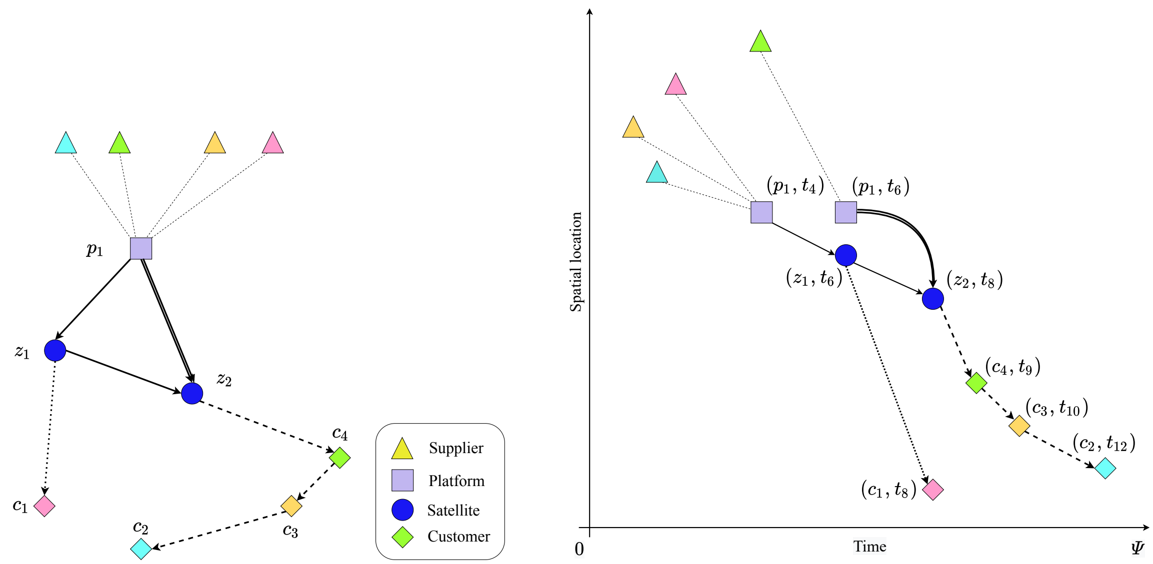
Let $\mathcal{G}^{ph} = (\mathcal{V}^{ph}, \mathcal{A}^{ph})$ be the weighted directed graph representing the physical network on which the problem is defined. The set of vertices $\mathcal{V}^{ph} = \mathcal{Q}^{ph} \cup \mathcal{P}^{ph} \cup \mathcal{Z}^{ph} \cup \mathcal{E}^{ph} \cup \mathcal{C}^{ph}$ is made up of five disjoint sets standing for the physical sites (known or among which locations are to be decided) of suppliers $\mathcal{Q}^{ph}$, potential platform sites $\mathcal{P}^{ph}$, possible satellite sites $\mathcal{Z}^{ph}$, vehicle garages $\mathcal{E}^{ph}$, and customers $\mathcal{C}^{ph}$, respectively. A fixed selection (opening) cost F_p and a capacity Θ_p are defined for each possible platform location $p \in \mathcal{P}^{ph}$. A fixed selection (opening) cost F_z is also defined for each potential satellite site.

The arc-set $\mathcal{A}^{ph} = \mathcal{A}_1^{ph} \cup \mathcal{A}_2^{ph}$ represents the direct links between locations, i.e., the vertices in $\mathcal{V}^{ph}$. A non-negative unit cost ζ_{ij} and a travel time τ_{ij} are associated with each arc $(i, j) \in \mathcal{A}^{ph}$. The set $\mathcal{A}_1^{ph}$ includes the arcs of the first echelon, corresponding to the connections between suppliers $\mathcal{Q}^{ph}$ and platforms $\mathcal{P}^{ph}$, between the latter and satellites $\mathcal{Z}^{ph}$, the arcs connecting pairs of satellites as well as the arcs connecting first-echelon garages to platforms and satellites. The set $\mathcal{A}_2^{ph}$ includes the arcs of the second echelon, that is, the connections between satellites $\mathcal{Z}^{ph}$ and the final customers $\mathcal{C}^{ph}$, the arcs connecting pairs of costumers, and the arcs connecting second-echelon garages to satellites and customers.

Platform facilities are capable to hold demands for a maximum holding time W_{max}^1 without any additional costs. Due to the lack of storage capacity at satellites and the time dependencies of demand, interacting vehicles from the first and second echelon must be synchronized at the satellite at a certain point in time, where first echelon vehicles can wait for a maximum time W_{max}^2. Moreover, it is assumed that each customer $c \in \mathcal{C}^{ph}$ has a (hard) time window $[a_c, b_c]$ (the time interval in which service must start at the node) and a service time σ_c. The distribution plan and the corresponding time-sensitive network are built for a given schedule length Ψ (e.g., a day or a week), The system, and the distribution plan, follow a cyclic and repetitive logistics operation over a certain planning horizon (e.g., a month or a season), during which demand and temporal properties of the system do not change (Andersen et al., 2009a,b; Zhu et al., 2014; Wang et al., 2019). Therefore, all transportation activities can only happen from time 0 to the given schedule length Ψ.

Let $\mathcal{K}$ denote the set of OD demands that must be transported from suppliers to customers. For each commodity $k \in \mathcal{K}$, let $vol(k)$ be its volume, $O(k) \in \mathcal{Q}^{ph}$ the associated supplier node, $D(k) \in \mathcal{C}^{ph}$ the associated customer node, and α^{pk} the time when commodity k would become ready for transportation if assigned to be shipped from platform $p \in \mathcal{P}^{ph}$. This parameter takes into consideration the time required for the transportation of each commodity from a supplier to a given platform. Two homogeneous fleets of vehicles $\mathcal{H} = H^1 \cup \mathcal{H}^2$, with limited load capacities cap_1 and cap_2, are available for the first and second echelon, respectively. Vehicle capacities are fixed. Vehicles can deliver any demand and are parked in strategically-located garages, $\mathcal{E}_1^{ph}$ for vehicles operating in the first echelon, and $\mathcal{E}_2^{ph}$ for vehicles operating in the second echelon.

The 2E-MALRPS consists in the selection of platform and satellite facilities, the allocation of demand from suppliers to platforms and customers to satellites, as well as the construction of a limited set of routes for the first and second echelons in such a way that: (i) The demand of each supplier is assigned to an open platform and satellite; (ii) every route of the first echelon starts and ends at the same vehicle garage ($\mathcal{E}_1^{ph}$); (iii) every route of the second echelon starts and ends at the same vehicle garage ($\mathcal{E}_2^{ph}$); (iv) all the customer demands are satisfied on time; (v) the load capacity of each vehicle is not exceeded; (vi) each customer is visited by only one vehicle; (vii) the total demand assigned to a facility (platforms and satellites) does not exceed its capacity at any time moment; (viii) the operations of vehicles operating at both echelons are synchronized at satellites; (ix) the time when the demand leaves platforms/satellites and the time when vehicles start on each echelon is defined; and (x) the sum of the fixed selection costs and the variable routing costs is minimized.



(a) (b)

Figure 2: Example of a feasible solution for the 2E-MALRPS

Figure 2 illustrates the dynamics of the system from a physical and a temporal point of view. Figure 2a shows a feasible solution where four OD demands are dispatched to their destination by means of platform facility p_1 and satellites z_1 and z_2, the full and dotted lines illustrating the first- and second-echelon vehicle movements, respectively. Operations are illustrated from a temporal point of view in Figure 2b, starting with the three OD demands, each with its own availability time, all being assigned to platform p_1 and ready to be shipped at time t_4. The fourth OD demand, available at time t_4, is also assigned to platform p_1, but it is ready to be shipped at time t_6. A first-echelon vehicle arrives at platform p_1, picks-up part of the available demand, and proceeds on its route to visit satellite z_1 at time t_6 and satellite z_2 at time t_8. A different first-echelon vehicle then picks-up the remaining demand at a later time, t_6, and starts its route arriving at satellite z_2 at time t_8. Two second-echelon vehicles leave their garage to arrive on time at satellites z_1 and z_2 to enable the freight transfer from the first-echelon vehicles and, then, deliver on time the freight to the appropriate customers. Multiple fleet synchronization activities take thus place at the two satellites. A first synchronization at satellite z_1, at time t_6, between one first-echelon vehicle and one second-echelon vehicle. A second synchronization takes places at satellite z_2, at time t_8, between the two first-echelon vehicles and another second-echelon vehicle. Vehicles returns to their respective garages once their respective routes are completed.

5 Time modeling

Time is a key aspect of the system. OD demands impose important constraints in time by means of availability restrictions at the suppliers, the need to pass through satellites of limited capacities, and of time windows at the final destinations, creating an interdependency in time throughout the whole distribution process. Excess in traveling or waiting times may result in operational infeasibilities and therefore become prohibitive. The time availability at the supplier nodes of the different commodities entail necessary scheduling decisions in the routing of those commodities from the platforms to the satellites. The lack of storage capacity and the time windows at the final destinations also result in the need to schedule vehicle routes departing from the satellites to visit the different destination sites. This becomes even more critical if ones assumes that storage at satellites is limited, which entails additional synchronization constraints between vehicles interacting at the satellites. All these decisions in time, when seen from the point of view of the commodities, take us to determine the *itinerary* for each commodity, from the moment in which it becomes available at the supplier node, until its delivery at its final destination, including the specific time periods associated to each of the events: arrival times at every location, departure times from every location. More formally, an itinerary for a given commodity is a tuple $\{(v_i, \mu_i, \nu_i) : i = 1 \dots k\}$ where $v_i \in \mathcal{V}^{ph}$ is the i-th node visited in the itinerary of the commodity, μ_i its arrival time to v_i, and ν_i its departure time from the node. In this section we present two modeling alternatives to include these timing decisions in our decision process, as discussed in the next few paragraphs.

The first type of modeling considers an implicit representation of these decisions by focusing on the time of operations, e.g., when vehicles arrive and depart facilities and customers to pick up or drop freight each echelon. As we present in Section 6, this representation leads to a compact, *continuous-time formulation* with a polynomial number of variables (indexed by the arcs and nodes of the network). Every time a vehicle visits a node v and picks up a commodity k, we use an allocation variable to match the commodity with the vehicle. The synchronization constraints are then imposed by restricting the difference in time between vehicles on different echelons that interact to transfer freight. Miller-Tucker-Zemlin type of constraints (Miller et al., 1960) are used to track the propagation of time and make consistent the arrival at and departure from each node in the network. The itinerary of each commodity can be constructed by matching the vehicle flow (visits of vehicles to nodes), allocation (what commodity is allocated to what vehicle), and time variables (at what time a node is visited).

The second modeling alternative is a “classical” discretization approach. The schedule length Ψ is partitioned into Δ time periods, each physical node $i \in \mathcal{V}^{ph}$ being duplicated at each possible time period within this discretization. For simplicity of presentation, but without loss of generality, we assume in the following that all time periods defined by the discretization granularity Δ are of equal length. This mechanism leads to a *time-space network*, where every node is a pair (i, t) with i representing a physical node in $\mathcal{V}^{ph}$ and $0 \leq t \leq \Psi$ a moment in time.

Physical arcs (i, j) in the original system now take the form $((i, t_i), (j, t_j))$ meaning that travel is performed between nodes i and j departing at time t_i and arriving at time t_j. In contrast to the representation of time discussed above, this new representation is explicit, in the sense that the nodes and arcs in the network encode the timing decisions entirely. Synchronization and other timing decisions and constraints are expressed as decisions or constraints in the resulting time-space network. In Section 7 we introduce a mathematical model for the 2E-MALRPS that is based on this time-space network representation of the problem. From a solution of the mathematical model associated, an itinerary can be easily constructed by following the paths of commodities and vehicles in the time-space network.

6 Compact Formulation for the 2E-MALRPS

The first formulation for the 2E-MALRPS is a three-index vehicle-flow formulation inspired by the works of Boccia et al. (2010) and Anderluh et al. (2019). Dealing with time in detail requires multiple sets of variables to record when and where each vehicle from each echelon arrives, departs, and waits at any given point in the network. Unlike platform and customer nodes, satellite facilities can be visited more than once by the first-echelon fleet. Therefore, to ensure fleet synchronization, we consider for each physical satellite $z \in \mathcal{Z}^{ph}$ a set of clone satellites $\tilde{\mathcal{Z}^{ph}}_z$, composed by the duplicate for each physical satellite $z \in \mathcal{Z}^{ph}$ as many times as the number of visits the facility can allow. The number of commodities $|K|$ represents an upper bound on the number of visits at each satellite. We therefore duplicate each satellite $|K|$ times. On the other hand, origin-destination demands are expressed as an additional index within flow variables to show how demands are transferred from platforms to customers.

Let $\alpha^{pk} \geq 0$ be the availability time of demand $k \in \mathcal{K}$ at each platform $p \in \mathcal{P}^{ph}$, if assigned to it, and M, a large integer number. The decision variables composing the formulation are defined as:

- $r_{ijh} \in \{0, 1\}, (i, j) \in \mathcal{A}^{ph}, h \in \mathcal{H}$: vehicle flow variable, 1 if arc (i, j) is used by vehicle h, and 0 otherwise;
- $f_{ps}^{hk} \in \{0, 1\}, p \in \mathcal{P}^{ph}, z \in \mathcal{Z}^{ph}, k \in \mathcal{K}, h \in \mathcal{H}^1$: flow of commodity k from platform p to satellite z by a vehicle $h \in \mathcal{H}^1$;
- $y_i \in \{0, 1\}, i \in (\mathcal{P}^{ph} \cup \mathcal{Z}^{ph})$: location variable, 1 if a facility i is open, 0 otherwise;
- $\gamma_{sc}^h \in \{0, 1\}, z \in \mathcal{Z}^{ph}, c \in \mathcal{C}^{ph}, h \in \mathcal{H}^2$: allocation variable, 1 if costumer c is allocated to satellite z with a given h 0 otherwise;
- $\mu_{ih}^1 \geq 0, i \in (\mathcal{P}^{ph} \cup \mathcal{Z}^{ph}), k \in \mathcal{H}^1$: arrival time of vehicle h at vertex i;
- $\mu_{ih}^2 \geq 0, i \in (\mathcal{Z}^{ph} \cup \mathcal{C}^{ph}), k \in \mathcal{H}^2$: arrival time of vehicle h at vertex i;
- $\nu_{ih}^1 \geq 0, i \in (\mathcal{Z}^{ph} \cup \mathcal{C}^{ph}), k \in \mathcal{H}^1$: departure time of vehicle h at vertex i;
- $\nu_{ih}^2 \geq 0, i \in (\mathcal{Z}^{ph} \cup \mathcal{C}^{ph}), k \in \mathcal{H}^2$: departure time of vehicle h at vertex i;
- $w_{ih}^1 \geq 0, i \in \mathcal{Z}^{ph}, k \in \mathcal{H}^1$: waiting time of vehicle h at satellite z;

- $w^2_{ih} \geq 0, i \in \mathcal{Z}^{ph}, k \in \mathcal{H}^2$: waiting time of vehicle h at satellite z;

The 2E-MALRPS can then be formulated as (the constraints enforcing the feasible range of the decisions variables are not shown as they are defined above):

$$\min \sum_{i \in \mathcal{P}^{ph}} F_i y_i + \sum_{i \in \mathcal{Z}^{ph}} F_i y_i + \sum_{h \in \mathcal{H}} \sum_{i \in (S \cup \mathcal{P}^{ph})} \sum_{j \in (S \cup \mathcal{P}^{ph})} \zeta_{ij} r_{ijh} + \sum_{h \in \mathcal{H}} \sum_{i \in (\mathcal{C}^{ph} \cup \mathcal{Z}^{ph})} \sum_{j \in (\mathcal{C}^{ph} \cup \mathcal{Z}^{ph})} \zeta_{ij} r_{ijh} \quad (1)$$

Subject to

$$\sum_{h \in \mathcal{H}^1} \sum_{j \in (\mathcal{E}_1^{ph} \cup \tilde{\mathcal{Z}^{ph}}_z), i \neq j} r_{ijh} \leq |\mathcal{H}^1| y_i \quad \forall i \in \mathcal{Z}^{ph} \quad (2)$$

$$\sum_{j \in \tilde{\mathcal{Z}^{ph}}, i \neq j} r_{ijh} - \sum_{j \in \mathcal{E}_1^{ph}, i \neq j} r_{jih} = 0 \quad \forall i \in \mathcal{P}^{ph}, h \in \mathcal{H}^1 \quad (3)$$

$$\sum_{j \in (\mathcal{E}_1^{ph} \cup \tilde{\mathcal{Z}^{ph}}), i \neq j} r_{ijh} - \sum_{j \in (\mathcal{P}^{ph} \cup \tilde{\mathcal{Z}^{ph}}), i \neq j} r_{jih} = 0 \quad \forall i \in \mathcal{Z}^{ph}, h \in \mathcal{H}^1 \quad (4)$$

$$\sum_{j \in \mathcal{P}^{ph}, i \neq j} r_{ijh} - \sum_{j \in \tilde{\mathcal{Z}^{ph}}, i \neq j} r_{jih} = 0 \quad \forall i \in \mathcal{E}_1^{ph}, h \in \mathcal{H}^1 \quad (5)$$

$$\sum_{i \in \mathcal{E}_1^{ph}} \sum_{j \in \mathcal{P}^{ph}} r_{ijh} \leq 1 \quad \forall h \in \mathcal{H}^1 \quad (6)$$

$$\sum_{h \in \mathcal{H}^2} \sum_{j \in (\tilde{\mathcal{Z}^{ph}} \cup \mathcal{C}^{ph}), i \neq j} r_{ijh} = 1 \quad \forall i \in \mathcal{C}^{ph} \quad (7)$$

$$\sum_{j \in \mathcal{E}_2^{ph}} r_{ijh} - \sum_{j \in \mathcal{C}^{ph}} r_{jih} = 0 \quad \forall i \in \mathcal{Z}^{ph}, h \in \mathcal{H}^2 \quad (8)$$

$$\sum_{j \in (\tilde{\mathcal{Z}^{ph}} \cup \mathcal{C}^{ph}), i \neq j} r_{ijh} - \sum_{j \in (\mathcal{C}^{ph} \cup \mathcal{E}_2^{ph}), i \neq j} r_{jih} = 0 \quad \forall i \in \mathcal{C}^{ph}, h \in \mathcal{H}^2 \quad (9)$$

$$\sum_{j \in \tilde{\mathcal{Z}^{ph}}} r_{ijh} - \sum_{j \in \mathcal{C}^{ph}} r_{jih} = 0 \quad \forall i \in \mathcal{E}_2^{ph}, h \in \mathcal{H}^2 \quad (10)$$

$$\sum_{i \in \mathcal{E}_2^{ph}} \sum_{j \in \tilde{\mathcal{Z}^{ph}}} r_{ijh} \leq 1 \quad \forall h \in \mathcal{H}^2 \quad (11)$$

$$\mu^1_{ih} + \tau_{ij} - \mu^1_{jh} \leq (1 - r_{ijh}) M \quad \forall h \in \mathcal{H}^1, (i,j) \in \mathcal{A}_1^{ph} \quad (12)$$

$$\nu^1_{ih} + \tau_{ij} - \nu^1_{jh} \leq (1 - r_{ijh}) M \quad \forall h \in \mathcal{H}^1, (i,j) \in \mathcal{A}_1^{ph} \quad (13)$$

$$\nu^1_{ih} \leq (\alpha^{ik} + W^1_{max}) + (1 - \sum_{j \in \tilde{\mathcal{Z}^{ph}}} f^{hk}_{ij}) M \quad \forall i \in \mathcal{P}^{ph}, h \in \mathcal{H}^1, k \in \mathcal{K} \quad (14)$$

$$\mu^1_{ih} \geq \alpha^{ih} \sum_{j \in \tilde{\mathcal{Z}^{ph}}} f^{hk}_{ij} \quad \forall i \in \mathcal{P}^{ph}, h \in \mathcal{H}^1, k \in \mathcal{K}\mathcal{H}^1, k \in \mathcal{K} \quad (15)$$

$$\mu^2_{ih} + \tau_{ij} - \mu^2_{jh} \leq (1 - r_{ijh}) M \quad \forall h \in \mathcal{H}^2, (i,j) \in \mathcal{A}_2^{ph}, i,j \in \mathcal{C}^{ph} \quad (16)$$

$$arg\,max\{\mu^2_{ih}, \mu^2_{ih} - w^2_{jh}\} + \tau_{ij} - \mu^2_{jh} \leq (1 - r_{ijh})M \quad \forall h \in \mathcal{H}^2, (i,j) \in \mathcal{A}^{ph}_2, i \in \tilde{\mathcal{Z}^{ph}}, j \in \mathcal{C}^{ph} \tag{17}$$

$$\nu^2_{ih} + \tau_{ij} - \nu^2_{jh} \leq (1 - r_{ijh})M \quad \forall h \in \mathcal{H}^2, (i,j) \in \mathcal{A}^{ph}_2 \tag{18}$$

$$w^1_{jh} \geq \mu^1_{jh} - \mu^2_{jb} - (2 - \gamma^h_{jc} - \sum_{i \in (\tilde{\mathcal{Z}^{ph}}_z \cup \mathcal{P}^{ph}), i \neq j} f^{hk_c}_{ij})M \quad \forall h \in \mathcal{H}^1, b \in \mathcal{H}^2, c \in \mathcal{C}^{ph}, j \in \tilde{\mathcal{Z}^{ph}} \tag{19}$$

$$w^2_{jb} \geq \mu^2_{jb} - \mu^1_{jh} - (2 - \gamma^h_{jc} - \sum_{i \in (\tilde{\mathcal{Z}^{ph}}_z \cup \mathcal{P}^{ph}), i \neq j} f^{hk_c}_{ij})M \quad \forall h \in \mathcal{H}^1, b \in \mathcal{H}^2, c \in \mathcal{C}^{ph}, j \in \tilde{\mathcal{Z}^{ph}} \tag{20}$$

$$\nu^2_{jb} \geq \mu^1_{jh} - (2 - \gamma^h_{jc} - \sum_{i \in (\tilde{\mathcal{Z}^{ph}}_z \cup \mathcal{P}^{ph}), i \neq j} f^{hk_c}_{ij})M \quad \forall h \in \mathcal{H}^1, b \in \mathcal{H}^2, c \in \mathcal{C}^{ph}, j \in \tilde{\mathcal{Z}^{ph}} \tag{21}$$

$$\nu^1_{jh} \geq \mu^2_{jb} - (2 - \gamma^h_{jc} - \sum_{i \in (\tilde{\mathcal{Z}^{ph}}_z \cup \mathcal{P}^{ph}), i \neq j} f^{hk_c}_{ij})M \quad \forall h \in \mathcal{H}^1, b \in \mathcal{H}^2, c \in \mathcal{C}^{ph}, j \in \tilde{\mathcal{Z}^{ph}} \tag{22}$$

$$a_i \leq \mu^2_{ih} \leq b_i \quad \forall i \in \mathcal{C}^{ph}, h \in \mathcal{H}^2 \tag{23}$$

$$\sum_{j \in (\mathcal{C}^{ph} \cup \mathcal{E}^{ph}_2), i \neq j} r_{ijh} + \sum_{j \in \mathcal{C}^{ph}} \sum_{b \in \tilde{\mathcal{Z}^{ph}}_z} r_{bjh} - \gamma^h_{bi} \leq 1 \quad \forall i \in \mathcal{C}^{ph}, z \in \mathcal{Z}^{ph}, h \in \mathcal{H}^2 \tag{24}$$

$$cap_1 \sum_{b \in (\tilde{\mathcal{Z}^{ph}}_j \cup \mathcal{E}^{ph}_1), b \neq j} r_{jbh} - \sum_{k \in \mathcal{K}} vol_k\, f^{hk}_{ij} \geq 0 \quad \forall h \in \mathcal{H}^1, i \in \mathcal{P}^{ph}, j \in \mathcal{Z}^{ph} \tag{25}$$

$$cap_1 \sum_{b \in \tilde{\mathcal{Z}^{ph}}_j} r_{ibh} - \sum_{k \in \mathcal{K}} vol_k\, f^{hk}_{ij} \geq 0 \quad \forall h \in \mathcal{H}^1, i \in \mathcal{P}^{ph}, j \in \mathcal{Z}^{ph} \tag{26}$$

$$\sum_{h \in \mathcal{H}^2} \sum_{i \in \tilde{\mathcal{Z}^{ph}}} \gamma^h_{ij} = 1 \quad \forall j \in \mathcal{C}^{ph} \tag{27}$$

$$\sum_{h \in \mathcal{H}^1} \sum_{i \in \mathcal{P}^{ph}} f^{hk_b}_{ij} = \gamma_{jb} \quad \forall j \in \tilde{\mathcal{Z}^{ph}}, b \in \mathcal{C}^{ph}; k_b = \{k \in \mathcal{K} | d_{bk} > 0\} \tag{28}$$

$$\sum_{h \in \mathcal{H}^1} \sum_{i \in \mathcal{P}^{ph}} \sum_{j \in \tilde{\mathcal{Z}^{ph}}} f^{hk}_{ij} = 1 \quad \forall k \in \mathcal{K} \tag{29}$$

$$\sum_{h \in \mathcal{H}^1} \sum_{k \in \mathcal{K}} \sum_{j \in \tilde{\mathcal{Z}^{ph}}} vol_k\, f^{hk}_{ij} \leq \Theta_i\, y_i \quad \forall i \in \mathcal{P}^{ph} \tag{30}$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{P}^{ph}} \sum_{j \in \tilde{\mathcal{Z}^{ph}}_z} vol_k\, f^{hk}_{ij} \leq cap_1 \quad \forall h \in \mathcal{H}^1 \tag{31}$$

$$\sum_{i \in \mathcal{C}^{ph}} d_i \sum_{j \in (\mathcal{Z}^{ph} \cup \mathcal{C}^{ph})} r_{ijh} \leq cap_2 \quad \forall h \in \mathcal{H}^2 \tag{32}$$

$$w^1_{ih} \leq W^2_{max} \quad \forall i \in \tilde{\mathcal{Z}^{ph}}, h \in \mathcal{H}^1 \tag{33}$$

$$w^2_{ih} \leq W^2_{max} \quad \forall i \in \tilde{\mathcal{Z}^{ph}}, h \in \mathcal{H}^2 \tag{34}$$

The objective function (1) minimizes the total transportation costs of the distribution network computed as the sum of the fixed cost of the selected facilities and the routing costs of the demand flows through the resulting network. Constraints (2) impose that outbound arcs from every open satellite must respect the total number of first-echelon vehicles. Constraints (3-5) are the flow conservation constraints for platforms, satellites, and first-echelon garage, respectively. Constraints (6) ensure that each active vehicle is assigned to one platform only. Constraints (7) ensure that every customer is visited by a single second-echelon vehicle. Constraints (8-10) are the flow conservation constraints for satellites, customers, and second-echelon garage, respectively. Constraints (11) ensure that each active vehicle is assigned to one satellite only. Constraints (12-13) handle the arrival and departure times of first-echelon vehicles.

Constraints (14) and (15) guarantee schedule feasibility with respect to demand availability and maximum holding time at platform facilities. Constraints (16-18) handle the arrival and departure times of second-echelon vehicles. Constraints (19) and (20) relate the arrival times at satellites of first- and second-echelon vehicles to guarantee fleet synchronization at satellites facilities. Constraints (21) and (22) relate the departure time and the arrival time of first and second echelon vehicles, respectively, at satellites. Constraints (23) ensure that second-echelon vehicles arrive within the customer time windows. Constraints (24), (25), and (26) link allocation and routing variables. Constraints (27) impose that each customer must be assigned to one satellite only. Constraints (29) ensure that each origin supplier is allocated to one platform. Constraints (28) are the flow conservation constraints at satellites. Constraints (30) ensure that the multicommodity flow going out from platforms is less than the platform capacity. Constraint (31) and (32) impose that the multicommodity flow carried by each vehicle, in the first and second echelon, respectively, is less that its capacity. Constraints (33) and (34) ensure that waiting times at satellite facilities respect the maximum permitted waiting time at each echelon.

7 Time-space Formulation for the 2E-MALRPS

Let $\mathcal{T}(\Delta)$ be the (ordered) set of time periods given by discretizing the schedule length Ψ according to the granularity Δ. Let also $\mathcal{T}_i(\Delta) \subseteq \mathcal{T}(\Delta)$ represent the set of time periods at which node $i \in \mathcal{V}^{ph}$ is relevant in the network because vehicles or commodity flows may access it at that time. Each system component has its own set of relevancy periods. For example, the time realizations of customers $i \in \mathcal{C}^{ph}$ must satisfy $\mathcal{T}_i(\Delta) \subseteq [a_i, b_i]$. Similarly, a platform $p \in \mathcal{P}^{ph}$ is only defined starting at the moment when the first commodity becomes available. This is illustrated in Figure 3 depicting the time periods of relevance for the nodes $i \in \mathcal{V}^{ph}$ appearing in Figure 2a. Note that copies in time are made at all periods for satellites and platforms, as these are available for the complete schedule length, compared to customers and suppliers for which copies are made for the time periods when they are relevant only.

The time-space network $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ for the 2E-MALRPS is then defined by the node sets $\mathcal{V} = \mathcal{Q} \cup \mathcal{E} \cup \mathcal{P} \cup \mathcal{Z} \cup \mathcal{C}$ reflecting the spatial and time position (i, t) of every node $i \in \mathcal{V}$ at time period $t \in \mathcal{T}_i(\Delta)$, where the sets in $\mathcal{V}$ are the time-space nodes for suppliers $\mathcal{Q}$, vehicle garages $\mathcal{E} = \mathcal{E}^1 \cup \mathcal{E}^2$, platforms $\mathcal{P}$, satellites $\mathcal{Z}$ and customers $\mathcal{C}$. To simplify the notation, let $\mathcal{V}_i$ stand for the set of time-space nodes $\{(i, t) : i \in V,\ t \in \mathcal{T}_i(\Delta)\}$, and $[a_i, b_i]$ be the time interval during which node $i \in \mathcal{V}$ is relevant in $\mathcal{G}$, i.e., $a_i = \min\{t : t \in \mathcal{T}_i(\Delta)\}$ and $b_i = \max\{t : t \in \mathcal{T}_i(\Delta)\}$.

Similar to the physical network, the set of arcs $\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2$ stands for connections between time-space nodes representing the various system components. At the first echelon, $\mathcal{A}^1$, one finds the connections between suppliers and platforms, platforms and satellites, pairs of satellites, as well as from first-echelon garages to platforms and from satellites to the former. The second

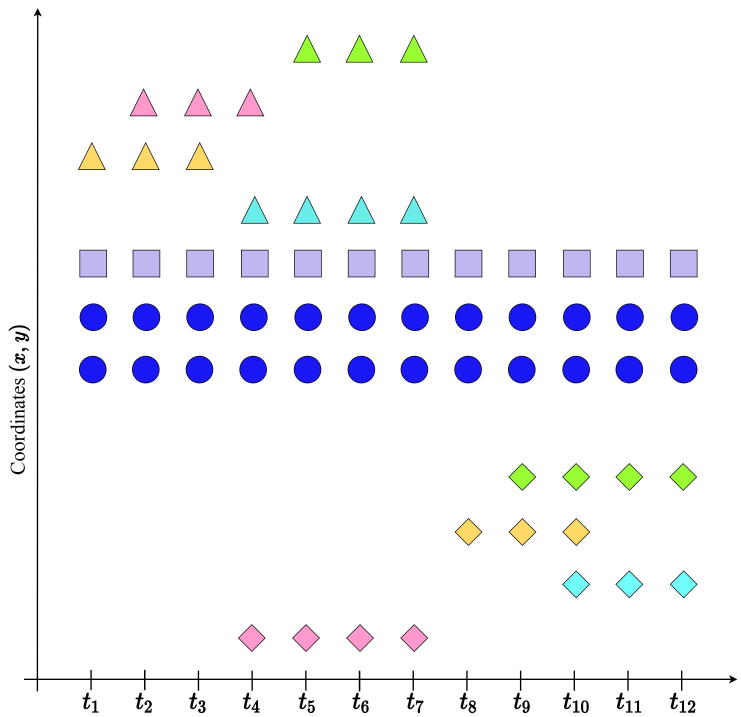


Figure 3: Time-space representation of the solution of Figure 2a

echelon arcs in $\mathcal{A}^2$ stand for the connections from satellites to customers, between pairs of the latter, as well as from second-echelon garages to satellites and from customer to the former. An arc $((i,t),(j,t')) \in \mathcal{A}$ is then defined for arc $(i,j) \in \mathcal{A}^{ph}$ with $t \in \mathcal{T}_i(\Delta)$ and $t' = t + \tau_{ij} \in \mathcal{T}_j(\Delta)$.

Commodities must be assigned to a single platform and satellite and the flow should not be split. Consequently, let $\mathcal{P}_0(k) = \{(p,t),(j,t') : p \in \mathcal{P}, j \in \mathcal{Z}, t \geq \alpha^{pk}\}$ be the set of platform to satellite arcs commodity k can be assigned to if passing through platform p to travel to a reachable satellite.

Waiting times at nodes $v_i \in \mathcal{V}$ are implicitly represented by the time difference between the departure of a vehicle and its prior arrival at the node, considering that freight can be held in any location at no cost. To enable waiting time at customers, the travel time of inbound arcs to customer nodes are considered to embed any necessary delays.

The resulting time-space network captures all physical and temporal characteristics of the problem. The size of the time-space network depends on $\mathcal{T}(\Delta)$ as it duplicates the nodes of the physical network at all relevant periods. It is worthwhile to mention that this modeling approach does not need to consider vehicle indexes, contrarily to the vehicle-indexed formulation. Define the following decision variables:

- $x_{ij} \in \{0,1\}, (i,j) \in \mathcal{A}$: 1 if an arc is selected, 0 otherwise;
- $f_{ij}^k \in \{0,1\}, (i,j) \in \mathcal{A}, k \in \mathcal{K}$: 1, if a commodity k goes through the arc (i,j), 0 otherwise;
- $y_i \in \{0,1\}, i \in (\mathcal{P}^{ph} \cup \mathcal{Z}^{ph})$: 1, if a facility is open at node i, 0 otherwise;

The time-space-based formulation then becomes (variable-range constraints are again not displayed):

$$\min \sum_{i\in\mathcal{P}^{ph}} F_i y_i + \sum_{i\in\mathcal{Z}^{ph}} F_i y_i + \sum_{(i,j)\in\mathcal{A}^1} \zeta_{ij} x_{ij} + \sum_{(i,j)\in\mathcal{A}^2} \zeta_{ij} x_{ij} \tag{35}$$

Subject to

$$\sum_{j\in\mathcal{C}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_c)\cup\mathcal{Z})} x_{ij} = 1 \quad \forall c \in C \tag{36}$$

$$\sum_{j\in\mathcal{C}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{Z})} x_{ij} = \sum_{j\in\mathcal{C}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{E}^2)} x_{ji} \quad \forall c \in \mathcal{C}^{ph} \tag{37}$$

$$\sum_{i\in((\mathcal{C}\setminus j)\cup\mathcal{Z})} x_{ij} \geq \sum_{\substack{h\in\mathcal{C}_j,\\ t(h)\leq t(j)+\sigma_c}} \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{E}^2)} x_{hi} \quad \forall j \in \mathcal{C} \tag{38}$$

$$\sum_{i\in\mathcal{E}^2} x_{ij} \leq \sum_{\substack{i\in\mathcal{C},\\ t(j)+w_{max}\geq t(i)\geq t(j)}} x_{ji} \quad \forall j \in \mathcal{Z} \tag{39}$$

$$\sum_{j\in\mathcal{Z}_s} \sum_{i\in\mathcal{E}^2} x_{ij} = \sum_{j\in\mathcal{Z}_s} \sum_{i\in\mathcal{C}} x_{ji} \quad \forall z \in \mathcal{Z}^{ph} \tag{40}$$

$$\sum_{i\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{P})} x_{ij} \leq y_{\mathcal{Z}_j} \quad \forall j \in \mathcal{Z} \tag{41}$$

$$\sum_{i\in((\mathcal{Z}\setminus\mathcal{Z}_z)\cup\mathcal{P})} x_{ij} \leq \sum_{\substack{i\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{E}^1),\\ t(j)+w_{max}\geq t(i)\geq t(j)}} x_{ji} \quad \forall j \in \mathcal{Z} \tag{42}$$

$$\sum_{i\in\mathcal{E}^1} x_{ij} = \sum_{i\in\mathcal{Z}} x_{ji} \quad \forall j \in \mathcal{P} \tag{43}$$

$$\sum_{j\in\mathcal{Z}} x_{ij} \leq |K^1| y_i \quad \forall i \in \mathcal{P} \tag{44}$$

$$\sum_{i\in P_0(k)} f_{ij}^k = 1 \quad \forall k \in \mathcal{K}, j \in \mathcal{Z} \tag{45}$$

$$\sum_{i\in\mathcal{P}_p} \sum_{j\in\mathcal{Z}} f_{ij}^k \leq y_p \quad \forall k \in \mathcal{K}, p \in \mathcal{P}^{ph} \tag{46}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{Z})} \sum_{j\in\mathcal{C}_j} f_{ij}^k - \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{E}^2)} \sum_{j\in\mathcal{C}_j} f_{ji}^k = 0 \quad \forall j \in \mathcal{C}^{ph}, k \in \mathcal{K}, j \neq d_k \tag{47}$$

$$\sum_{j\in\mathcal{C}_j} f_{ij}^k - \sum_{j\in\mathcal{C}_j} f_{ji}^k \leq 1 - \sum_{j\in\mathcal{C}_j} x_{ji} \quad \forall j \in \mathcal{C}^{ph}, k \in \mathcal{K}, j \neq d_k \tag{48}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{Z})} \sum_{j\in\mathcal{C}_j} f_{ij}^k - \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{E}^2)} \sum_{j\in\mathcal{C}_j} f_{ji}^k = \sum_{i\in((\mathcal{C}\setminus\mathcal{C}_j)\cup\mathcal{Z})} \sum_{j\in\mathcal{C}_j} x_{ij}$$

$$\forall j \in \mathcal{C}^{ph}, k \in \mathcal{K}, j = d_k \tag{49}$$

$$\tag{50}$$

$$\sum_{j\in\mathcal{Z}_z}\sum_{i\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{P})} f_{ij}^k = \sum_{j\in\mathcal{Z}_z}\sum_{h\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{E}^1)} f_{jh}^k + \sum_{j\in\mathcal{Z}_z}\sum_{l\in(\mathcal{C})} f_{jl}^k \quad \forall z\in\mathcal{Z}^{ph}, k\in\mathcal{K} \tag{51}$$

$$\sum_{i\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{P})} f_{ij}^k \leq \sum_{\substack{h\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{E}^1),\\ t(j)+w_{max}\geq t(h)\geq t(j)}} f_{jh}^k + \sum_{\substack{l\in(\mathcal{C}),\\ t(j)+w_{max}\geq t(l)\geq t(j)}} f_{jl}^k \quad \forall j\in\mathcal{Z}, k\in\mathcal{K} \tag{52}$$

$$\sum_{k\in\mathcal{K}} w^k \sum_{i\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{P})} f_{ij}^k \leq cap_1 \sum_{i\in((\mathcal{Z}\setminus\mathcal{Z}_j)\cup\mathcal{P})} x_{ij} \quad \forall j\in\mathcal{Z} \tag{53}$$

$$\sum_{k\in\mathcal{K}} w^k \sum_{i\in\mathcal{Z}} f_{ij}^k \leq cap_2 \quad \forall j\in\mathcal{C}, \tag{54}$$

$$\sum_{k\in\mathcal{K}} w^k \sum_{j\in(\mathcal{Z}\setminus\mathcal{Z}_j)} f_{ij}^k \leq \Theta_p y_i \quad \forall i\in\mathcal{P} \tag{55}$$

$$f_{ij}^k \leq x_{ij} \quad \forall k\in\mathcal{K}, (i,j)\in\mathcal{A}^1 \tag{56}$$

$$f_{ij}^k \leq x_{ij} \quad \forall k\in\mathcal{K}, (i,j)\in\mathcal{A}^2 \tag{57}$$

The objective function (35) minimizes the total cost of the system, including the fixed cost of selecting (opening) facilities on both echelons and the variable travel costs of the vehicles on both echelons to move the demand flows. Constraints (36) ensure that each customer can be visited only once. Equations (37) are conservation constraints on routing variables for customers. Constraints (38) impose that outbound connections to customers must take place after their respective service time node. Constraints (39) enforce that, for each outbound connection from a satellite to a customer, there is an inbound connection from a second-level vehicle garage to the satellite, including the waiting times at the satellite. Equations (40) are conservation constraints on routing variables for each satellite facility for the second echelon. Because routing variables are not indexed by the vehicle identification, freight flow could have been transferred between first echelon vehicles, which is not considered as a feature of the problem setting. Constraints (41) aim to avoid such transfers by ensuring that each open satellite is visited at most once for each time period, while constraints (42) ensure that for each outbound connection from a satellite to other satellite or garage, there is an inbound connection from a platform or from a different satellite. Constraints (43) and (44) enforce the routing conservation of first-level routing variables and impose the maximum outbound connections from platform facilities in terms of the fleet size for the first echelon, respectively.

Constraints (45) and (46) impose that each demand departs from the assigned open platform after their availability time, and assures that each demand is not split. Constraints (47) and (48) impose flow conservation for commodities Constraints (49) guarantee that each commodity flow reaches its destination customer. Constraints (51) impose flow conservation at satellites. Constraints (52) complement constraints (51) by ensuring spatial and temporal synchronization at time-space satellites, considering waiting time. Constraints (53) and (54) ensure that the total flow assigned to each route is not larger than the vehicle capacity for the first and second echelons, respectively. Similarly, constraints (55) impose that the assigned routes to each platform do not exceed the facility capacity. Constraints (56) and (57) link flow and routing variables.

8 Dynamic Discretization Discovery for the 2E-MALRPS

The temporal dimension of the time-space formulation provides a more realistic model for the problem setting. On the other hand, its pseudo-polynomial size also makes it less scalable as the time granularity gets smaller. We therefore propose a *dynamic discretization discovery (DDD)* framework for the time-space model to address this scalability drawback, building on the method introduced by Boland et al. (2017) for service network design problems. Notice that, demand in the problem setting we study generates compulsory time moments that must be explicitly included in the network representation, which both increases the difficulty of the problem and somehow facilitates the discretization of time. We provide in this section the foundations and notation for the proposed solution framework, as well as a more in-depth description of the inner procedures.

8.1 Preliminary notation

The proposed dynamic discretization solution framework employs a relaxation of the time-space representation of the 2E-MALRPS in order to provide feasible solutions to the problem without creating the *complete* time-space network. Without loss of generality, we assume that the *complete time-space network* is defined by $\mathcal{T}(\bar{\Delta})$, with $\bar{\Delta}$ being the largest number of time periods possible, necessary to capture all the relevant time moments within the system. Notice that this complete time-space network contains all feasible solutions for the 2E-MALRPS, but its dimension may be prohibitively large. A discretization parameter Δ, with $1 < \Delta < \bar{\Delta}$, can then be defined to generate a *reduced time-space network* by decreasing the number of time periods which are relevant for each vertex. We provide in the following the necessary assumptions to ensure that the reduced time-space network is a relaxation for the 2E-MALRPS, and derive lower bounds for the 2E-MALRPS regardless of the granularity of the discretization.

We define the *reduced time-space network* $\mathcal{G}_\Delta = (\mathcal{V}_\Delta, \mathcal{A}_\Delta)$, for $1 < \Delta < \bar{\Delta}$, composed by a reduced set of integer time points $\mathcal{T}_i(\Delta) \cap [a_i, b_i]$, for each vertex $i \in \mathcal{V}^{ph}$. In this context, inbound arcs to customers no longer embed waiting times. Rather, we consider a set of time-space nodes before each time window to represent early arrivals to customers, while the original travel times are maintained. Unlike a complete time-space network, the reduced time-space network is an aggregated network derived from $\mathcal{G}$, where $|\mathcal{G}_\Delta| \leq |\mathcal{G}|$. Consequently, the arc-length τ_{ij} for each arc $(i,j) \in \mathcal{A}^{ph}$ is also aggregated in terms of Δ. The aggregation ensures that there is a time-space arc (i,t) and (j,t') in $\mathcal{G}_\Delta$ for each arc $(i,j) \in \mathcal{A}^{ph}$, with $t' \leq t + \tau_{ij}$. An arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ is then considered to be *too short* when $t' \leq t + \tau_{ij}$ as it might model negative ($t' < t$) or zero ($t' = t$) travel times. We assume that the travel costs remain unchanged when the travel time of an arc in $\mathcal{A}_\Delta$ is *too short*.

Constraints (41), limiting to one the number of first-echelon in-bound movements to satellites, require reformulation in the present context. Indeed, while these constraints ensure that no freight transfer takes place between first echelon routes, they may also yield too many infeasible solutions when a coarse time discretization is considered. We therefore rather define constraints (58) to limit the inbound connections at time-space satellites in terms of the time interval Δ.

$$\sum_{i \in ((\mathcal{Z} \setminus \mathcal{Z}_j) \cup \mathcal{P})} x_{ij} \leq (\bar{\Delta} - \Delta)\, y_{\mathcal{Z}_j} \quad \forall j \in \mathcal{Z} \tag{58}$$

In order to derive lower bounds for the 2E-MALRPS based on the reduced time-space network, our solution relies on a formulation defined by the objective function (35) and constraints (36) - (40), (58) and (42)-(57). To simplify the notation, we refer to the solution of this time-space formulation as TEF($\mathcal{G}_\Delta$) for a reduced time-space network $\mathcal{G}_\Delta$. In addition, we show that the following properties are satisfied by the time-space network.

- **Property 1.** $\forall i \in \mathcal{V}^{ph}$, there is a set of time-space nodes (i,t) in $\mathcal{V}_\Delta$ for every time interval $\mathcal{T}_i(\Delta) \cap [a_i, b_i]$, where $[a_i, b_i]$ refers to the time windows for each customer node $i \in \mathcal{C}^{ph}$, the complete schedule length (e.g. $a_i = 0$ and $b_i = \Psi$) for each node $i \in \mathcal{Z}^{ph} \cup \mathcal{E}^{ph}$, as well as the availability time of each commodity and the complete schedule length on each platform facility $\mathcal{P}^{ph}$.
- **Property 2.** $\forall (i,t) \in \mathcal{V}_\Delta$ and arc $(i,j) \in \mathcal{A}^{ph}$, there is a time-space arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ with $t' \leq t + \tau_{ij}$. If arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$, there is no time-space node $(j,t'') \in \mathcal{V}_\Delta$ with $t' < t'' \leq t + \tau_{ij}$.
- **Property 3.** $\forall (i,t) \in \mathcal{Z}_\Delta$, there is a waiting time-space arc to the same time-space satellite $(i,t') \in \mathcal{Z}_\Delta$ with $t' \leq t + W^2_{max}$.
- **Property 4.** $\forall (i,t) \in \mathcal{C}_\Delta$, there is at least one time-space node before each customer's lower time window, exclusively to allow early inbound arrivals but not early outbound from said customer. Furthermore, any active early customer connection leads to an outbound connection coming from the time-space node at or after the customer earliest time windows limit. This represents the potential waiting time that can take place before the customer's time window.

For a given commodity $k \in \mathcal{K}$ and a complete time-expanded network $\mathcal{G}$, we define an *itinerary* in $\mathcal{G}$ as a path $r = (v_i, t_i)_{i=1}^{l}$ connecting the node of the initial vehicle arrival and the departure of the commodity at a platform (thus $v_1 \in \mathcal{P}^{ph}$) to the node of the arrival and departure time at its destination (meaning $v_l = D(k)$), including the times of arrival and departure at each intermediary node in the time-expanded network. We assume that the transfer between the first and second echelon occurs at a satellite $1 < j < l-1$ such that $v_{j+w} \in \mathcal{Z}^{ph}$ with $w = \{0,1,2,3\}$, meaning that t_j, t_{j+1} and t_{j+2}, t_{j+3} represent the arrival time to and departure time from the transfer satellite at each echelon, respectively. Hence, we have that for an itinerary $r = (v_i, t_i)_{i=1}^{l}$ in $\mathcal{G}$, arcs $((v_i,t_i),(v_{i+1},t_{i+1})) \in \mathcal{A}$ for every i; except for $i = j+1$, as the arc $((v_{j+1},t_{j+1}),(v_{j+2},t_{j+2}))$ represent the departure and arrival of different routes at each echelon. Note that connections to and from garages are omitted since vehicles are assumed to move empty on those legs. We let $R^k_{\mathcal{G}}$ be the set of itineraries that can be used to move commodity $k \in \mathcal{K}$ from its origin $O(k)$ to its destination $D(k)$ throughout the network $\mathcal{G}$, and we use $R_{\mathcal{G}} = \cup_k R^k_{\mathcal{G}}$ to refer to the set of all possible itineraries.

Lemma 1. Let $\mathcal{G}_\Delta$ a reduced time-space network that satisfies properties 1, 2, 3 and 4. Then for each commodity $k \in \mathcal{K}$ and for each itinerary $r = (v_i, t_i)_{i=1}^{l} \in R^k_{\mathcal{G}}$ there exists an itinerary $r' = (v_i, t'_i)_{i=1}^{l} \in R^k_{\mathcal{G}_\Delta}$ such that $t'_i \leq t_i$ for every $i = 1 \ldots l$.

Proof Lemma 1. We conduct the proof by induction on i for the itinerary $r = (v_i, t_i)_{i=1}^{l}$ in $R_{\mathcal{G}}^{k}$. For $i = 1$, let $t'_1 = a_{v_1}$ be the temporal lower bound of v_1. By Property 1, we have that the time-space node (v_1, t'_1) with $t'_1 = a_{v_1}$ yields to $t'_1 \leq t_1$, as there is no $(v_1, t''_1) \in \mathcal{G}_\Delta$ with $t''_1 < a_{v_1}$. From the time-space node (v_1, t'_1), we can map the remainder of the itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^{k}$ by defining an *equivalent* time-space node (v_i, t'_i) in $\mathcal{G}_\Delta$, for each $(v_i, t_i) \in R_{\mathcal{G}}^{k}$ with $t'_i = arg\,max\{d \in \mathcal{T}_i(\Delta) | d \leq t_i\}$. By Property 2, there is an arc $((v_i, t'_i), (v_{i+1}, t'_{i+1})) \in \mathcal{A}_\Delta$ with $t'_{i+1} \leq t'_i + \tau_{v_i v_{i+1}} \leq t_i + \tau_{v_i v_{i+1}}$, while Property 3 and 4, enables early waiting times at satellites as well early arrival at customers, respectively.

Assuming that $i = w$ is true, we can prove that our condition holds for $i = w + 1$. Hence, by the inductive assumption, there is an itinerary $r = [(v_1, t'_1), (v_2, t'_2), ..., (v_w, t'_w)]$ with $(v_w, t'_w) \in \mathcal{G}_\Delta$ and $t'_w \leq t_w$. By Property 1, the set of integer time points $\mathcal{T}_{v_i(\Delta)}$ representing the time moments at which node $i = w+1$ becomes relevant must exists in $\mathcal{V}_\Delta$. By Property 2, 3 and 4, arc $((v_w, t'_w), (v_{w+1}, t'_{w+1})) \in \mathcal{A}_\Delta$ with $t'_{w+1} \leq t'_w + \tau_{v_w v_{w+1}} \leq t_w + \tau_{v_w v_{w+1}}$. By Property 3, there must exist a waiting time at satellites with a lesser of equal value to the original waiting time, while Property 4, ensures that there is an early arrival point in time for customers. Consequently, we can ensure that the defined conditions can be verified for each connection within $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^{k}$, and thus for each itinerary in $R_{\mathcal{G}}$. □

Lemma 2. If a reduced time-space network $\mathcal{G}_\Delta$ satisfies properties 1, 2, 3 and 4, then the optimal solution of the 2E-MALRPS on the reduced time-space network $(\mathcal{G}_\Delta)$ is a lower bound for the solution of the 2E-MALRPS on the complete time-space network $(\mathcal{G})$.

Proof Lemma 2. To prove this lemma, we will show that each time-space arc representing the optimal solution for the 2E-MALRPS in a complete time-space network, can be mapped onto a reduced time-space network, with an equal or lesser operational cost.

Consider $Z_{\mathcal{G}}^{*} = (x_{\mathcal{G}}^{*}, f_{\mathcal{G}}^{*}, y_{\mathcal{G}}^{*})$ an optimal integer solution of the 2E-MALRPS in a complete time-space network $(\mathcal{G})$, with $A_{\mathcal{G}}^{*} = \{((v_i, t_i), (v_j, t_j)) \in \mathcal{A}_{\mathcal{G}} \mid x_{(v_i,t_i),(v_j,t_j)} = 1\}$. Let $R_{\mathcal{G}}$ be the set of itineraries $r \in R_{\mathcal{G}}$ dispatching each commodity $k \in \mathcal{K}$ from its origin $O(k)$ to its destination $D(k)$ throughout the system with the arcs in $A_{\mathcal{G}}^{*}$. In what follows, we will show that each arc in $A_{\mathcal{G}}^{*}$ can be mapped to a unique set of arcs $A_{\mathcal{G}_\Delta}$ of a reduced time-space network $(\mathcal{G}_\Delta)$, so we can construct $Z_{\mathcal{G}_\Delta} = (x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$ in respect to each arc in $A_{\mathcal{G}_\Delta}$.

By Lemma 1, for each arc $((v_i, t_i)(v_j, t_j)) \in A_{\mathcal{G}}^{*}$ in $R_{\mathcal{G}}$ (excluding garages connections), there exists $(v_i, t'_i) \in \mathcal{G}_\Delta$ with $t'_i \leq t_i$ and a t'_j such that $((v_i, t'_i)(v_j, t'_j)) \in A_{\mathcal{G}_\Delta}$. Hence, for each itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}$, there is an *equivalent* itinerary $r' = (v_i, t'_i)_{i=1}^{l} \in R_{\mathcal{G}_\Delta}$ such that $t'_i \leq t_i$. Because the number of both platform and satellite facilities must hold for each $r \in R_{\mathcal{G}}$ mapped to $\mathcal{G}_\Delta$, we have that $y_{\mathcal{G}_\Delta} = y_{\mathcal{G}}^{*}$. Now we can track each commodity flow from its origin to its destination in $R_{\mathcal{G}_\Delta}$ to derive both routing and flow decisions to $x_{\mathcal{G}_\Delta}$ and $f_{\mathcal{G}_\Delta}$. Notice that by Lemma 1 and Property 3, fleet synchronization within each $r \in R_{\mathcal{G}_\Delta}$ holds, but takes place at the same or earlier point in time on the same satellite.

Recall that every route in the first and second echelon must start and end at a vehicle garage. We have that for each path $r \in R_{\mathcal{G}}$, every origin, satellite serving as the transfer point and destination of each commodity $k \in \mathcal{K}$ are known. Notice that, for some of these time-space nodes, there exists an unique time-space arc in $A_{\mathcal{G}}^{*}$ that represents the leg used for each route as the *start point* after a vehicle leaves the garage or the *end point* before the vehicle returns to the garage. By Lemma 1 and Properties 1 and 2, there must exist a time-space node

$(e_1, t'_{e_1}) \in \mathcal{E}^1_\Delta$ and $(e_2, u'_{e_2}) \in \mathcal{E}^2_\Delta$, such that $((z, t'_z), (e_1, t'_{e_1}))$ and $((c, t'_c), (e_2, t'_{e_2}))$ exists in $A_{\mathcal{G}_\Delta}$ for each *end point* at the first and second echelon, with (z, t'_z) and (c, t'_c) in $R_{\mathcal{G}_\Delta}$, $z \in \mathcal{Z}^{ph}$ and $c \in \mathcal{C}^{ph}$. Similarly, there are time-space nodes $(e'_1, t'_{e'_1}) \in \mathcal{E}^1_\Delta$ and $(e'_2, u'_{e'_2}) \in \mathcal{E}^2_\Delta$, such that $((e'_1, t'_{e'_1}), (p, t'_p))$ and $((e'_2, t'_{e'_2}), (z', t''_{z'}))$ exists in $A_{\mathcal{G}_\Delta}$ for each *start point* at the first and second echelon, with (p, t'_p) and $(z', t''_{z'})$ in $R_{\mathcal{G}_\Delta}$, $p \in \mathcal{P}^{ph}$ and $z' \in \mathcal{Z}^{ph}$. Thus, we can then derive the routing decisions to $x_{\mathcal{G}_\Delta}$ for the resulting inbound and outbound for each garage.

Now, the solution $Z_{\mathcal{G}_\Delta} = (x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$ constructed in this way is feasible for the 2E-MALRPS onto the reduced time-space network ($\mathcal{G}_\Delta$) while routing and location costs holds. Therefore, we have that $Z_{\mathcal{G}_\Delta}$ is identical to $Z^*_{\mathcal{G}}$ with arrival and departure times taking earlier or equal values than the ones on the complete time-space network, but with the same operational cost. Consequently, the optimal solution for the 2E-MALRPS on a reduced time-space network ($\mathcal{G}_\Delta$) is a lower bound of the optimal solution obtained in a complete time-space network ($\mathcal{G}$). □

8.2 Algorithm outline

The proposed solution framework, illustrated in Figure 4, iteratively refines a reduced time-space network and solves the integer program defined by this network configuration to extract lower bounds for the 2E-MALRPS, until the problem is solved to optimality up to a specified tolerance. The descriptions of the main components of the algorithm are provided in the following sections as indicated in the figure.

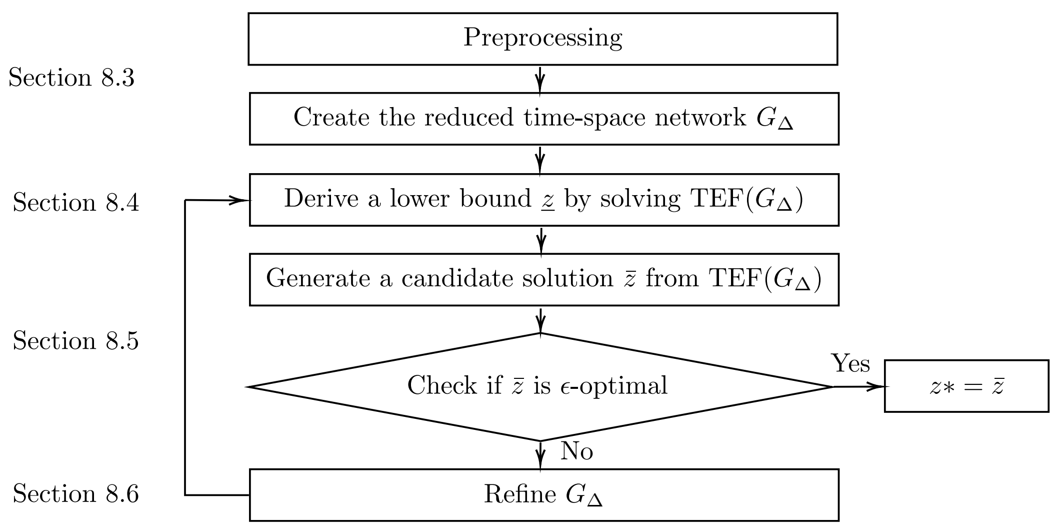


Figure 4: Dynamic discretization framework for the 2E-MALRPS

8.3 Initial reduced network

The first step for the dynamic discretization framework is to create an initial reduced network $\mathcal{G}_\Delta$ satisfying Properties 1-4. Before this creation, our method uses a specialized preprocessing analysis to prune arcs and tighten time windows on the original static network ($\mathcal{G}^{ph}$). Indeed, due to the time dependency of demand, both availability time at platforms and customer time windows can be tightened, as some time instants might be unreachable when linking the origin to the destination of each demand.

An iterative exploration is performed for each commodity in order to perform this tightening, by enumerating the possible combinations of single platform and satellite facilities that could link each demand origin and destination, including garage connections for each echelon. The algorithm repeats this process for each time period when the commodity would be available at each platform. The resulting set of feasible partial paths traced for each commodity defines the possible unreachable time periods for both platforms and customers, which can then be excluded from the static network. After preprocessing the static network, the initial reduced time-space network $\mathcal{G}_\Delta$ is then generated by duplicating each node and arc in $\mathcal{G}^{ph}$ at each relevant time periods, while Properties 1-4 are satisfied.

8.4 Derive a 2E-MALRPS lower bound on $\mathcal{G}_\Delta$

To derive lower bounds for the 2E-MALRPS, the proposed dynamic discretization framework solves the integer program defined by the modified time-space formulation TEF($\mathcal{G}_\Delta$) (Section 8.1), on the reduced time-space network $\mathcal{G}_\Delta$. Compounding our problem, routing decisions boost the iterative growth of the reduced time-space network. In general, due to the presence of short arcs, numerous candidate solutions on the reduced time-space network can have an identical objective value and also share location and routing decisions while differing on the vehicle schedules. Hence, although the reduced network is refined multiple times, some of these candidate solution configurations may not change in terms of cost and structure, while the reduced network size grows with each refinement iteration. This characteristic leads to a certain degeneracy of the solution, which, in turn, increases the complexity of the optimization problem addressed at each iteration.

We propose a specialized procedure to address the potential degeneracy in the reduced time-space network. The objective of this procedure is to handle degenerate solutions within the reduced time-space network, while mitigating the impact arising from the growth underling network. To do so, our approach identifies whether a solution is considered degenerate, to intensively utilize its location and routing configurations to potentially prune the lower bound for the 2E-MALRPS. As illustrated in Algorithm 1, the procedure relies on an integer solution Sol provided by TEF($\mathcal{G}_\Delta$) and the values of its decision variables $(z, x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$.

The proposed procedure solves TEF($\mathcal{G}_\Delta$) if no solution is provided; or explores a given solution configuration within TEF($\mathcal{G}_\Delta$), otherwise. When a solution Sol is provided, the algorithm uses the location, routing and flow decisions as the starting point to solve TEF($\mathcal{G}_\Delta$). The resulting integer solution Sol_{local}, can potentially match the current objective function and structure of Sol with different vehicle schedules. In this scenario, the current solution is updated and the procedure ends. When the objective function of Sol does not match the local solution Sol_{local}, it can be assumed that Sol_{local} cannot be a candidate lower bound of the problem, and cannot be further explored. Hence, the current solution Sol is no longer feasible for the current refined reduced network.

Algorithm 1: Degeneracy($\mathcal{G}_\Delta, Sol$)

input: $\mathcal{G}_\Delta, Sol = (z, x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$
if $Sol \neq \varnothing$ **then**
  $Sol_{local} \leftarrow TEF(\mathcal{G}_\Delta, Sol)$;
  if $z(Sol_{local}) \neq z(Sol)$ **then**
    $Sol \leftarrow TEF(\mathcal{G}_\Delta)$;
  else
    $Sol \leftarrow Sol_{local}$;
  end
else
  $Sol \leftarrow TEF(\mathcal{G}_\Delta)$;
end

8.5 Obtain a 2E-MALRPS upper bound

To determine a feasible upper bound for the 2E-MALRPS, the optimal solution obtained on the reduced time-space network ($\mathcal{G}_\Delta$) must remain feasible when evaluated with the actual travel times for all arcs. A relaxed form of the compact model introduced in Section 6 is proposed for this purpose. The relaxed compact formulation is defined by isolating the sets of timing constraints (12)-(23), (33), and (34), that is, the vehicle scheduling and synchronization constraints, as well as constraints concerning the availability and due time of each OD demand. The relaxed compact model also excludes the constraints (2)-(11) and (24)-(32) related to routing, location, vehicle capacity, and commodity allocation since these decisions remain feasible for the 2E-MALRPS on the reduced time-space network.

The procedure then consists of solving this relaxed compact formulation using the optimal solution structure, i.e., the location, routing, and allocation decisions, obtained by the time-space model on the reduced network $\mathcal{G}_\Delta$, as well as the actual travel times. Obtaining an optimal solution to this problem means the solution structure of the of the reduced network is feasible with the actual travel times and, thus, that it is a feasible solution and an upper bound for the 2E-MALRPS. The algorithm then stops and advances to the refinement step.

When an optimal solution to the relaxed compact formulation is not found, the procedure iteratively examines the integer solutions identified while computing the lower bound (Section 8.4), performing the same evaluation described above for the optimal solution. If an optimal solution to the relaxed problem is found and if the corresponding solution value is better (lower) than the current upper bound (if any), then the upper bound is updated and the procedure terminates. Otherwise, one proceeds to the next integer solution. For each such solution, this procedure allows the dynamic discretization framework to define potentially good quality upper bounds, without the necessity to have a well-refined reduced time-space network.

8.6 Refinement

The final step of the dynamic discretization framework is to refine the reduced time-space network. Whether the integer solution is feasible for the 2E-MALRPS or not, short arcs may still be found within the routing decisions, thus potentially violating some of the temporal constraints of the system. By Lemma 2, we have that the solution is a lower bound for the 2E-MALRPS and thus, the reduced time-space network must be refined. Insights on how to refine the reduced time-space network may be derived form the lower bound obtained by TEF($\mathcal{G}_\Delta$) and the short arcs in the integer solution it entails. Refining the reduced network in terms of these short arcs, extracted from the lower bound obtained at each iteration, is crucial to strengthen the reduced time-space network and improve the precision of the lower bound for the 2E-MALRP in future iterations.

The proposed refinement procedure, Algorithm 2, then consists in extending short arcs, that is, the arcs $((i,t),(j,t'))$ in TEF($\mathcal{G}_\Delta$) with short travel times $t' < t + \tau_{ij}$ relative to the pair of nodes $((i,t),(j,t+\tau_{ij}))$ in the initial problem, while ensuring that Properties 1-4 remain valid for the reduced network after being refined.

Algorithm 2: Refinement($\mathcal{G}_\Delta, Sol$)

input: $\mathcal{G}_\Delta, Sol = (z^*, (\mathcal{V}^*_\Delta, \mathcal{A}^*_\Delta))$
for $((i,t),(j,t')) \in \mathcal{A}^*_\Delta$ **do**
  if $t' \leq t + \tau_{ij}$ **then**
    if $isFeasible(\mathcal{G}_\Delta, (i, t' - \tau_{ij}))$ **then**
      if $j \in \mathcal{Z}^{ph}$ ***AND*** $\delta^-((j,t')) > 1$ **then**
        AddSatellite($((i,t),(j,t'))$);
      end
      AddNode($\mathcal{G}_\Delta, (i, t' - \tau_{ij})$);
      Restore($\mathcal{G}_\Delta, (i, t' - \tau_{ij})$);
    end
    if $isFeasible(\mathcal{G}_\Delta, (j, t + \tau_{ij}))$ **then**
      if $j \in \mathcal{Z}^{ph}$ ***AND*** $\delta^-((j,t')) > 1$ **then**
        AddCut($(j, t + \tau_{ij})$);
      end
      AddNode($\mathcal{G}_\Delta, (j, t + \tau_{ij})$);
      Restore($\mathcal{G}_\Delta, (j, t + \tau_{ij})$);
    end
  end
end

Notice that, due to the degeneracy of the solution that routing decisions bring to the dynamic discretization framework, refining a short arc in terms of one of its extreme points does not exclude its counterpart to potentially appear in the following iterations. We, therefore, propose a two-way refinement procedure to extend short arcs of the reduced time-space network based on the analysis of the integer solution TEF($\mathcal{G}_\Delta$) and the two extreme points of the arc. The general procedure, Algorithm 2, examines each arc of the solution to the reduced network TEF($\mathcal{G}_\Delta$), searching for short arcs which can be extended. For each such short arc, the procedure checks whether the extended arc is feasible with respect to the temporal properties of the nodes involved. When extending a short arc, Algorithm 2 relies on additional procedures, to add new nodes to the reduced time-space network, Algorithms 3 and 4, and to restore the reduced network, that is, to update the arc connections on the reduced network given the new nodes added and the corresponding waiting times, Algorithms 5 and 6, respectively, while ensuring that Properties 1-4 hold.

Algorithm 3: AddSatellite$(((i,t),(j,t')),\delta^-((j,t')))$

input: $((i,t),(j,t'))$
$\Phi \leftarrow$ OffSet$((j,t'),\delta^-((j,t')))$;
for $(j,\bar{t}) \in \Phi$ **do**
  if $(i,t') \notin \mathcal{G}_\Delta$ ***AND*** *isFeasible*$(\mathcal{G}_\Delta,(j,\bar{t}))$ **then**
    AddNode$(\mathcal{G}_\Delta,(j,\bar{t}))$;
    Restore$(\mathcal{G}_\Delta,(j,\bar{t}))$;
  end
  UpdateWaitingArcs$((j,\bar{t}))$;
  AddCut$((j,\bar{t}))$;
end

Algorithm 3 ensures that no flow transshipment takes place between first echelon routes. Recall (Section 8.1) that, even though constraint (58) is introduced as a mean to reduce solution errors in the reduced time-space network under different discretization granularity levels, it might not avoid solutions where commodities are transshipped among routes at satellites. The objective of the proposed procedure is twofold. First, to add supplementary time-space nodes of the respective satellite, both backward and forward in time with an offset of 1 (as the finer granularity possible) in terms of the number of inbound connections $\delta^-((v,t))$ of the time-space satellite (v,t) being refined. The procedure then sets their feasible outbound connections, Algorithm 5 and updates the corresponding waiting times, Algorithm 6. Second, to mark each time-space satellite added and replace their respective constraints (58) with constraints (41) for the next iteration of the solution framework.

Algorithm 4 adds time-space nodes to the reduced network and then sets their feasible outbound connection and updates the corresponding waiting times. By Property 4, we assume that waiting at customers is possible before their respective time windows. Therefore, any new time-space customer added to the reduced time-space network within the customer's time windows can connect with other time-space customers within and before (but never after) their time windows. Time-space customers located before the time windows are set to receive inbound connections with outbound connections at or after the lower bound of the customer's time window.

Algorithm 4: AddNode$((i,t'))$

input: $\mathcal{G}_\Delta,(i,t') \quad (i,t') \quad with \quad t_b < t' < t_{b+1}$
if $(i,t') \notin \mathcal{G}_\Delta$ **then**
  $\mathcal{G}_\Delta \leftarrow \mathcal{G}_\Delta \cup (i,t')$;
  for $((i,t_b),(j,t)) \in \mathcal{A}_\Delta$ **do**
    AddArc$(((i,t'),(j,t)))$;
  end
  UpdateWaitingArcs$((i,t'))$;
end

Algorithm 5 updates the reduced time-space network arcs given the new time-space nodes added, while maintaining the feasible longest distance between nodes and Properties 1-4 are satisfied.

Algorithm 5: Restore($\mathcal{G}_\Delta, (i, t')$)

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input: $\mathcal{G}_\Delta, (i, t') with t_b < t' < t_{b+1}$

if $(i, t') \notin \mathcal{G}_\Delta$ **then**
    for $((i, t_b), (j, t)) \in \mathcal{A}_\Delta$ **do**
        $t" \leftarrow arg\,max\{d \in \mathcal{T}_j(\Delta) | d \leq t' + \tau_{ij}\}$;
        if $t" \neq t'$ **then**
            $Delete(((i, t'), (j, t)))$;
            $AddArc(((i, t'), (j, t")))$;
        end
    end
    for $((j, t), (i, t_b)) \in \mathcal{A}_\Delta with t + \tau_{ij} \geq t'$ **do**
        $Delete(((j, t), (i, t_b)))$;
        $AddArc(((j, t), (i, t')))$;
    end
end

Waiting times are updated separately by Algorithm 6, as each node type handles waiting time differently. Notice that, waiting times for the entire system are constrained by the availability times at platform facilities, the maximum waiting time at satellite facilities, and the time before getting to customers, and at the customer time windows.

9 Computational results

This section presents and discusses the results of experiments conducted to asses the performance of the proposed mathematical formulations and solution framework for the 2E-MALRPS. We first introduce the instances used in the computational study in Section 9.1. We then compare, in Section 9.2, the compact and time-space formulations in terms of computational efficiency and quality of upper and lower bounds, when addressed directly with CPLEX. We also illustrate in that section some of the challenges and impact of granularity on the discretization of time in the time-space formulation. We present the calibration analysis of the dynamic discretization discovery solution framework, with respect to the level of granularity and the impact of degeneracy, Section 9.3. The results of a series of experiments illustrating the performance of the solution framework and the effects of problem instance characteristics are then analyzed in Section 9.4.

The experiments were conducted on a single machine with Intel(R) Core(TM) i7-7800X with 128 GB of RAM running Linux. The mathematical formulation and the proposed solution framework are implemented in C++ using IBM ILOG CPLEX concert technology 20.1. The MIPs used within the solution framework were solved with an optimality gap tolerance of 1% as the stopping criterion. Computation times reported are in seconds. The lower bound values

Algorithm 6: UpdateWaitingArcs((i, t'))

input: $\mathcal{G}_\Delta, (i, t') \quad with \quad t_b < t' < t_{b+1}$

if $i \in \mathcal{P}^{ph}$ **then**
    $Delete(((i, t_b), (i, t_{b+1})))$;
    $AddArc(((i, t_b), (i, t')))$;
    $AddArc(((i, t'), (i, t_{b+1})))$;
end
if $i \in \mathcal{Z}^{ph}$ **then**
    for $(i, t) \in \mathcal{T}_i(\Delta)$ **do**
        if $t < t' \leq t + W^2_{max}$ **then**
            $AddArc(((i, t), (i, t')))$;
        end
    end
end
if $i \in \mathcal{C}^{ph}$ **then**
    for $(i, t) \in \mathcal{T}_i(\Delta)$ **do**
        if $t \geq a_i$ **then**
            $AddArc(((i, t'), (i, t)))$;
        end
    end
end

provided on results obtained by the proposed solution framework correspond to, either the optimal solution obtained by the time-space model on the reduced network, or the best linear relaxation obtained througouth the optimization process, when the optimally gap tolerance is not reached within given time limit.

9.1 Instances

We define our testbed based on the instances introduced by Dellaert et al. (2019) for the 2EVRPTW, since no instances are available in the literature involving the integrated treatment of the attributes considered in the 2E-MALRPS. The testbed instances include the 2E-MALRPS set of attributes, e.g., time-dependent origin-destination demands, vehicle garages, and fleet synchronization, which are not included in the original instance configuration proposed by Dellaert et al. (2019). The testbed consists on five sets of instances differentiated by the number of customers. We extend three instance sets with 15, 30, and 50 customers (Dellaert et al., 2019), and create two small-sized instance sets with 5 and 10 customers by extracting the customers with the minimum distance to satellites from the set of instances with 100 customers of Dellaert et al. (2019).

Each instance set is composed of 60 instances, divided into four categories CA, CB, CC, and CD, differing in the time windows width and customer-demand values. The instances simulate a circular urban area, divided into three concentric sections for platforms, satellites, and customers. To introduce OD demands, we randomly located supplier points within the platform section for each instance, and assigned an unique OD demand to each supply point. Availability time for each OD demand on each platform is defined by the ceiling (nearest higher integer) of the Euclidean distance between the supply point and the platform. Availability times are then included in the temporal components of the systems as a global temporal offset ρ based on the latest availability time in the system. The offset ρ is added to each customers' time window to include the additional time when the demand is available at platforms, yielding $[a_i + \rho, b_i + \rho]$. The maximum waiting time at satellites is set to 4, while service time at customers is set to 2. Load capacities for vehicles given in Dellaert et al. (2019) are considered to be fixed, where first-level vehicles have a capacity of $cap_1 = 200$ and second-level vehicles have a capacity of $cap_2 = 50$. Travel costs are computed as the ceiling of Euclidean distances. The proposed 2E-MALRPS testbed is available at https://github.com/davesco24/2emalrpslib

9.2 Compact vs Time-expanded models and the effect of granularity

We first focus on the effectiveness of the compact and time-space formulations, as well as on the impact of the granularity of the time discretization on the behaviour of the time-space formulation, in terms of solution quality and computational efficiency. Three sets of small instances, with 5, 10 and 15 customers, are used for benchmarking and the formulations are solved directly using CPLEX. The time-space formulation is solved for a complete time-space network with $\bar{\Delta}$ time periods.

Table 1 displays the comparative performance results. It provides the number of OD demands for each instance set, the schedule length (Ψ), the total number of instances for each set (NI), the number of instances for which feasible (FUB) and optimal (OUB) solutions (upper bounds) were found for each formulation within the maximum time limit of 2.5 hours, the average run-time taken to address an instance of the set (CPUsec), and the average root gap (RG) for each formulation. The root-gap values aim to measure the quality of the lower bounds obtained by each formulation, and are computed as the percentage difference between the initial lower bound, obtained by the LP relaxation of the respective model at the root of the branch-and-bound tree, and the best integer solution obtained for the instance.

The results reported in Table 1 show the expected performance similarity with respect to the upper bounds, but remarkable differences on the lower-bound and run-time values. When compared to the compact formulation, the linear relaxation of the time-space formulation provides much better lower bounds (with an average improvement of some 23%), but is usually slower at proving optimality. Nevertheless, as the problem size increases, its behavior is appears more robust, providing feasible solutions for 51 instances, as compared to only 11 for the compact formulation on instances with 15 OD demands. We observe that the overall lower bound improvements by the time-space formulation result from the exclusion of big-M coefficients and vehicle indexes in the flow variables, which help reduce noise in the mathematical model and the symmetries in the routing, respectively.

Our experiments on instances with 15 OD demands show a significant reduction in the numbers of feasible and optimal solutions for both formulations. Multiple factors contribute to this behavior. On the one hand, the compact formulation presents a poor LP-relaxation of the MIP model. On the other hand, the time-space model, despite providing better LP-relaxations, suffers from scalability issues provoked from the size of the time-space representation of the

Instances			Compact				Time-expanded			
OD	**Ψ**	**NI**	**FUB**	**OUB**	**CPUsec**	**RG (%)**	**FUB**	**OUB**	**CPUsec**	**RG (%)**
5	100	60	56	48	2816.21	30	60	50	1706.21	20
10	100	60	7	2	8759.18	49	60	33	4453.92	27
15	200	60	11	0	9000.00	59	51	1	8977.97	33

Table 1: Performance comparison of compact and time-expanded formulations

underlying network, which, in turn, leads to larger, and thus, harder to solve, integer programs than those defined by the compact formulation. This characteristic does not only influence the quality of the optimization solutions but also makes it more challenging to tackle problems with a larger number of OD demands or longer schedule lengths.

The discretization granularity plays a key role in the trade off between the accuracy and performance of the time-space formulation. On the one hand, as seen in the results of Table 1, a finely discretized time-space network provides an accurate representation of the system, but at the price of a large integer problem. On the other hand, the size of the time-space representation and integer problem can be reduced by lowering the number of time instants (see also Section 8), but at the price of a decrease in solution quality. Figure 5 illustrates this trade off, by displaying average performance measures of the time-space formulation under different granularity levels for the set of instances with 15 OD demands and a schedule length of 200. Results are displayed the complete time-space network, with $\bar{\Delta} = 200$ and four coarser granularity levels. Four performance measures are considered, namely, the number of incumbent (best feasible integer) solutions found, **NIS**, measuring the network accuracy; the optimality gap %, **OGP**, obtained within a time limit of 2.5 hours, the solution-cost increment %, **SCI**, computed with respect to the complete time-space network, and the model-size reduction %, **MSR**.

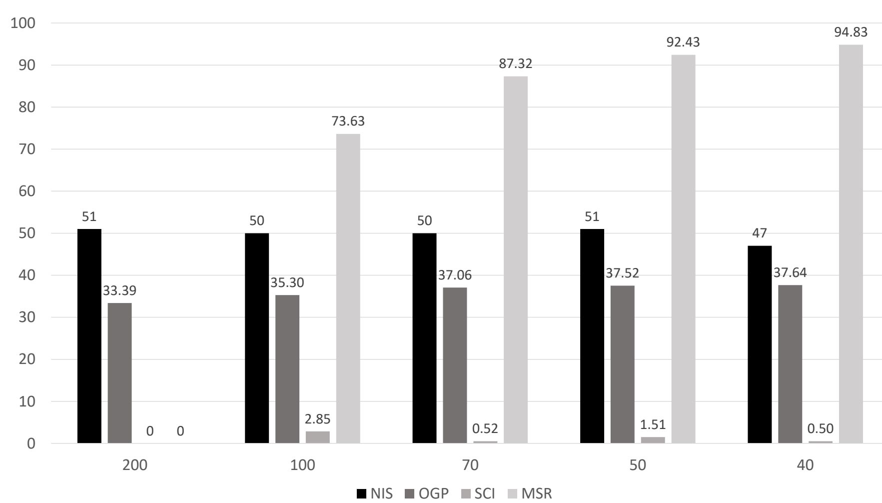

Figure 5: Number of incumbent solutions (NIS), optimality gap % (OGP), solution-cost increment % (SCI), and model-size reduction % (MSR) versus granularity level

The set of time instants under consideration and corresponding coarser granularity levels enable a considerable reduction in the dimension of the integer program (MRS), without a significant degradation of performance (NIS, OGP, and SCI). The observed accuracy loss is driven by two main aspects: network aggregation and loss of consolidation. The aggregation of nodes and arcs resulting from time discretization, not only leads to significantly smaller optimization problems, as depicted by the MSR (with an average model reduction of about 87%), but also adversely affects the temporal preciseness of the network. Notice that, the standalone time-space formulation is set to limit satellites to one inbound per time period to avoid freight transshipment between first-echelon vehicles (see Section 7). This, along with coarser granularity, often renders infeasible a number of commodity flows, as freight consolidation becomes more restrictive with satellite aggregation. The latter is reflected with the greater cost increment of the integer solution, which, on average, tends to grow by 1.3% compared to the complete time-space network. This reflects the loss of feasible low-cost connections and satellite options for freight consolidation leading to generally higher solution costs. Observe that, while the number of incumbent values is found to generally increase as the granularity of the discretization increases, the quality of the optimality gap is not improved, mostly due to the loss on accuracy on the optimization problem and increased solution costs.

9.3 Calibrating the dynamic discretization discovery solution framework

This section presents the experimental results of the DDD solution framework under different parametric configurations. First, we evaluate its performance of the DDD under different levels of granularity. Second, we investigate the effects of degeneracy (Section 8.4). Experiments are performed on the instances with 15 OD demands. We impose a time limit of two and a half hours and an optimality gap of 1% to the DDD, and a time limit of one hour to address the reduced time-space formulation at each iteration. As we focus on the DDD, the set of properties introduced in Section 8.1 are part of these experiments, which allows the construction of time-space networks with coarser granularities without introducing infeasibility.

The results of the level-of-granularity experiments are graphically shown in Figures 6 and 7, according to the type of test instances (i.e., CA, CB, CC, and CD) and the granularity level, namely, fine ($\Delta = 18$), moderate ($\Delta = 6$), and coarse ($\Delta = 2$). Figure 6 displays the distributions of the optimality gap % (**OGP**) and the number of incumbent solutions (**NIS**) obtained by the DDD for the three granularity values considered. Figure 7, on the other hand, displays the same information for the average time used by the reduced time-space formulation at each iteration to obtain the best integer solution (**TUB**), and for the average total run time (**ATT**).

The experimental results show, again, that coarse granularity leads to a good general performance, in terms of both root gap and run time, while a finer granularity leads to larger integer programs, e.g., 30% larger, on average, under moderate granularity compared to the coarse-granularity case. This yields programs which are difficult to address in a reasonable time (Figure 7), resulting in weaker lower bounds (Figure 6). This effect is most noticeable for instances with wide and sparse time windows (e.g., types CC and CD).

While coarse granularity generally yields tighter lower bounds, it leads, however, to a very inaccurate representation of the system. Moderate granularity, on the other hand, provides sufficient temporal preciseness and thus, a more accurate representation of the system; however,

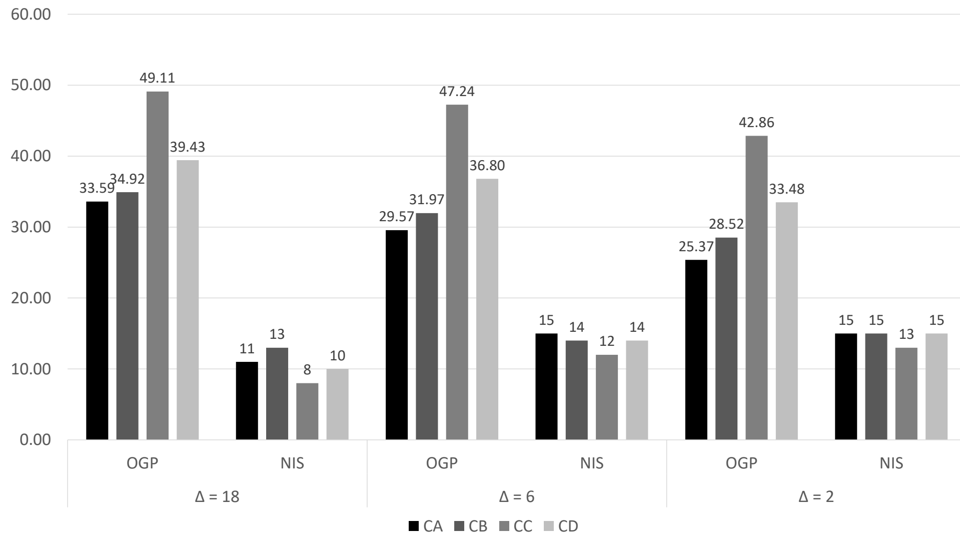

Figure 6: Dynamic discretization discovery: Number of incumbent solutions (NIS) and optimality gap % (OGP) versus granularity levels and instance type

it also generally results in weaker lower bounds due to the large size of the integer problem. The latter makes the use of moderate granularity in a solution framework less suitable since it can negatively affect the performance of the DDD, in particular, when the size of the reduced time-space network grows with each iteration.

The lack of preciseness present in a reduced time-space network derived with coarse granularity tends to lead to degenerate solutions. This results from the wide number of candidate solutions that can be obtained on the reduced time-space network with identical objective value and distribution structure, but different vehicle schedules. We observe that, unless a sufficiently fine granularity is considered, time-space formulations with moderate and coarse discretization are more susceptible to have degenerate solutions, which can impact the complexity of the optimization problems on each iteration of the DDD. Figure 8 illustrates the impact of the degeneracy procedure using a coarse granularity ($\Delta = 2$).

The performance of the DDD is illustrated by contrasting the average optimality gap obtained by the DDD using the degeneracy procedure, for each instance type CA, CB, CC, and CD, to that of the DDD without the degeneracy procedure, identified as NCA, NCB, NCC, and NCD. The experimental results show that, using the degeneracy procedure leads to a general improvement of the optimality gap over the entire instance set. One also observes that, larger numbers of platform and satellite facilities, as well as wider customer time windows, are the two key factors to drive degeneracy on the integer problem. Both of these components reduce the temporal preciseness of the reduced time-space network and provide broader refining options for short arcs. Instances with tighter time windows, most notably, instances of CA and CB types, tend to benefit from the degeneracy procedure, displaying optimality-gap improvements of 17% and 30%, respectively, compared to the results on same instances types without the degeneracy procedure. On the other hand, instances with broader and sparse time windows, e.g., instances of CC and CD types, display improvements of 14% and 6%, respectively. The latter reflects the efficiency of the proposed degeneracy procedure in supporting the DDD to

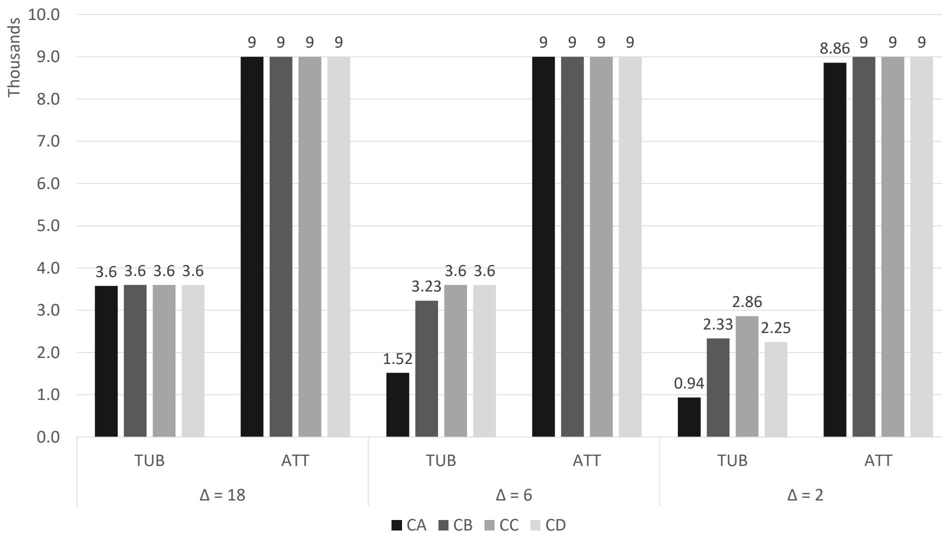

Figure 7: Dynamic discretization discovery: Reduced time-space network run times (TUB) and DDD run times (TUB) versus granularity levels and instance type

avoid being trapped on lower bounds values for several iterations and enabling tightening the general lower-bound values obtained by the DDD.

9.4 Performance of the dynamic discretization discovery solution framework

We investigate the performance of the DDD solution framework for the 2E-MALRPS on the instances with 5, 10, 15, 30, and 50 OD demands. Based on the experiments presented in Section 9.3, computational tests are performed using the coarsest discretization granularity possible for each instance to decrease the size of the underlying network, thus enabling a further reduction of the time required to solve the integer program. The stopping criteria are an optimality gap of less or equal to 1% and a maximum run time of 2.5 hours for small-sized instances (5 and 10 OD demands), 5 hours for medium-sized instances (15 OD demands), and 10 hours for large-sized instances with more than 15 OD demands. Tables 2 and 3 summarize the results of the experiments, the latter corresponding to the case when availability times are disabled. The tables display the number of platforms, satellites, and OD demands for each instance set, the schedule length (Ψ), the granularity of the time discretization (Δ), the total number of instances for each instance set (NI), the number of instances for which feasible (FUB) and optimal (OUB) solutions (upper bounds) are found, as well as the average optimality gap (OG(%)) and the average run-time (CPUsec) for the respective instance set.

The results of the computational experiments displayed in Table 2 show that the DDD clearly outperforms the compact formulation and the time-expanded one addressed head on. We see that the DDD is able to provide feasible solutions for the complete set of instances with less than 50 OD demands as well as providing the optimal solution for the complete set of instances with 5 and 10 OD demands. Concerning instances with 15 OD demands, the DDD is able to solve 4 instances to optimality, with an average optimality gap of 18% for the remaining

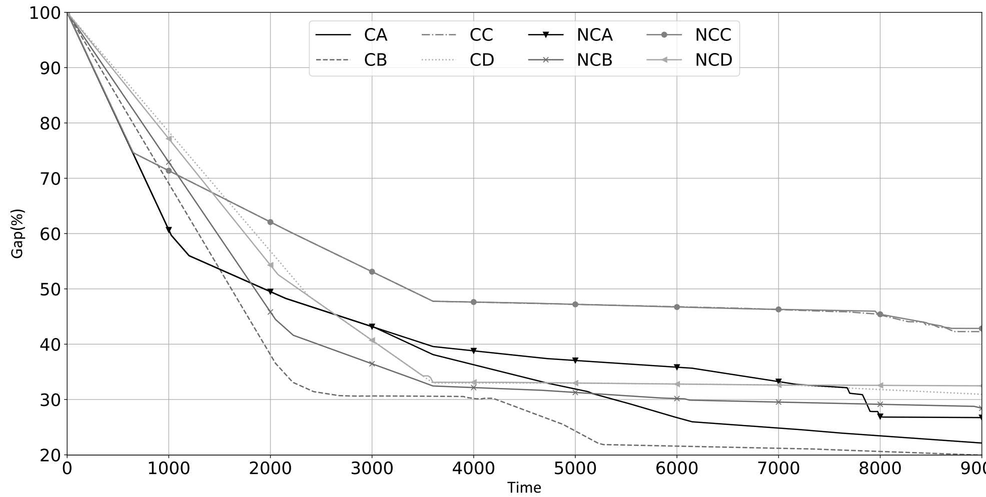

Figure 8: Degeneracy: Optimality gap (%) versus run time and instance type

instances, while the number of optimal solutions for instances with 30 OD demands is one. We observe that, despite a very coarse granularity, which can significantly reduce the size of the optimization problem, the inaccuracy of the network causing degeneracy reduces the rate at which the lower bound can increase at each iteration. This phenomenon is better captured by results on instances with 30 and 50 OD demands, where the optimality of most solutions obtained by the DDD remains unproven. As expected, the proposed degeneracy procedure significantly reduces the effects of the degeneracy on the solution, in particular in cases with tight time windows, as depicted in Figure 8. However, while the degeneracy method helps to avoid the solution framework being trapped in a fixed lower bound, the lower bound values do not increase significantly in several iterations. The procedure is shown to mainly help achieve a good upper bound faster, rather than improving the lower bound at a fast rate. Therefore, although the solution framework can converge to provably high-quality upper bounds values for larger instances, the slow incremental rate of the lower bound makes it harder to assess the true quality of the solution.

Further performance analysis is provided by disabling the availability times in all instances. The results summarized in Table 3 show that the DDD algorithm is able to provide good quality solutions for all instances sets in this case as well. Similar to the results obtained on instances with tight availability times, the computational experiments shows that the DDD is able to achieve optimality for 20 out of 20 instances with 5 and 10 OD demands with disabled availability times. At the same time, the DDD converges to feasible solutions for the complete set of instances with 15 and 30 OD demands, while obtaining feasible solutions for 17 out of 20 instances for instances with 50 OD demands. In terms of computational efficiency, the solution framework presents a more robust performance on instances with tighter availability times compared to the results obtained on instances with disabled availability times. The broader number of availability options resulting from the lack of precise time moments yields larger time-space networks and harder integer problems at each iteration of the DDD. As already

Platforms	Satellites	OD	Ψ	Δ	NI	FUB	OUB	OG(%)	CPUsec
2	3	5	100	2	20	20	20	0.00	1123.51
3	5	5	100	2	20	20	20	0.00	1076.03
6	4	5	100	2	20	20	20	0.00	1390.17
2	3	10	100	2	20	20	20	0.22	4451.20
3	5	10	100	2	20	20	20	0.27	4780.29
6	4	10	100	2	20	20	20	0.29	4841.81
2	3	15	200	2	20	20	3	18.55	16346.97
3	5	15	200	2	20	20	1	20.09	17545.86
6	4	15	200	2	20	20	0	24.19	18000.00
2	3	30	200	2	20	20	1	47.93	34374.49
3	5	30	200	2	20	20	0	47.77	36000.00
6	4	30	200	2	20	20	0	51.52	36000.00
2	3	50	200	2	20	18	0	42.46	36000.00

Table 2: Performance of DDD solution framework for 15, 30 and 50 OD demands

mentioned, this reduces the quality of the lower bounds, impacting the number of optimal solutions found for instances with 15 and 30 OD demands.

Cost-wise, the solutions on instances with disabled availability times tend to be cheaper, an average cost reduction of at least 3% is observed, than those obtained on instances with tighter availability times. This operational cost reduction can be attributed to the sets of low-cost routes that can only be used at early time moments, but are unreachable when tighter availability times are present. At the same time, disabling the availability times enables far more options for consolidation of demand at satellite facilities. This greater level of consolidation, leading to a lower number of facilities selected, lowers the system fixed costs related to facility usage. This is more noticeable on instances with late availability times and early due times (as in the case of instances types CA and CB), whereby a greater number of nearby platforms and satellites needs to be open and connected to meet the final destination of each OD demand on time. We thus conclude that, the lack of consideration of availability times can lead to an inaccurate representation of the distribution system, in particular for time-driven systems. This makes the DDD capable of providing not only good quality solutions for decision makers but also avoids the inclusion of routes that might be unreachable when availability times are not being considered.

10 Conclusions and Perspectives

This paper introduces the multi-attribute two-echelon location-routing problem with synchronization constraints (MA-2ELRPS) and presents two mixed-integer programming formulations, involving different modeling techniques to capture time. We also present an exact solution framework that iteratively refines a reduced time-space network and solves the integer program defined by its network configuration to extract lower bounds, in order to solve the problem to optimality or up to a specified tolerance.

Platforms	Satellites	OD	Ψ	Δ	NI	FUB	OUB	OG(%)	CPUsec
2	3	5	100	2	20	20	20	0.34	2013.26
3	5	5	100	2	20	20	20	0.32	2095.67
6	4	5	100	2	20	20	20	0.23	2105.53
2	3	10	100	2	20	20	20	0.50	4689.99
3	5	10	100	2	20	20	20	0.34	4490.47
6	4	10	100	2	20	20	20	0.29	4936.37
2	3	15	200	2	20	20	0	15.21	18000.00
3	5	15	200	2	20	20	0	18.52	18000.00
6	4	15	200	2	20	20	0	15.35	18000.00
2	3	30	200	2	20	20	0	46.66	36000.00
3	5	30	200	2	20	20	0	55.01	36000.00
6	4	30	200	2	20	20	0	52.77	36000.00
2	3	50	200	2	20	17	0	45.09	36000.00

Table 3: Performance of DDD solution framework for 15, 30 and 50 OD demands with no availability time

The computational study reveals the effectiveness of the mathematical formulations and the solution framework. Comparative analyses show that the proposed time-space formulation obtains the same or better upper bounds, and improves the lower bounds for small-instances, relative to the compact formulation. Similarly, the experimental results obtained by the proposed dynamic discretization discovery framework show its effectiveness compared to addressing the compact and time-space models directly. The results also show the capability of the proposed DDD to address large instance sizes. The success of the dynamic discretization discovery framework relies on the proposed degeneracy mitigation procedure, to avoid the solution framework being trapped in a fixed lower bound for multiple iterations, as well as the efficient use of the compact formulation to complement the framework by enhancing the search for feasible upper bounds for the MA-2ELRPS in shorter times.

The bottleneck of the dynamic discretization discovery framework depends on the growing rate of the reduced time-space network at each iteration and more complex degeneracy aspects. Hence, future research should be directed toward studying acceleration techniques and valid inequalities to tighten lower bounds without affecting the complexity of the integer problem. We also consider promising to generalize the usage of the dynamic discretization discovery framework to location-routing problem variants and other related problem settings involving time aspects. The study of uncertainty in several problem parameters is certainly a challenging research avenue. We aim to present contributions in this area soon.

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Complete Result Tables

Tables 4 - 6 display the detailed results of the first set of experiments focusing on the effectiveness of the compact and time-space formulations when addressed directly using CPLEX. Three sets of small instances, with 5, 10 and 15 customers, are used for benchmarking. The time-space formulation is solved for a complete time-space network with $\bar{\Delta}$ time periods.

The tables provide the instance **ID**, the schedule length (Ψ), the number of instances for which feasible (FUB) and optimal (OUB) solutions (upper bounds) were found for each formulation within the maximum time limit of 2.5 hours, the run-time (CPUsec), and the root gap (RG) for each formulation computed as the percentage difference between the initial lower bound, obtained by the LP relaxation of the respective model at the root of the branch-and-bound tree, and the best integer solution obtained for the instance.

Tables 7 - 10 display the detailed results of the set of experiments focusing on the performance of the dynamic discretization discovery (DDD) solution framework for the 2E-MALRPS. Test results are shown for the instances with 5 and 10 OD demands in Table 7, and for the same instances with disabled availability times in Table 8. Tables 9 and 10 display the same type of results for the instances with 15, 30, and 50 OD demands. The experiments are performed using a coarse discretization granularity $\Delta = 2$. The stopping criteria are an optimality gap of less or equal to 1% and a maximum run time of 2.5 hours for small-sized instances (5 and 10 OD demands), 5 hours for medium-sized instances (15 OD demands), and 10 hours for large-sized instances with more than 15 OD demands. The tables display the instance **ID**, the schedule length (Ψ), the best upper bound (**UB**), the run-time (CPUsec), the lower bound (**LB**), and the optimality gap (OG(%)).

Instances		Compact				Time-expanded			
ID	Ψ	FUB	OUB	CPUsec	RG (%)	FUB	OUB	CPUsec	RG (%)
Ca1-2,3,5	100	280	9000.00	133.16	52.44	280	0.14	169.00	39.64
Ca1-3,5,5	100	222	5882.37	150.77	32.09	222	0.88	190.91	14.01
Ca1-6,4,5	100	286	9000.00	138.63	48.84	271	0.37	210.49	22.33
Ca2-2,3,5	100	152	136.68	131.50	13.49	152	0.27	121.45	20.10
Ca2-3,5,5	100	N.A	9000.00	171.20	39.72	284	0.15	174.00	38.73
Ca2-6,4,5	100	219	1567.85	158.27	27.73	219	1.26	160.26	26.82
Ca3-2,3,5	100	287	9000.00	129.20	54.98	287	0.05	246.00	14.29
Ca3-3,5,5	100	220	2122.07	148.70	32.41	220	0.46	191.20	13.09
Ca3-6,4,5	100	226	3702.22	149.31	33.93	226	0.64	176.19	22.04
Ca4-2,3,5	100	358	9000.00	167.00	53.35	358	0.07	190.00	46.93
Ca4-3,5,5	100	168	1637.53	121.61	27.62	168	0.42	138.60	17.50
Ca4-6,4,5	100	239	2242.18	155.49	34.94	239	1.55	202.21	15.39
Ca5-2,3,5	100	199	285.97	158.40	20.40	199	0.48	176.00	11.56
Ca5-3,5,5	100	N.A	9000.00	122.91	33.92	186	0.32	121.00	34.95
Ca5-6,4,5	100	231	1653.81	161.06	30.28	231	54.79	163.53	29.21
Cb1-2,3,5	100	152	347.05	109.96	27.66	152	0.23	131.90	13.23
Cb1-3,5,5	100	168	3045.10	127.26	24.25	168	1.98	121.45	27.71
Cb1-6,4,5	100	N.A	9000.00	155.06	49.16	305	0.31	179.00	41.31
Cb2-2,3,5	100	137	9.49	117.22	14.44	137	0.14	125.60	8.32
Cb2-3,5,5	100	154	1485.30	138.66	9.96	154	0.22	134.25	12.82
Cb2-6,4,5	100	212	1651.25	169.50	20.05	212	1.04	135.13	36.26
Cb3-2,3,5	100	332	9000.00	111.74	66.34	332	0.06	237.00	28.61
Cb3-3,5,5	100	160	2136.07	111.09	30.57	160	0.52	129.13	19.30
Cb3-6,4,5	100	273	1374.15	165.00	39.56	273	0.35	212.07	22.32
Cb4-2,3,5	100	N.A	9000.00	123.36	55.94	280	9000.00	121.16	56.73
Cb4-3,5,5	100	142	628.19	115.94	18.35	142	0.82	118.49	16.56
Cb4-6,4,5	100	267	9000.00	107.58	59.71	267	0.64	209.46	21.55
Cb5-2,3,5	100	129	212.55	106.90	17.13	129	0.47	112.54	12.76
Cb5-3,5,5	100	179	9000.00	128.97	27.95	179	0.71	152.45	14.83
Cb5-6,4,5	100	272	1449.36	114.15	58.03	272	0.43	209.86	22.84
Cc1-2,3,5	100	129	70.72	106.50	17.44	129	9000.00	112.38	12.89
Cc1-3,5,5	100	135	561.41	104.88	22.31	135	3622.10	109.89	18.60
Cc1-6,4,5	100	215	2664.15	138.88	35.41	215	9000.00	147.67	31.32
Cc2-2,3,5	100	122	20.85	99.99	18.04	122	51.00	96.21	21.14
Cc2-3,5,5	100	175	1031.37	119.67	31.62	175	9000.00	149.45	14.60
Cc2-6,4,5	100	190	2296.00	120.47	36.59	190	9000.00	127.08	33.11
Cc3-2,3,5	100	136	47.18	109.67	19.36	136	208.75	115.97	14.73
Cc3-3,5,5	100	142	469.91	113.52	20.06	142	3906.78	128.83	9.28
Cc3-6,4,5	100	228	2241.48	135.00	40.79	228	9000.00	178.48	21.72
Cc4-2,3,5	100	171	76.03	144.86	15.29	171	3160.12	161.00	5.85
Cc4-3,5,5	100	154	360.16	106.23	31.02	154	9000.00	125.80	18.31
Cc4-6,4,5	100	213	2288.94	140.73	33.93	213	9000.00	163.95	23.03
Cc5-2,3,5	100	123	113.27	113.76	7.51	123	9000.00	116.31	5.44
Cc5-3,5,5	100	124	530.07	108.78	12.27	124	246.22	113.17	8.73
Cc5-6,4,5	100	202	1572.93	152.22	24.64	202	9000.00	168.91	16.38
Cd1-2,3,5	100	155	391.97	131.99	14.85	155	1.31	135.22	12.76
Cd1-3,5,5	100	170	545.91	144.00	15.29	170	0.75	159.60	6.12
Cd1-6,4,5	100	254	4119.62	135.40	46.69	254	99.91	187.61	26.14
Cd2-2,3,5	100	140	109.62	118.78	15.16	140	1.72	126.76	9.46
Cd2-3,5,5	100	158	468.17	136.32	13.72	158	6.96	138.20	12.53
Cd2-6,4,5	100	210	4144.27	128.52	38.80	210	25.38	162.90	22.43
Cd3-2,3,5	100	142	120.44	114.20	19.58	142	3.05	117.35	17.36
Cd3-3,5,5	100	147	500.62	109.23	25.70	147	11.45	119.09	18.99
Cd3-6,4,5	100	231	804.23	134.29	41.87	231	600.47	195.98	15.16
Cd4-2,3,5	100	202	146.58	153.80	23.86	202	0.58	168.88	16.40
Cd4-3,5,5	100	162	1542.11	128.99	20.38	162	1.70	138.15	14.72
Cd4-6,4,5	100	222	9000.00	139.23	37.29	222	156.39	184.01	17.11
Cd5-2,3,5	100	178	239.92	134.29	24.56	178	1.13	159.08	10.63
Cd5-3,5,5	100	178	706.87	133.80	24.83	178	5.14	146.54	17.68
Cd5-6,4,5	100	228	1218.31	158.99	30.27	228	191.79	206.46	9.45
Averages			2816.21		30.24		1706.21		20.06

Table 4: Direct solving of both formulations on instances with 5 OD demands

Instances		Compact				Time-expanded			
ID	Ψ	FUB	OUB	CPUsec	RG (%)	FUB	OUB	CPUsec	RG (%)
Ca1-2,3,10	100	N.A	9000.00	101.28	70.21	340	2.81	252.99	25.59
Ca1-3,5,10	100	N.A	9000.00	119.56	61.56	311	84.24	242.91	21.89
Ca1-6,4,10	100	N.A	9000.00	140.54	57.80	333	7.62	233.20	29.97
Ca2-2,3,10	100	N.A	9000.00	136.30	44.82	247	2210.60	192.42	22.10
Ca2-3,5,10	100	N.A	9000.00	164.97	51.48	340	1.21	215.00	36.76
Ca2-6,4,10	100	N.A	9000.00	155.05	54.13	338	2537.13	249.87	26.07
Ca3-2,3,10	100	N.A	9000.00	177.31	48.46	344	0.30	219.00	36.34
Ca3-3,5,10	100	N.A	9000.00	144.49	54.56	318	4.29	298.43	6.16
Ca3-6,4,10	100	N.A	9000.00	157.17	48.81	307	4.15	264.01	14.00
Ca4-2,3,10	100	N.A	9000.00	192.00	56.06	437	0.96	288.50	33.98
Ca4-3,5,10	100	N.A	9000.00	108.11	64.90	308	485.53	226.41	26.49
Ca4-6,4,10	100	N.A	9000.00	135.00	58.97	329	9.32	268.37	18.43
Ca5-2,3,10	100	N.A	9000.00	203.42	53.13	434	8.18	239.07	44.92
Ca5-3,5,10	100	N.A	9000.00	156.13	47.43	297	13.21	228.66	23.01
Ca5-6,4,10	100	N.A	9000.00	150.54	55.59	339	9000.00	276.89	18.32
Cb1-2,3,10	100	N.A	9000.00	144.76	20.02	181	3.35	159.09	12.11
Cb1-3,5,10	100	N.A	9000.00	160.72	42.80	281	3913.71	252.11	10.28
Cb1-6,4,10	100	N.A	9000.00	131.69	56.40	302	4.71	244.43	19.06
Cb2-2,3,10	100	N.A	9000.00	158.80	28.79	223	0.99	198.31	11.07
Cb2-3,5,10	100	N.A	9000.00	148.08	53.87	321	2.54	247.19	22.99
Cb2-6,4,10	100	N.A	9000.00	174.42	31.06	253	1017.00	219.09	13.40
Cb3-2,3,10	100	N.A	9000.00	168.94	49.87	337	0.70	277.90	17.54
Cb3-3,5,10	100	N.A	9000.00	124.96	60.70	318	1061.83	205.65	35.33
Cb3-6,4,10	100	N.A	9000.00	147.48	57.50	347	11.74	230.19	33.66
Cb4-2,3,10	100	N.A	9000.00	179.28	45.84	331	1.04	210.57	36.38
Cb4-3,5,10	100	N.A	9000.00	160.19	51.01	327	806.72	209.83	35.83
Cb4-6,4,10	100	N.A	9000.00	204.76	52.71	433	2519.29	236.08	45.48
Cb5-2,3,10	100	N.A	9000.00	147.70	52.66	312	30.08	233.63	25.12
Cb5-3,5,10	100	N.A	9000.00	148.55	50.65	301	1579.01	206.68	31.33
Cb5-6,4,10	100	N.A	9000.00	155.02	60.05	388	4.77	335.31	13.58
Cc1-2,3,10	100	288	9000.00	175.04	38.58	285	9000.00	195.98	31.24
Cc1-3,5,10	100	N.A	9000.00	170.80	33.02	255	9000.00	186.47	26.88
Cc1-6,4,10	100	N.A	9000.00	214.78	31.38	313	9000.00	202.80	35.21
Cc2-2,3,10	100	268	9000.00	168.17	37.25	271	9000.00	192.73	28.09
Cc2-3,5,10	100	N.A	9000.00	121.90	62.38	324	9000.00	217.08	33.00
Cc2-6,4,10	100	N.A	9000.00	192.73	38.03	311	9000.00	190.65	38.70
Cc3-2,3,10	100	275	2846.39	103.33	62.43	276	9000.00	203.63	25.95
Cc3-3,5,10	100	N.A	9000.00	198.86	38.62	324	9000.00	208.04	35.79
Cc3-6,4,10	100	N.A	9000.00	186.09	32.33	275	9000.00	199.53	27.44
Cc4-2,3,10	100	225	704.69	211.00	6.22	226	9000.00	173.93	22.70
Cc4-3,5,10	100	N.A	9000.00	199.46	36.48	314	9000.00	198.43	36.81
Cc4-6,4,10	100	N.A	9000.00	178.24	43.23	314	9000.00	197.62	37.06
Cc5-2,3,10	100	289	9000.00	150.35	46.68	282	9000.00	132.89	52.88
Cc5-3,5,10	100	279	9000.00	164.20	37.33	262	9000.00	192.31	26.60
Cc5-6,4,10	100	N.A	9000.00	193.91	52.82	411	9000.00	199.93	51.35
Cd1-2,3,10	100	N.A	9000.00	150.64	47.51	287	9000.00	238.43	16.92
Cd1-3,5,10	100	N.A	9000.00	112.75	62.04	297	9000.00	257.23	13.39
Cd1-6,4,10	100	N.A	9000.00	143.80	64.14	401	9000.00	253.06	36.89
Cd2-2,3,10	100	N.A	9000.00	152.94	43.98	273	3145.09	232.92	14.68
Cd2-3,5,10	100	N.A	9000.00	140.98	55.53	317	9000.00	233.32	26.40
Cd2-6,4,10	100	N.A	9000.00	147.20	59.67	365	9000.00	264.63	27.50
Cd3-2,3,10	100	N.A	9000.00	157.92	42.16	273	4709.40	224.49	17.77
Cd3-3,5,10	100	N.A	9000.00	106.47	65.09	305	9000.00	211.74	30.58
Cd3-6,4,10	100	N.A	9000.00	118.58	56.40	272	9000.00	218.97	19.49
Cd4-2,3,10	100	257	9000.00	162.44	36.79	257	5.25	232.53	9.52
Cd4-3,5,10	100	N.A	9000.00	119.79	63.03	324	9000.00	221.01	31.79
Cd4-6,4,10	100	N.A	9000.00	132.04	50.36	266	9006.83	206.24	22.47
Cd5-2,3,10	100	N.A	9000.00	161.04	46.32	300	41.39	199.46	33.51
Cd5-3,5,10	100	N.A	9000.00	139.42	53.68	301	9000.00	260.13	13.58
Cd5-6,4,10	100	N.A	9000.00	200.86	40.75	339	9000.00	220.84	34.86
Averages			8759		48.77		4453.92		26.77

Table 5: Direct solving of both formulations on instances with 10 OD demands

Instances		Compact				Time-expanded			
ID	Ψ	FUB	OUB	CPUsec	RG (%)	FUB	OUB	CPUsec	RG (%)
Ca1-2,3,15	200	415	9001.73	122.35	63.48	335	9000.06	238.22	28.89
Ca1-3,5,15	200	N.A	9000.83	148.71	58.23	356	9000.11	274.37	22.93
Ca1-6,4,15	200	N.A	9000.06	222.82	44.16	399	9000.15	282.2	29.27
Ca2-2,3,15	200	N.A	9000.08	173.73	54.64	383	9000.06	255.09	33.4
Ca2-3,5,15	200	N.A	9000.1	159.7	61.8	418	9000.18	290.96	30.39
Ca2-6,4,15	200	N.A	9000.1	185.41	54.89	411	9000.14	286.52	30.29
Ca3-2,3,15	200	N.A	9000.01	148.6	57.18	347	9000.35	282.37	18.63
Ca3-3,5,15	200	N.A	9000.15	125.84	N.A	N.A	9000.29	301.53	N.A
Ca3-6,4,15	200	N.A	9000.08	203.94	49.52	404	9000.29	252.92	37.4
Ca4-2,3,15	200	372	9001.51	146.4	60.22	368	9000.33	278.64	24.28
Ca4-3,5,15	200	N.A	9000.09	117.95	63.6	324	7656.37	282.78	12.72
Ca4-6,4,15	200	N.A	9000.16	198.48	46.36	370	9000.43	279.8	24.38
Ca5-2,3,15	200	N.A	9000.19	138.9	N.A	377	9000.2	322.81	N.A
Ca5-3,5,15	200	N.A	9000.77	127	65.11	364	9001	342.65	5.86
Ca5-6,4,15	200	N.A	9000.12	170.39	51.04	348	9000.88	219.21	37.01
Cb1-2,3,15	200	N.A	9000.06	137.14	N.A	369	9000.21	303.62	N.A
Cb1-3,5,15	200	N.A	9000.08	142.6	61.97	375	9000.2	311.2	17.01
Cb1-6,4,15	200	N.A	9000.09	201.5	53.36	432	9000.2	266.34	38.35
Cb2-2,3,15	200	N.A	9000.07	136.38	N.A	N.A	9000.23	324.83	N.A
Cb2-3,5,15	200	N.A	9000.89	135.95	65.41	393	9000.17	244.29	37.84
Cb2-6,4,15	200	N.A	9000.12	206.03	51.29	423	9000.08	291.12	31.18
Cb3-2,3,15	200	N.A	9000.08	134.33	63.3	366	9000.1	288.16	21.27
Cb3-3,5,15	200	N.A	9000.18	123.95	62.89	334	9000.28	245.48	26.5
Cb3-6,4,15	200	N.A	9000.06	189.21	53.17	404	9000.11	239.55	40.7
Cb4-2,3,15	200	383	9000.05	132.56	62.87	357	9000.1	244.67	31.46
Cb4-3,5,15	200	N.A	9000.13	124.11	64.44	349	9000.16	302.74	13.26
Cb4-6,4,15	200	N.A	9000.05	197.66	53.38	424	9000.08	241.71	42.99
Cb5-2,3,15	200	N.A	9000.09	167.51	56.15	382	9000.16	286.44	25.02
Cb5-3,5,15	200	N.A	9000.1	142.28	58.88	346	9000.17	252.27	27.09
Cb5-6,4,15	200	N.A	9000.12	198.82	46.41	371	9000.16	199.91	46.12
Cc1-2,3,15	200	330	9000.08	124.39	62.08	328	9000.75	197.01	39.94
Cc1-3,5,15	200	N.A	9000.16	150.26	56.95	349	9001.78	265.58	23.9
Cc1-6,4,15	200	N.A	9000.12	206.61	N.A	N.A	9000.21	264.94	N.A
Cc2-2,3,15	200	376	9000.07	132.7	64.71	N.A	9000.87	219.49	41.63
Cc2-3,5,15	200	N.A	9000.17	146.01	N.A	N.A	9000.26	264.3	N.A
Cc2-6,4,15	200	N.A	9000.15	193.12	88.11	1624	9000.62	265.28	83.67
Cc3-2,3,15	200	N.A	9000.07	124.64	65.57	362	9000.4	157.95	56.37
Cc3-3,5,15	200	330	9000.06	111.49	66.22	723	9000.2	248.01	24.85
Cc3-6,4,15	200	N.A	9000.15	186.43	N.A	N.A	9000.24	220.21	N.A
Cc4-2,3,15	200	371	9001.46	136.49	63.21	397	9000.3	164.98	55.53
Cc4-3,5,15	200	N.A	9000.09	109.04	85.28	741	9000.24	230.93	68.84
Cc4-6,4,15	200	N.A	9000.12	175.79	N.A	N.A	9004.36	186.01	N.A
Cc5-2,3,15	200	324	9004.63	114.83	64.56	336	9000.33	221.03	31.78
Cc5-3,5,15	200	N.A	9000.08	124.09	N.A	N.A	9000.48	245.88	N.A
Cc5-6,4,15	200	N.A	9000.15	164.94	N.A	N.A	9000.34	245.88	N.A
Cd1-2,3,15	200	330	9000.26	140.24	57.11	327	9000.74	223.64	31.61
Cd1-3,5,15	200	N.A	9000.11	152.67	54.02	332	9000.25	233.22	29.75
Cd1-6,4,15	200	N.A	9000.08	185.32	54.24	405	9000.3	266.85	34.11
Cd2-2,3,15	200	N.A	9000.04	152.37	57.08	355	9000.1	225.22	36.56
Cd2-3,5,15	200	N.A	9000.08	157.95	57.77	374	9000.15	268.27	28.27
Cd2-6,4,15	200	N.A	9000.09	204.93	50.02	410	9000.13	299.23	27.02
Cd3-2,3,15	200	348	9000.08	132.98	59.95	332	9000.09	204.26	38.48
Cd3-3,5,15	200	N.A	9000.19	138.47	53.22	296	9000.16	196.86	33.49
Cd3-6,4,15	200	N.A	9000.08	192.96	52	402	9000.2	267.6	33.43
Cd4-2,3,15	200	385	9000.04	159.58	58.33	383	9000.06	238.7	37.68
Cd4-3,5,15	200	N.A	9000.06	118.15	62.13	312	9000.18	202.29	35.16
Cd4-6,4,15	200	N.A	9000.06	165.67	56.52	381	9000.26	243.74	36.03
Cd5-2,3,15	200	N.A	9000.07	145.88	58.44	351	9000.08	238.75	31.98
Cd5-3,5,15	200	N.A	9000.07	157.8	55.04	351	9000.13	238.55	32.04
Cd5-6,4,15	200	N.A	9000.14	164.94	57.71	390	9000.7	221.03	43.33
Averages			9000.29		58.88		8854.65		33.39

Table 6: Direct solving of both formulations on instances with 15 OD demands

ID	Ψ	UB	CPUsec	LB	OG (%)	ID	Ψ	UB	CPUsec	LB	OG (%)
Ca1-2,3,5	100	280	84.48	280	0.00	Ca1-2,3,10	100	340	2632.89	340	0.00
Ca1-3,5,5	100	222	240.83	222	0.00	Ca1-3,5,10	100	311	2665.06	311	0.00
Ca1-6,4,5	100	271	326.86	271	0.00	Ca1-6,4,10	100	333	2757.28	333	0.00
Ca2-2,3,5	100	152	259.12	152	0.00	Ca2-2,3,10	100	247	4610.19	247	0.00
Ca2-3,5,5	100	284	146.15	284	0.00	Ca2-3,5,10	100	340	2719.61	340	0.00
Ca2-6,4,5	100	219	145.68	219	0.00	Ca2-6,4,10	100	338	4683.19	338	0.00
Ca3-2,3,5	100	287	131.13	287	0.00	Ca3-2,3,10	100	344	2724.71	344	0.00
Ca3-3,5,5	100	220	112.50	220	0.00	Ca3-3,5,10	100	318	2981.32	318	0.00
Ca3-6,4,5	100	226	318.72	226	0.00	Ca3-6,4,10	100	307	2815.94	307	0.00
Ca4-2,3,5	100	358	338.31	358	0.00	Ca4-2,3,10	100	437	2262.91	437	0.00
Ca4-3,5,5	100	168	148.84	168	0.00	Ca4-3,5,10	100	308	2838.09	308	0.00
Ca4-6,4,5	100	239	314.10	239	0.00	Ca4-6,4,10	100	329	2320.93	329	0.00
Ca5-2,3,5	100	199	266.40	199	0.00	Ca5-2,3,10	100	434	2629.16	434	0.00
Ca5-3,5,5	100	186	248.09	186	0.00	Ca5-3,5,10	100	297	2260.71	297	0.00
Ca5-6,4,5	100	231	396.00	231	0.00	Ca5-6,4,10	100	339	3313.91	339	0.00
Cb1-2,3,5	100	152	374.55	152	0.00	Cb1-2,3,10	100	181	2027.53	181	0.00
Cb1-3,5,5	100	168	174.64	168	0.00	Cb1-3,5,10	100	281	6253.40	281	0.00
Cb1-6,4,5	100	305	161.17	305	0.00	Cb1-6,4,10	100	302	2480.83	302	0.00
Cb2-2,3,5	100	137	349.67	137	0.00	Cb2-2,3,10	100	223	2472.83	223	0.00
Cb2-3,5,5	100	154	207.78	154	0.00	Cb2-3,5,10	100	321	2996.65	321	0.00
Cb2-6,4,5	100	212	154.05	212	0.00	Cb2-6,4,10	100	253	3246.87	253	0.00
Cb3-2,3,5	100	332	82.09	332	0.00	Cb3-2,3,10	100	337	2323.08	337	0.00
Cb3-3,5,5	100	160	145.04	160	0.00	Cb3-3,5,10	100	318	3183.35	318	0.00
Cb3-6,4,5	100	273	348.96	273	0.00	Cb3-6,4,10	100	347	2892.64	347	0.00
Cb4-2,3,5	100	280	374.37	280	0.00	Cb4-2,3,10	100	331	2557.21	331	0.00
Cb4-3,5,5	100	142	211.63	142	0.00	Cb4-3,5,10	100	327	3793.88	327	0.00
Cb4-6,4,5	100	267	76.34	267	0.00	Cb4-6,4,10	100	433	4662.86	433	0.00
Cb5-2,3,5	100	129	163.68	129	0.00	Cb5-2,3,10	100	312	2941.38	312	0.00
Cb5-3,5,5	100	179	177.29	179	0.00	Cb5-3,5,10	100	301	4108.44	301	0.00
Cb5-6,4,5	100	272	293.33	272	0.00	Cb5-6,4,10	100	388	2948.55	388	0.00
Cc1-2,3,5	100	129	1680.53	128	0.78	Cc1-2,3,10	100	281	7045.98	281.00	0.00
Cc1-3,5,5	100	135	3017.01	134	0.74	Cc1-3,5,10	100	254	7115.80	252.00	0.79
Cc1-6,4,5	100	215	4611.40	214	0.47	Cc1-6,4,10	100	306	8154.46	304.00	0.65
Cc2-2,3,5	100	122	3103.75	121	0.82	Cc2-2,3,10	100	271	5205.04	269.00	0.74
Cc2-3,5,5	100	175	1826.48	174	0.57	Cc2-3,5,10	100	319	5704.49	317.00	0.63
Cc2-6,4,5	100	190	5372.75	189	0.53	Cc2-6,4,10	100	312	6928.43	310.00	0.64
Cc3-2,3,5	100	136	4375.43	135	0.74	Cc3-2,3,10	100	276	7236.21	274.00	0.72
Cc3-3,5,5	100	142	5571.93	141	0.70	Cc3-3,5,10	100	309	7442.47	307.00	0.65
Cc3-6,4,5	100	228	5797.59	227	0.44	Cc3-6,4,10	100	266	6100.56	265.00	0.38
Cc4-2,3,5	100	171	3868.22	170	0.58	Cc4-2,3,10	100	225	7142.61	223.00	0.89
Cc4-3,5,5	100	154	6169.20	153	0.65	Cc4-3,5,10	100	310	6011.60	309.00	0.32
Cc4-6,4,5	100	213	2283.80	212	0.47	Cc4-6,4,10	100	312	5794.55	310.00	0.64
Cc5-2,3,5	100	123	5796.96	122	0.81	Cc5-2,3,10	100	182	7180.07	181.00	0.55
Cc5-3,5,5	100	124	2007.81	123	0.81	Cc5-3,5,10	100	254	6752.82	253.00	0.39
Cc5-6,4,5	100	202	5759.53	201	0.50	Cc5-6,4,10	100	339	7379.21	337.00	0.59
Cd1-2,3,5	100	155	225.53	154	0.65	Cd1-2,3,10	100	282	4246.68	280.00	0.71
Cd1-3,5,5	100	170	188.82	169	0.59	Cd1-3,5,10	100	270	4882.50	269.23	0.29
Cd1-6,4,5	100	254	274.39	253	0.39	Cd1-6,4,10	100	380	4894.86	377.00	0.79
Cd2-2,3,5	100	140	230.87	139	0.71	Cd2-2,3,10	100	273	5569.29	271.00	0.73
Cd2-3,5,5	100	158	342.02	157	0.63	Cd2-3,5,10	100	317	7149.74	316.00	0.32
Cd2-6,4,5	100	210	184.80	209	0.48	Cd2-6,4,10	100	350	6953.20	347.00	0.86
Cd3-2,3,5	100	142	330.62	141	0.70	Cd3-2,3,10	100	273	5185.51	273	0.00
Cd3-3,5,5	100	147	360.91	146	0.68	Cd3-3,5,10	100	300	5367.44	297.00	1.00
Cd3-6,4,5	100	231	305.07	230	0.43	Cd3-6,4,10	100	272	6742.48	270.00	0.74
Cd4-2,3,5	100	202	309.09	201	0.50	Cd4-2,3,10	100	257	6361.45	257	0.00
Cd4-3,5,5	100	162	133.25	161	0.62	Cd4-3,5,10	100	324	7290.98	322.00	0.62
Cd4-6,4,5	100	222	355.73	221	0.45	Cd4-6,4,10	100	266	5421.55	266.00	0.00
Cd5-2,3,5	100	178	125.49	177	0.56	Cd5-2,3,10	100	300	6669.32	300	0.00
Cd5-3,5,5	100	178	90.38	177	0.56	Cd5-3,5,10	100	290	4087.47	289.00	0.34
Cd5-6,4,5	100	228	323.12	227	0.44	Cd5-6,4,10	100	339	6344.08	337.00	0.59
Averages			1196.57		0.30	*Averages*			4901.96		4.76

Table 7: DDD results on instances with 5 and 10 OD demands

ID	Ψ	UB	CPUsec	LB	OG (%)	ID	Ψ	UB	CPUsec	LB	OG (%)
Ca1-2,3,5	100	171	339.59	171	0.00	Ca1-2,3,10	100	340	3077.29	340	0.00
Ca1-3,5,5	100	185	417.31	185	0.00	Ca1-3,5,10	100	290	2054.96	290	0.00
Ca1-6,4,5	100	194	316.68	194	0.00	Ca1-6,4,10	100	313	4572.98	313	0.00
Ca2-2,3,5	100	136	333.80	136	0.00	Ca2-2,3,10	100	247	4920.98	247	0.00
Ca2-3,5,5	100	199	417.76	199	0.00	Ca2-3,5,10	100	340	2644.54	340	0.00
Ca2-6,4,5	100	219	478.67	219.00	0.00	Ca2-6,4,10	100	338	4982.71	338	0.00
Ca3-2,3,5	100	287	456.24	287.00	0.00	Ca3-2,3,10	100	344	5257.40	344	0.00
Ca3-3,5,5	100	198	160.01	198.00	0.00	Ca3-3,5,10	100	299	3220.29	299	0.00
Ca3-6,4,5	100	226	237.57	226.00	0.00	Ca3-6,4,10	100	289	3087.53	289	0.00
Ca4-2,3,5	100	197	292.89	197.00	0.00	Ca4-2,3,10	100	306	5380.30	306	0.00
Ca4-3,5,5	100	150	474.32	150.00	0.00	Ca4-3,5,10	100	308	2055.86	308	0.00
Ca4-6,4,5	100	236	305.21	236.00	0.00	Ca4-6,4,10	100	317	3520.54	317	0.00
Ca5-2,3,5	100	185	440.84	185.00	0.00	Ca5-2,3,10	100	422	3701.48	422	0.00
Ca5-3,5,5	100	141	215.40	141.00	0.00	Ca5-3,5,10	100	296	4681.03	296	0.00
Ca5-6,4,5	100	231	437.18	231.00	0.00	Ca5-6,4,10	100	339	3774.23	339	0.00
Cb1-2,3,5	100	139	393.61	139.00	0.00	Cb1-2,3,10	100	181	2055.20	181	0.00
Cb1-3,5,5	100	147	181.29	147.00	0.00	Cb1-3,5,10	100	281	4634.56	281	0.00
Cb1-6,4,5	100	257	395.55	257.00	0.00	Cb1-6,4,10	100	302	2370.37	302	0.00
Cb2-2,3,5	100	137	131.13	137.00	0.00	Cb2-2,3,10	100	199	2891.89	199	0.00
Cb2-3,5,5	100	152	231.99	152.00	0.00	Cb2-3,5,10	100	321	5075.24	321	0.00
Cb2-6,4,5	100	212	111.85	212.00	0.00	Cb2-6,4,10	100	253	2395.87	253	0.00
Cb3-2,3,5	100	332	234.12	332.00	0.00	Cb3-2,3,10	100	337	4405.89	337	0.00
Cb3-3,5,5	100	133	296.53	133.00	0.00	Cb3-3,5,10	100	300	3001.60	300	0.00
Cb3-6,4,5	100	273	375.23	273.00	0.00	Cb3-6,4,10	100	331	5357.60	331	0.00
Cb4-2,3,5	100	280	179.89	280.00	0.00	Cb4-2,3,10	100	315	3547.27	315	0.00
Cb4-3,5,5	100	135	227.81	135.00	0.00	Cb4-3,5,10	100	327	4925.07	327	0.00
Cb4-6,4,5	100	265	262.36	265.00	0.00	Cb4-6,4,10	100	424	3333.37	424	0.00
Cb5-2,3,5	100	129	161.27	129.00	0.00	Cb5-2,3,10	100	306	2460.66	306	0.00
Cb5-3,5,5	100	146	384.40	146.00	0.00	Cb5-3,5,10	100	270	2625.74	270	0.00
Cb5-6,4,5	100	272	449.97	272.00	0.00	Cb5-6,4,10	100	296	2339.10	296	0.00
Cc1-2,3,5	100	129	3573.50	128.00	0.78	Cc1-2,3,10	100	281	6381.30	280.00	0.36
Cc1-3,5,5	100	135	4862.27	134.00	0.74	Cc1-3,5,10	100	252	6759.99	250.00	0.79
Cc1-6,4,5	100	215	2849.92	214.00	0.47	Cc1-6,4,10	100	304	5704.65	304.00	0.00
Cc2-2,3,5	100	122	3629.07	121.00	0.82	Cc2-2,3,10	100	268	4957.86	266.00	0.75
Cc2-3,5,5	100	175	3268.61	174.00	0.57	Cc2-3,5,10	100	319	5764.52	317.00	0.63
Cc2-6,4,5	100	190	3462.29	189.00	0.53	Cc2-6,4,10	100	312	7214.03	310.00	0.64
Cc3-2,3,5	100	136	4988.50	135.00	0.74	Cc3-2,3,10	100	275	4953.59	275	0.00
Cc3-3,5,5	100	142	4346.75	141.00	0.70	Cc3-3,5,10	100	309	5004.70	307	0.65
Cc3-6,4,5	100	228	5020.70	227.00	0.44	Cc3-6,4,10	100	266	6660.94	264	0.75
Cc4-2,3,5	100	171	5366.22	170.00	0.58	Cc4-2,3,10	100	225	4629.25	225	0.00
Cc4-3,5,5	100	154	3354.18	153.00	0.65	Cc4-3,5,10	100	309	4646.84	307	0.65
Cc4-6,4,5	100	213	3881.94	212.00	0.47	Cc4-6,4,10	100	311	5418.83	309	0.64
Cc5-2,3,5	100	123	4075.29	122.00	0.81	Cc5-2,3,10	100	182	5004.48	181	0.55
Cc5-3,5,5	100	124	5136.12	123.00	0.81	Cc5-3,5,10	100	254	4982.86	252	0.79
Cc5-6,4,5	100	202	4561.48	201.00	0.50	Cc5-6,4,10	100	331	6250.35	329	0.60
Cd1-2,3,5	100	155	2981.80	154.00	0.65	Cd1-2,3,10	100	287	6241.41	271.00	5.57
Cd1-3,5,5	100	170	4250.96	169.00	0.59	Cd1-3,5,10	100	297	5491.05	295	0.67
Cd1-6,4,5	100	254	3723.68	253.00	0.39	Cd1-6,4,10	100	401	6966.86	399	0.50
Cd2-2,3,5	100	140	4352.56	139.00	0.71	Cd2-2,3,10	100	273	5075.12	271	0.73
Cd2-3,5,5	100	158	3313.12	157.00	0.63	Cd2-3,5,10	100	317	5590.91	315	0.63
Cd2-6,4,5	100	210	2131.37	209.00	0.48	Cd2-6,4,10	100	365	6256.97	363	0.55
Cd3-2,3,5	100	142	2044.07	141.00	0.70	Cd3-2,3,10	100	273	5850.61	271	0.73
Cd3-3,5,5	100	147	3034.68	146.00	0.68	Cd3-3,5,10	100	305	6275.87	303	0.66
Cd3-6,4,5	100	231	4773.71	230.00	0.43	Cd3-6,4,10	100	272	4567.17	270	0.74
Cd4-2,3,5	100	202	2931.52	201.00	0.50	Cd4-2,3,10	100	243	5921.58	241	0.82
Cd4-3,5,5	100	162	2753.56	161.00	0.62	Cd4-3,5,10	100	324	6062.00	322	0.62
Cd4-6,4,5	100	222	5275.05	221.00	0.45	Cd4-6,4,10	100	266	6602.67	264	0.75
Cd5-2,3,5	100	178	3359.22	177.00	0.56	Cd5-2,3,10	100	300	7086.39	298	0.67
Cd5-3,5,5	100	178	4586.33	177.00	0.56	Cd5-3,5,10	100	301	4311.83	299	0.66
Cd5-6,4,5	100	228	3060.29	227.00	0.44	Cd5-6,4,10	100	339	7350.65	337	0.59
Averages			1196.57		0.30	*Averages*			4901.96		4.76

Table 8: Results of the DDD on instances with 5 and 10 OD demands.

ID	Ψ	UB	CPUsec	LB	OG (%)	ID	Ψ	UB	CPUsec	LB	OG (%)	ID	Ψ	UB	CPUsec	LB	OG (%)
Ca1-2,3,15	200	335	17580.00	331.69	0.99	Ca1-2,3,30	200	828	36000	465.31	43.80	Ca1-2,3,50	200	1067	36000.00	563.05	47.23
Ca1-3,5,15	200	336	18000.00	329.09	2.06	Ca1-3,5,30	200	940	36000	480.22	48.91	Ca1-3,5,50	200	N.A	36000.00	N.A	N.A
Ca1-6,4,15	200	396	18000.00	324.71	18.00	Ca1-6,4,30	200	813	36000	456.79	43.81	Ca1-6,4,50	200	N.A	36000.00	N.A	N.A
Ca2-2,3,15	200	383	18000.00	328.04	14.35	Ca2-2,3,30	200	739	36000	401.29	45.70	Ca2-2,3,50	200	1083	36000.00	480.89	55.60
Ca2-3,5,15	200	388	18000.00	352.08	9.26	Ca2-3,5,30	200	639	3489.89	632.58	1.00	Ca2-3,5,50	200	N.A	36000.00	N.A	N.A
Ca2-6,4,15	200	404	18000.00	324.59	19.66	Ca2-6,4,30	200	900	36000	439.48	51.17	Ca2-6,4,50	200	N.A	36000.00	N.A	N.A
Ca3-2,3,15	200	337	1635.98	333.91	0.92	Ca3-2,3,30	200	806	36000	446.53	44.60	Ca3-2,3,50	200	1165	36000.00	505.35	56.62
Ca3-3,5,15	200	410	18000.00	301.53	26.46	Ca3-3,5,30	200	805	36000	441.91	45.10	Ca3-3,5,50	200	N.A	36000.00	N.A	N.A
Ca3-6,4,15	200	402	18000.00	338.25	15.86	Ca3-6,4,30	200	753	36000	411.67	45.33	Ca3-6,4,50	200	N.A	36000.00	N.A	N.A
Ca4-2,3,15	200	368	18000.00	326.70	11.22	Ca4-2,3,30	200	803.	36000	414.40	48.39	Ca4-2,3,50	200	N.A	36000.00	456.91	N.A
Ca4-3,5,15	200	324	8917.19	324.00	0.00	Ca4-3,5,30	200	753	36000	466.34	38.07	Ca4-3,5,50	200	N.A	36000.00	N.A	N.A
Ca4-6,4,15	200	365	18000.00	331.88	9.07	Ca4-6,4,30	200	840	36000	424.33	49.48	Ca4-6,4,50	200	N.A	36000.00	N.A	N.A
Ca5-2,3,15	200	361	18000.00	318.22	11.85	Ca5-2,3,30	200	707	36000	426.15	39.72	Ca5-2,3,50	200	1144	36000.00	541.88	52.63
Ca5-3,5,15	200	345	18000.00	339.64	1.55	Ca5-3,5,30	200	808	36000	410.26	49.23	Ca5-3,5,50	200	N.A	36000.00	N.A	N.A
Ca5-6,4,15	200	345	18000.00	288.22	16.46	Ca5-6,4,30	200	891	36000	404.97	54.55	Ca5-6,4,50	200	N.A	36000.00	N.A	N.A
Cb1-2,3,15	200	335	1723.45	333.92	0.32	Cb1-2,3,30	200	885	36000	426.16	51.85	Cb1-2,3,50	200	1104	36000.00	502.51	54.48
Cb1-3,5,15	200	355	18000.00	320.04	9.85	Cb1-3,5,30	200	976	36000	445.84	54.32	Cb1-3,5,50	200	N.A	36000.00	N.A	N.A
Cb1-6,4,15	200	405	18000.00	329.79	18.57	Cb1-6,4,30	200	1006	36000	431.71	57.09	Cb1-6,4,50	200	N.A	36000.00	N.A	N.A
Cb2-2,3,15	200	403	18000.00	324.83	19.40	Cb2-2,3,30	200	786	36000	360.38	54.15	Cb2-2,3,50	200	869	36000.00	671.51	22.73
Cb2-3,5,15	200	390	18000.00	331.31	15.05	Cb2-3,5,30	200	924	36000	406.92	55.96	Cb2-3,5,50	200	N.A	36000.00	N.A	N.A
Cb2-6,4,15	200	423	18000.00	355.46	15.97	Cb2-6,4,30	200	947	36000	401.35	57.62	Cb2-6,4,50	200	N.A	36000.00	N.A	N.A
Cb3-2,3,15	200	356	18000.00	320.42	10.00	Cb3-2,3,30	200	736	36000	409.92	44.30	Cb3-2,3,50	200	1157	36000.00	574.27	50.37
Cb3-3,5,15	200	327	18000.00	297.88	8.91	Cb3-3,5,30	200	796	36000	405.99	49.00	Cb3-3,5,50	200	N.A	36000.00	N.A	N.A
Cb3-6,4,15	200	403	18000.00	296.48	26.43	Cb3-6,4,30	200	968	36000	419.06	56.71	Cb3-6,4,50	200	N.A	36000.00	N.A	N.A
Cb4-2,3,15	200	337	18000.00	288.73	14.32	Cb4-2,3,30	200	761	36000	413.39	45.68	Cb4-2,3,50	200	N.A	36000.00	537.47	N.A
Cb4-3,5,15	200	313	18000.00	281.53	10.05	Cb4-3,5,30	200	807	36000	365.58	54.70	Cb4-3,5,50	200	N.A	36000.00	N.A	N.A
Cb4-6,4,15	200	383	18000.00	358.90	6.29	Cb4-6,4,30	200	874	36000	477.00	45.42	Cb4-6,4,50	200	N.A	36000.00	N.A	N.A
Cb5-2,3,15	200	356	18000.00	311.01	12.64	Cb5-2,3,30	200	739	36000	393.36	46.77	Cb5-2,3,50	200	1145	36000.00	595.06	48.03
Cb5-3,5,15	200	343	18000.00	252.27	26.45	Cb5-3,5,30	200	849	36000	576.00	32.16	Cb5-3,5,50	200	N.A	36000.00	N.A	N.A
Cb5-6,4,15	200	371	18000.00	299.91	19.16	Cb5-6,4,30	200	931	36000	589.00	36.73	Cb5-6,4,50	200	N.A	36000.00	N.A	N.A
Cc1-2,3,15	200	325	18000.00	213.51	34.30	Cc1-2,3,30	200	783	36000	420.53	46.29	Cc1-2,3,50	200	1122	36000.00	515.26	54.08
Cc1-3,5,15	200	336	18000.00	196.96	41.38	Cc1-3,5,30	200	966	36000	485.52	49.74	Cc1-3,5,50	200	N.A	36000.00	N.A	N.A
Cc1-6,4,15	200	518	18000.00	266.65	48.52	Cc1-6,4,30	200	824	36000	539.00	34.59	Cc1-6,4,50	200	N.A	36000.00	N.A	N.A
Cc2-2,3,15	200	392	18000.00	250.80	36.02	Cc2-2,3,30	200	724	36000	371.69	48.66	Cc2-2,3,50	200	963	36000.00	555.32	42.33
Cc2-3,5,15	200	367	18000.00	235.07	35.95	Cc2-3,5,30	200	801	36000	356.73	55.46	Cc2-3,5,50	200	N.A	36000.00	N.A	N.A
Cc2-6,4,15	200	430	18000.00	270.28	37.14	Cc2-6,4,30	200	944	36000	458.48	51.43	Cc2-6,4,50	200	N.A	36000.00	N.A	N.A
Cc3-2,3,15	200	362	18000.00	213.82	40.93	Cc3-2,3,30	200	733	36000	353.46	51.78	Cc3-2,3,50	200	1007	36000.00	639.39	36.51
Cc3-3,5,15	200	311	18000.00	183.02	41.15	Cc3-3,5,30	200	864	36000	342.07	60.41	Cc3-3,5,50	200	N.A	36000.00	N.A	N.A
Cc3-6,4,15	200	458	18000.00	255.04	44.31	Cc3-6,4,30	200	922	36000	350.45	61.99	Cc3-6,4,50	200	N.A	36000.00	N.A	N.A
Cc4-2,3,15	200	392	18000.00	199.03	49.23	Cc4-2,3,30	200	767	36000	310.94	59.46	Cc4-2,3,50	200	948	36000.00	531.09	43.98
Cc4-3,5,15	200	330	18000.00	168.67	48.89	Cc4-3,5,30	200	820	36000	470.63	42.61	Cc4-3,5,50	200	N.A	36000.00	N.A	N.A
Cc4-6,4,15	200	416	18000.00	263.54	36.65	Cc4-6,4,30	200	858	36000	354.58	58.67	Cc4-6,4,50	200	N.A	36000.00	N.A	N.A
Cc5-2,3,15	200	336	18000.00	205.49	38.84	Cc5-2,3,30	200	714	36000	392.68	45.00	Cc5-2,3,50	200	1082	36000.00	694.58	35.81
Cc5-3,5,15	200	356	18000.00	220.53	38.05	Cc5-3,5,30	200	764	36000	364.33	52.31	Cc5-3,5,50	200	N.A	36000.00	N.A	N.A
Cc5-6,4,15	200	420	18000.00	245.70	41.50	Cc5-6,4,30	200	910	36000	421.95	53.63	Cc5-6,4,50	200	N.A	36000.00	N.A	N.A
Cd1-2,3,15	200	326	18000.00	280.48	13.96	Cd1-2,3,30	200	806	36000	411.96	48.89	Cd1-2,3,50	200	1199	36000.00	607.82	49.31
Cd1-3,5,15	200	332	18000.00	264.99	20.18	Cd1-3,5,30	200	1017	36000	412.31	59.46	Cd1-3,5,50	200	N.A	36000.00	N.A	N.A
Cd1-6,4,15	200	404	18000.00	333.72	17.40	Cd1-6,4,30	200	918	36000	418.40	54.42	Cd1-6,4,50	200	N.A	36000.00	N.A	N.A
Cd2-2,3,15	200	345	18000.00	300.35	12.94	Cd2-2,3,30	200	818	36000	398.58	51.27	Cd2-2,3,50	200	1128	36000.00	580.44	48.54
Cd2-3,5,15	200	372	18000.00	334.87	9.98	Cd2-3,5,30	200	822	36000	380.94	53.66	Cd2-3,5,50	200	N.A	36000.00	N.A	N.A
Cd2-6,4,15	200	408	18000.00	327.94	19.62	Cd2-6,4,30	200	969	36000	456.07	52.93	Cd2-6,4,50	200	N.A	36000.00	N.A	N.A
Cd3-2,3,15	200	332	18000.00	268.81	19.03	Cd3-2,3,30	200	776	36000	428.45	44.79	Cd3-2,3,50	200	1117	36000.00	610.04	45.39
Cd3-3,5,15	200	296	18000.00	242.23	18.17	Cd3-3,5,30	200	942	36000	449.23	52.31	Cd3-3,5,50	200	N.A	36000.00	N.A	N.A
Cd3-6,4,15	200	402	18000.00	317.55	21.01	Cd3-6,4,30	200	896	36000	411.49	54.07	Cd3-6,4,50	200	N.A	36000.00	N.A	N.A
Cd4-2,3,15	200	376	18000.00	306.87	18.39	Cd4-2,3,30	200	826	36000	419.38	49.23	Cd4-2,3,50	200	1171	36000.00	512.37	56.25
Cd4-3,5,15	200	312	18000.00	262.13	15.98	Cd4-3,5,30	200	844	36000	447.36	47.00	Cd4-3,5,50	200	N.A	36000.00	N.A	N.A
Cd4-6,4,15	200	381	18000.00	262.91	30.99	Cd4-6,4,30	200	887	36000	406.15	54.21	Cd4-6,4,50	200	N.A	36000.00	N.A	N.A
Cd5-2,3,15	200	351	18000.00	310.64	11.50	Cd5-2,3,30	200	727	36000	376.68	48.19	Cd5-2,3,50	200	1099	36000.00	555.58	49.45
Cd5-3,5,15	200	344	18000.00	266.92	22.41	Cd5-3,5,30	200	816	36000	375.89	53.93	Cd5-3,5,50	200	N.A	36000.00	N.A	N.A
Cd5-6,4,15	200	339	18000.00	266.92	21.26	Cd5-6,4,30	200	991	36000	431.44	56.46	Cd5-6,4,50	200	N.A	36000.00	N.A	N.A
Averages			17297.61		20.95	*Averages*			35458.16		49.07	*Averages*			36000		47.18

Table 9: DDD results on instances with 15, 30 and 50 OD demands.

ID	Ψ	UB	CPUsec	LB	OG (%)	ID	Ψ	UB	CPUsec	LB	OG (%)	ID	Ψ	UB	CPUsec	LB	OG (%)
Ca1-2,3,15	200	331	18000	298.43	9.84	Ca1-2,3,30	200	842	36000	425.44	49.47	Ca1-2,3,50	200	1067	36000	562.93	47.24
Ca1-3,5,15	200	326	18000	286.32	12.17	Ca1-3,5,30	200	967	36000	456.79	52.76	Ca1-3,5,50	200	N.A	36000	N.A	N.A
Ca1-6,4,15	200	404	18000	315.96	21.79	Ca1-6,4,30	200	813	36000	456.79	43.81	Ca1-6,4,50	200	N.A	36000	N.A	N.A
Ca2-2,3,15	200	355	18000	322.24	9.23	Ca2-2,3,30	200	739	36000	401.29	45.70	Ca2-2,3,50	200	1095	36000	519.38	52.57
Ca2-3,5,15	200	390	18000	320.59	17.80	Ca2-3,5,30	200	847	36000	362.94	57.15	Ca2-3,5,50	200	N.A	36000	N.A	N.A
Ca2-6,4,15	200	356	18000	329.61	7.41	Ca2-6,4,30	200	900	36000	429.48	52.28	Ca2-6,4,50	200	N.A	36000	N.A	N.A
Ca3-2,3,15	200	341	18000	314.73	7.70	Ca3-2,3,30	200	806	36000	446.53	44.60	Ca3-2,3,50	200	1105	36000	541.54	50.99
Ca3-3,5,15	200	293	18000	239.69	18.19	Ca3-3,5,30	200	805	36000	441.91	45.10	Ca3-3,5,50	200	N.A	36000	N.A	N.A
Ca3-6,4,15	200	372	18000	309.11	16.90	Ca3-6,4,30	200	753	36000	411.67	45.33	Ca3-6,4,50	200	N.A	36000	N.A	N.A
Ca4-2,3,15	200	323	18000	310.88	3.75	Ca4-2,3,30	200	762	36000	414.40	45.62	Ca4-2,3,50	200	1115	36000	497.32	55.40
Ca4-3,5,15	200	314	18000	261.62	16.68	Ca4-3,5,30	200	753	36000	366.34	51.35	Ca4-3,5,50	200	N.A	36000	N.A	N.A
Ca4-6,4,15	200	367	18000	321.82	12.31	Ca4-6,4,30	200	840	36000	404.33	51.87	Ca4-6,4,50	200	N.A	36000	N.A	N.A
Ca5-2,3,15	200	337	18000	299.69	11.07	Ca5-2,3,30	200	707	36000	366.15	48.21	Ca5-2,3,50	200	1120	36000	550.19	50.88
Ca5-3,5,15	200	377	18000	291.42	22.70	Ca5-3,5,30	200	808	36000	370.26	54.18	Ca5-3,5,50	200	N.A	36000	N.A	N.A
Ca5-6,4,15	200	348	18000	303.26	12.86	Ca5-6,4,30	200	891	36000	404.97	54.55	Ca5-6,4,50	200	N.A	36000	N.A	N.A
Cb1-2,3,15	200	335	18000	300.42	10.32	Cb1-2,3,30	200	885	36000	426.16	51.85	Cb1-2,3,50	200	1070	36000	548.26	48.76
Cb1-3,5,15	200	326	18000	281.35	13.70	Cb1-3,5,30	200	976	36000	445.84	54.32	Cb1-3,5,50	200	N.A	36000	N.A	N.A
Cb1-6,4,15	200	406	18000	297.83	26.64	Cb1-6,4,30	200	1006	36000	371.07	63.11	Cb1-6,4,50	200	N.A	36000	N.A	N.A
Cb2-2,3,15	200	355	18000	307.87	13.28	Cb2-2,3,30	200	786	36000	360.38	54.15	Cb2-2,3,50	200	931	36000	510.15	45.20
Cb2-3,5,15	200	396	18000	322.25	18.62	Cb2-3,5,30	200	924	36000	406.92	55.96	Cb2-3,5,50	200	N.A	36000	N.A	N.A
Cb2-6,4,15	200	427	18000	320.59	24.92	Cb2-6,4,30	200	947	36000	401.35	57.62	Cb2-6,4,50	200	N.A	36000	N.A	N.A
Cb3-2,3,15	200	356	18000	292.73	17.77	Cb3-2,3,30	200	736	36000	409.92	44.30	Cb3-2,3,50	200	1073	36000	536.76	49.98
Cb3-3,5,15	200	293	18000	239.69	18.19	Cb3-3,5,30	200	796	36000	405.99	49.00	Cb3-3,5,50	200	N.A	36000	N.A	N.A
Cb3-6,4,15	200	410	18000	284.79	30.54	Cb3-6,4,30	200	968	36000	419.06	56.71	Cb3-6,4,50	200	N.A	36000	N.A	N.A
Cb4-2,3,15	200	357	18000	287.12	19.57	Cb4-2,3,30	200	761	36000	413.39	45.68	Cb4-2,3,50	200	N.A	36000	490.20	N.A
Cb4-3,5,15	200	314	18000	261.62	16.68	Cb4-3,5,30	200	807	36000	365.58	54.70	Cb4-3,5,50	200	N.A	36000	N.A	N.A
Cb4-6,4,15	200	382	18000	321.82	15.75	Cb4-6,4,30	200	874	36000	378.33	56.71	Cb4-6,4,50	200	N.A	36000	N.A	N.A
Cb5-2,3,15	200	352	18000	304.75	13.42	Cb5-2,3,30	200	739	36000	393.36	46.77	Cb5-2,3,50	200	1042	36000	529.40	49.19
Cb5-3,5,15	200	345	18000	243.57	29.40	Cb5-3,5,30	200	849	36000	385.51	54.59	Cb5-3,5,50	200	N.A	36000	N.A	N.A
Cb5-6,4,15	200	377	18000	291.42	22.70	Cb5-6,4,30	200	931	36000	414.66	55.46	Cb5-6,4,50	200	N.A	36000	N.A	N.A
Cc1-2,3,15	200	335	18000	228.19	31.88	Cc1-2,3,30	200	783	36000	350.45	55.24	Cc1-2,3,50	200	993	36000	387.51	60.98
Cc1-3,5,15	200	330	18000	208.32	36.87	Cc1-3,5,30	200	966	36000	305.54	68.37	Cc1-3,5,50	200	N.A	36000	N.A	N.A
Cc1-6,4,15	200	520	18000	266.63	48.73	Cc1-6,4,30	200	824	36000	424.70	48.46	Cc1-6,4,50	200	N.A	36000	N.A	N.A
Cc2-2,3,15	200	411	18000	222.50	45.86	Cc2-2,3,30	200	724	36000	371.69	48.66	Cc2-2,3,50	200	1020	36000	374.01	63.33
Cc2-3,5,15	200	425	18000	234.22	44.89	Cc2-3,5,30	200	801	36000	356.73	55.46	Cc2-3,5,50	200	N.A	36000	N.A	N.A
Cc2-6,4,15	200	509	18000	362.73	28.74	Cc2-6,4,30	200	944	36000	354.43	62.45	Cc2-6,4,50	200	N.A	36000	N.A	N.A
Cc3-2,3,15	200	346	18000	213.64	38.25	Cc3-2,3,30	200	733	36000	353.46	51.78	Cc3-2,3,50	200	978	36000	388.26	60.30
Cc3-3,5,15	200	295	18000	239.88	18.69	Cc3-3,5,30	200	864	36000	342.07	60.41	Cc3-3,5,50	200	N.A	36000	N.A	N.A
Cc3-6,4,15	200	436	18000	249.64	42.74	Cc3-6,4,30	200	922	36000	350.45	61.99	Cc3-6,4,50	200	N.A	36000	N.A	N.A
Cc4-2,3,15	200	418	18000	298.17	28.67	Cc4-2,3,30	200	767	36000	310.94	59.46	Cc4-2,3,50	200	0	36000	384.45	N.A
Cc4-3,5,15	200	338	18000	294.07	13.00	Cc4-3,5,30	200	820	36000	261.50	68.11	Cc4-3,5,50	200	N.A	36000	N.A	N.A
Cc4-6,4,15	200	519	18000	312.47	39.79	Cc4-6,4,30	200	858	36000	354.58	58.67	Cc4-6,4,50	200	N.A	36000	N.A	N.A
Cc5-2,3,15	200	324	18000	281.29	13.18	Cc5-2,3,30	200	714	36000	392.68	45.00	Cc5-2,3,50	200	1099	36000	357.89	67.43
Cc5-3,5,15	200	344	18000	284.66	17.25	Cc5-3,5,30	200	764	36000	364.33	52.31	Cc5-3,5,50	200	N.A	36000	N.A	N.A
Cc5-6,4,15	200	369	18000	306.48	16.94	Cc5-6,4,30	200	910	36000	421.95	53.63	Cc5-6,4,50	200	N.A	36000	N.A	N.A
Cd1-2,3,15	200	327	18000	276.16	15.55	Cd1-2,3,30	200	806	36000	411.96	48.89	Cd1-2,3,50	200	1057	36000	569.37	46.13
Cd1-3,5,15	200	333	18000	259.86	21.96	Cd1-3,5,30	200	1017	36000	412.31	59.46	Cd1-3,5,50	200	N.A	36000	N.A	N.A
Cd1-6,4,15	200	403	18000	328.98	18.37	Cd1-6,4,30	200	918	36000	418.40	54.42	Cd1-6,4,50	200	N.A	36000	N.A	N.A
Cd2-2,3,15	200	362	18000	306.49	15.33	Cd2-2,3,30	200	818	36000	398.58	51.27	Cd2-2,3,50	200	1065	36000	546.68	48.67
Cd2-3,5,15	200	376	18000	321.14	14.59	Cd2-3,5,30	200	822	36000	380.94	53.66	Cd2-3,5,50	200	N.A	36000	N.A	N.A
Cd2-6,4,15	200	408	18000	321.20	21.28	Cd2-6,4,30	200	969	36000	366.58	62.17	Cd2-6,4,50	200	N.A	36000	N.A	N.A
Cd3-2,3,15	200	336	18000	293.29	12.71	Cd3-2,3,30	200	776	36000	428.45	44.79	Cd3-2,3,50	200	1118	36000	540.49	51.66
Cd3-3,5,15	200	329	18000	232.75	29.25	Cd3-3,5,30	200	942	36000	449.23	52.31	Cd3-3,5,50	200	N.A	36000	N.A	N.A
Cd3-6,4,15	200	410	18000	311.84	23.94	Cd3-6,4,30	200	896	36000	411.49	54.07	Cd3-6,4,50	200	N.A	36000	N.A	N.A
Cd4-2,3,15	200	380	18000	273.66	27.98	Cd4-2,3,30	200	826	36000	419.38	49.23	Cd4-2,3,50	200	1110	36000	518.81	53.26
Cd4-3,5,15	200	315	18000	255.13	19.01	Cd4-3,5,30	200	844	36000	447.36	47.00	Cd4-3,5,50	200	N.A	36000	N.A	N.A
Cd4-6,4,15	200	383	18000	299.55	21.79	Cd4-6,4,30	200	887	36000	406.15	54.21	Cd4-6,4,50	200	N.A	36000	N.A	N.A
Cd5-2,3,15	200	356	18000	319.59	10.23	Cd5-2,3,30	200	727	36000	376.68	48.19	Cd5-2,3,50	200	N.A	36000	590.85	N.A
Cd5-3,5,15	200	339	18000	272.04	19.75	Cd5-3,5,30	200	816	36000	375.89	53.93	Cd5-3,5,50	200	N.A	36000	N.A	N.A
Cd5-6,4,15	200	372	18000	279.51	24.86	Cd5-6,4,30	200	991	36000	431.44	56.46	Cd5-6,4,50	200	N.A	36000	N.A	N.A
Averages			18000		20.90	*Averages*			36000		53.05	*Averages*			36000		53.06

Table 10: DDD results on instances with 15, 30 and 50 OD demands and disabled availability time