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International High-Frequency Arbitrage for Cross-Listed Stocks

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Abstract. We explore latency arbitrage activities with a new arbitrage strategy that we test with high-frequency data during the first six months of 2019. We study the profitability of mean-reverting arbitrage activities of 74 cross-listed stocks involving three exchanges in Canada and the United States. Our arbitrage strategy is a hybrid between triangular arbitrage and pairs trading. We synchronize the high-frequency data feeds from the three exchange venues considering explicitly the latency that comes from the transportation of information between the exchanges and its treatment time. Other trading costs and arbitrage risks are also considered. The annual net profit of an HFT firm that uses limit orders is around CAD $8 million (USD $6 million), a result that we consider reasonable when compared with the previous literature. International latency arbitrage with market orders is never profitable.

Keywords: Latency arbitrage, cross-listed stock, high-frequency trading, limit order, market order, synthetic hedging instrument, mean-reverting arbitrage, international arbitrage, supervised machine learning.

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1. Introduction

We study the profitability of mean-reverting arbitrage activities of cross-listed stocks involving three exchanges in Canada and the United States. Our main research question is the following: Is high-speed arbitrage profitable for High-Frequency Traders (HFTs) under strong competition and when all potential arbitrage costs and risks are considered?

Stock exchanges in different countries often use distinct market microstructures, while many large public firms employ cross-border listing to reduce their cost of capital and have access to more liquidity. The current market structure of stock exchanges in North America and Europe is very competitive, fragmented, and very fast (Biais and Woolley, 2011; Jones, 2013; Goldstein et al, 2014; O'Hara, 2015; Wah, 2016). Changes in regulation, particularly the Regulation NMS in the US and the IIROC rules in Canada[1], led to an increase in the number of trading venues, thus further fragmenting financial markets (Garriott et al, 2013; Chao et al, 2019). In 2019, there were more than twenty designated exchanges in North America, and competition related to trading fees, rebates, and colocation fees has increased significantly in recent years (Thomson Reuters, 2019).

The existence of multiple venues means that the price for a given asset need not always be the same across all venues for a very short period, opening the door to high-speed arbitrage across markets (O'Hara, 2015; Foucault and Biais, 2014). Given that this form of arbitrage can be done by creating portfolios that result from spatial arbitrage, traders must thus appraise intra-market liquidity and analyze the assets' serial correlation. But serial correlation may break down in very

[1] Regulation NMS in the US: SEC Exchange Act Release No. 34-51808 (June 9, 2005). IIROC rules in Canada: CSE Trading Rules and the Universal Market Integrity Rules, of the Investment Industry Regulatory Organization of Canada (IIROC, 2015). See also The MiFID Directive in Europe: Directive 2004/39/EC of the European Parliament and of the Council of April 21, 2004 on markets in financial instruments.

short periods of time, which further increases the possibility of high-speed spatial arbitrage opportunities (Budish et al, 2015).

With market fragmentation, traders need to search for liquidity across many venues in the same country or across countries. High speed can be crucial when there is strong competition. The ability of HFTs to enter and cancel orders very rapidly makes it hard for many traders to discern where liquidity really exists, which creates more opportunities for HFTs to exploit profitable trading opportunities.

International latency arbitrage opportunities may also arise because of different market models used in local exchanges, different regulations, transient supply and demand shocks, and because of the arrival of new local information generating asynchronous adjustments in asset prices. These additional arbitrage possibilities terminate either when an arbitrageur exploits the new opportunity or market makers update their quotes to reflect the new information (Foucault et al, 2017). However, different local market makers are not always harmonized in real time. High-speed international arbitrage may then benefit all market participants (those with high speed and without) by reducing inter-markets bid-ask spreads, a measure of market quality (Hendershott et al, 2011; Riordan and Storkenmaier, 2012). In that sense, HFTs may even become inter-market makers who provide liquidity with their arbitrage activities.

While arbitrage forces should drive prices to attain an equilibrium, the exchange acting as a price leader could attract a significant portion of order flow if the adjustment takes time. In this case, it is reasonable to assume that price discovery will tend to occur primarily in the original stock exchange of a cross-listed stock. Empirical evidence suggests that prices on Canadian and U.S. exchanges are mutually adjusting for Canadian-based cross-listed stocks (Eun and Sabherwal, 2003; Chouinard and D’Souza, 2003).

Considering a cross-contry environment, we revisit latency arbitrage strategies and we propose a new model of international mean-reverting arbitrage activities with FX rate hedging. We test the model across three North American exchanges during the first six months of 2019: the New York Stock Exchange (NYSE) and the Chicago Mercantile Exchange (CME) in the United States, and the Toronto Stock Exchange (TSX) in Canada.

This is the first contribution that examines stocks' cross-country mean-reverting arbitrage with FX rate hedging. We are working from the perspective of a unique temporal frame of reference, meaning that we synchronize the data feeds from the three exchange venues considering explicitly the latency that comes from the transportation of information between the exchanges and its treatment time.

Our strategy is a hybrid between triangular arbitrage and pairs trading. We construct a portfolio of synthetic instruments from pairs of cross-listed Canadian stocks of the same company traded on both the TSX and the NYSE and compute their relative spread (SPRD), defined as the ratio of a synthetic instrument of stock prices and a hedging position in the CAD/USD futures (CADUS). The relative spread deviation resulting from a variation between the synthetic instrument and the hedging instrument is expected to be mean reverting. We analyze this intraday reverting behavior in detail for each pair of stocks between the TSX and the NYSE. Significant deviations of the relative spread from its target value could lead to arbitrage opportunities. We develop different arbitrage strategies to exploit these deviations and to demonstrate the potential profitability of mean-reverting arbitrage opportunities that exist between the TSX, NYSE, and CME markets.

Potential important arbitrage profits or realized opportunity costs presented in the literature are often based on strong (and sometimes unrealistic) assumptions about the functioning of financial markets. The most prevalent costs are latency costs, direct trading fees, rebates on trading fees, and colocation and proprietary data feed costs. Moreover, the closing of positions is not always coherent with market reality. Also, mean-reversion risk, execution risk, and non-execution risk are additional cost components that may affect arbitrage profits. We propose a methodology to introduce all of them and adjust our algorithm performance accordingly.

High-frequency trading is very competitive and fast, so there is a great risk of the market moving between the time of observing an arbitrage opportunity and the time of the exchange receiving orders sent by a trader's algorithm (i.e., execution risk when using market orders, non-execution risk when using limit orders). Latency costs for the transportation and the treatment of information may matter when exchanges are distant, and assets quoted in different currencies are present. Moreover, gains per trade for high-frequency traders are relatively small given their short holding periods, so trading costs and rebates may be significant in the computation of net profits, even more so when considering the enormous quantity of trades per day that HFTs perform. The colocation and the proprietary data feed costs are also significant at many exchanges, although they have decreased due to recent competition between exchanges. The non-consideration of all these potential costs may have generated an undeniable overestimation of the latency arbitrage profitability previously presented in the literature (Wah, 2016, Budish et al, 2015, Tivnan et al, 2019 and Dewhurst et al, 2019, among others).

Our approach utilizes a professional trading software suite provided by Deltix EPAM Systems (Deltix, 2020) that enables us to apply our trading strategies in the most realistic market environment possible, just like real HFTs do. This approach, coupled with the inclusion of latency and trading costs, and trading risks in our methodology, generates more realistic results compared with those in previous studies, thus further separating us from past studies.

As mentioned by Chen, Da and Huang (2019), the understanding of arbitrage activity from empirical research is still limited. We analyze various arbitrage strategy performances with state-of-the-art trading and quoting behaviors that can be deployed in a real-time environment by institutional investors, professional arbitrageurs, market makers, and hedgers[2]. Our approach allows a contemporary understanding of an economically viable arbitrage approach that helps restore equilibrium in financial markets.

To the best of our knowledge, we are the first to quantify the importance and the economic value of providing liquidity in the context of arbitrage while considering the limit order book (LOB) queue positions and limit orders instead of only market orders. Our approach is consistent with the revisited HFT market maker definition proposed by O'Hara (2015): "HFT market making differs from traditional market making in that it is often implemented across and within markets, making it akin to statistical arbitrage."[3]

Our results report a net annual profit of about CAD $8 million (USD $6 million) for the year 2019 which seems reasonable for this international arbitrage activity with 36 profitable cross-listed stocks that can be easily managed by one trader in a large trading firm. The 36 profitable pairs of stocks were selected from the 74 potential cross-listed stocks by using a dynamic decision tree machine learning model. The gross annual profit was about CAD $19 million and the main difference between the gross and the net annual profits is explained by latencies in the transportation and the treatment of information, and the non-execution risk because we used limit orders. Trading fees were consequently not important while rebates were significant. We have also shown that international arbitrage opportunities with market orders and transaction fees were not profitable mainly due to transaction fees and the execution risk that comes with latency.

[2] Our software allows switching from simulations to real time trading with the exact same codes.

[3] See also Rein et al. (2021) and Krauss (2017) on statistical arbitrage.

The rest of our contribution is organized as follows. Section 2 presents the literature on arbitrage trading with high-frequency data. An emphasis is put on empirical studies that have estimated the profitability of this trading activity in an HFT environment. We then propose, in Section 3, our methodology based on a mean-reverting model of arbitrage that can be executed with market orders or limit orders. We show the main differences between the two approaches with an emphasis on trading fees versus trading rebates. We also consider other costs and latencies. Section 4 documents the real latency costs for the transportation and treatment of information between the three exchanges as well as the trading fees and rebates. We also document the co-location and the proprietary data feed costs at the TSX, our trading location, and the different risks associated to arbitrage including execution risk, non-execution risk, and mean-reversion risk. The next two sections detail different practical considerations for putting in place our trading strategies and data management. Section 7 is dedicated to our empirical results and Section 8 discusses the performance of our arbitrage strategy. Section 9 concludes.

2. Related literature

Two main issues are at the heart of research on high-frequency trading (HFT): profitability and fairness in trading. Both are interconnected and require appropriate research approaches that are fundamental to understanding the behavior of trading participants and making adequate policy recommendations when necessary. The structure of exchange markets, such as the Toronto Stock Exchange (TSX) or the New York Stock Exchange (NYSE), has been radically transformed by new technology over the last 25 years. HFT is executed by extremely fast computers, and software programming for trading is often strategic.

Liquidity and price discovery now arise in a more complex way, often owing to high speed. These changes have affected the market microstructure and the formation of capital in financial markets. They may also have reduced fairness between market participants, warranting new regulatory rules. However, conclusions on the private net benefits of high-frequency trading and its fairness are not always based on solid academic research, according to O'Hara (2015) and Chen, Da and Huang (2019). In fact, the debate about the high-frequency trading arms race is still open (Aquilina et al, 2020).

Academic interest in latency arbitrage is a relatively recent phenomenon, and available studies have investigated it from different angles. The idea that price dislocations exist in fragmented markets is not new. In fact, contributions from the 1990s highlighted the issue in the US, even when market fragmentation was not as prevalent as it is today (Blume and Goldstein, 1991; Lee, 1993; Hasbrouck, 1995). More recent studies on that matter include Shkilko et al (2008) and Ding et al (2014). Soon after, other articles began mentioning the possibility for high-speed traders to exploit these market anomalies. Foucault and Biais (2014) and O'Hara (2015) both mention that HFTs can capitalize on latency arbitrage opportunities but they did not provide strong empirical evidence.

Hasbrouck and Saar (2013) are among the first to investigate trading with activities in the millisecond environment. Menkveld (2014, 2016) analyzes the behavior of a HFT who is a market maker. He shows that the market maker reduces price variations for the same stock on different exchanges by doing arbitrage activities across trading venues. Budish et al (2015) document the latency arbitrage opportunities they found between the CME and the NYSE from 2005 to 2011. They demonstrate that correlation between the assets breaks down as speed between transactions increases. They show that these breakdowns roughly yield an average of USD $75 million a year from a simple latency strategy of arbitraging the spread of one pair of highly correlated assets: The S&P 500 exchange traded fund (ticker SPY) traded in New York and the S&P 500 E-mini futures contract (ticker ES) traded in Chicago. That pair of instruments had an average of 800 daily arbitrage opportunities during that period, and the authors notice that the arbitrage frequency tracks the overall volatility of the market, with a higher number of opportunities during the financial crisis in 2008, the Flash Crash on May 6, 2010, and the European crisis in summer 2011.

Budish et al (2015) also find that the median ES-SPY arbitrage opportunities duration declines drastically from 97 milliseconds in 2005 to 7 milliseconds in 2011, which is explained by the high-speed arms race led by HFT firms. The median profits per arbitrage opportunity remain relatively constant over time even though competition clearly reduced the duration of arbitrage opportunities. Budish et al (2015) mention the latency issue, but in a rather incomplete fashion. Their approach does not consider latencies such as the real information transportation cost between the two exchanges nor the information treatment time of a round trip. They may have overestimated the real profits generated by their trading strategy and underestimated the execution risk. In their study, around 85% of latency arbitrage opportunities had a duration of less than 10 milliseconds in 2011. It is possible that this proportion has grown since then, given the technology developments since 2011. This emphasizes the importance of including new latency assumptions for our more recent period of analysis. Finally, as they mention, their strategy only considers bid-ask spread costs, whereas a richer estimate of arbitrage opportunities must also include, at least, exchange fees, and all latency costs.

Wah (2016) examines latency arbitrage opportunities on a larger scale for cross-listed stocks of the S&P 500 in eleven US stock exchanges in 2014. Her strategy uses crossed market prices (i.e., when the bid price in an exchange is higher than the ask price in another exchange for the same stock) to locate arbitrage opportunities documented in MIDAS trades and quotes data from the SEC.[4] Considering one infinitely fast arbitrageur operating on these eleven markets, the author estimates that arbitrage opportunity profits were USD $3.03 billion in 2014 for the S&P 500 tickers alone. However, round trip information transportation and information treatment time are not considered in the profitability of the strategy, nor are the other trading costs (except for the bid-ask spread cost, due to the use of market orders).

Tivnan et al (2019) and Dewhurst et al (2019) also examine latency arbitrage on cross-listed stocks in the US National Market System, but with data in 2016 from MIDAS. These two studies consider actionable dislocation segments in their computations, i.e., latency arbitrage opportunities that last longer than the two-way travel time for a fiber optic cable between exchanges' servers. At this trading speed, the transportation time assumption is especially important, even more so when exchanges are far apart, as in our application. Tivnan et al (2019) and Dewhurst et al (2019) have a more realistic approach when compared with Budish et al (2015) and Wah (2016) but they do not consider information treatment time, nor trading costs.

Tivnan et al (2019) find 65 million actionable latency arbitrage opportunities on the Dow 30 in 2016, totaling over USD $160 million of realized opportunity cost, which is comparable to the figures found by Wah (2016) on a per-ticker basis. During that period, approximately 22% of all trades happened during a price dislocation. Dewhurst et al (2019) extended their application to the S&P 500 and the Russell 3000 stocks during the same period. They found 3.1 billion latency arbitrage opportunities in the Russell 3000, resulting in a realized opportunity cost greater than USD $2 billion, with up to 23% of observed trades happening during price dislocations. This confirms the presumption of Budish et al (2015) that potential latency arbitrage opportunities are

[4] MIDAS is the US Securities and Exchange Commission's Market Information Data Analytics System.

plentiful in the US market. We extend these contributions by analyzing cross-country mean-reverting arbitrage between the US and Canadian markets using different arbitrage strategies, and by considering execution risk for market orders, non-execution risk for limit orders, latency costs, trading costs, rebates, and exchanges fixed costs. Also, our strategies could be implemented by practitioners, contrary to those from the papers mentioned above.

The main sources of violations of the law of one price (LOP) are the market models trading costs and the transient supply and demand shocks due to information diffusion delay creating asynchronous adjustments in asset prices (O'Hara, 2016; Tassel, 2020). The use of a professional trading software suite should limit the effects of these dislocations by considering explicitly the market trading costs, the information diffusion delays, and the continuous price discoveries between the NYSE, the TSX, and the CME.

3. Methodology

Arbitrage process

We propose an innovative hybrid approach involving pairs trading and triangular arbitrage for cross-listed stocks between the TSX and the NYSE. In its simplest form, this approach is based on the identification of mean-reverting arbitrage opportunities from a basket of Canadian equities traded on the TSX, their cross-listed NYSE peers,[5] and the CME CAD/USD futures (CADUS, for short), a hedging instrument for the synthetic exchange rate.

We first compute a synthetic instrument calculated as the ratio of the stock's simultaneous prices in the NYSE and the TSX (the synthetic, henceforth) obtained from the combination of opposite positions (long USD and short CAD, for example) of the same stock being traded on both

[5] Canadian cross-listed stocks on the NYSE (Canadian ordinaries henceforth) have an identical certificate in the United States and Canada. They do not include any legal restrictions on ownership and cross-border transactions.

sides of the border. As for internationally cross-listed stocks, the stock prices share two underlying factors: the firm's fundamental value and the exchange rate (Scherrer, 2018). Given that we use the same stock in the two exchanges, the idiosyncratic differences are minimal and should not affect the convergence in pairs trading, contrary to what is often observed with different stocks in the literature (Frazzini et al, 2018; Engelberg et al, 2009; Pontiff, 2006).

Second, we hedge the synthetic with an opposite position in the CADUS. Defining the relative spread (SPRD) as equal to the ratio of the synthetic over the CADUS, we test for the SPRD stationarity, a *sine qua non* condition for mean-reverting strategies. At equilibrium, SPRD must converge to a value close to 1.0 for each pair in all trading days, with very few exceptions. Spot and futures prices should diverge slightly, only by the basis value, which accounts for maturity differences in the two instruments.

As a distance criterion, we propose a non-parametric threshold rule adjusted for the strategies' net costs in order to uncover economically relevant opportunities. This is an alternative to standard deviation multiples (Stübinger and Bredthauer, 2017; Gatev et al, 2006). The chosen distance approach is simple and transparent, and allows for large-scale empirical applications (Krauss, 2017).

As the FX, TSX and NYSE market makers are not perfectly integrated, we have to consider the differences between the functioning of the US and Canadian microstructures. These sources of divergences may influence limit order books (depth, granularity, imbalance, and bid-ask spread) and marketable orders (trade intensity and potential directional or bouncing behavior).

Data from the three geographically distant exchanges are asynchronous. We apply synchronization procedures to the arbitrageur information processing lag. We implement a two-regime shift incurred by transport delays of information to and from the exchange servers, and we correct the timestamps for the exchanges' processing time and matching delays. The synchronization is effective at a TSX colocation server.

Our methodology does not allow holding overnight positions. This prevents hedging overnight gap risk and tying up capital due to end-of-day margin requirements (Menkveld, 2014). This also avoids being forced to unwind positions due to margin squeezes (Brunnermeier and Pedersen, 2008). We use the exchanges' appropriate trading fees and rebates to evaluate net arbitrage performances, as well as colocation expenses. Details on these costs are provided in Table 1 of Section 4.

Relative spread

Arbitrage opportunities are identified by constructing a relative spread (SPRD) equal to the ratio of a synthetic spread calculated as the ratio of a stock's simultaneous prices in the NYSE and the TSX (the synthetic, hereafter), to the hedging instrument, the CADUS futures:

$$\gamma_t \equiv \frac{S_{us,t}/S_{cad,t}}{r_t},$$

where γ_t is the mathematical notation for SPRD value at time t, $S_{us,t}$ and $S_{cad,t}$ are the cross-listed stock values on the US market (NYSE) and the Canadian market (TSX), at time t, and r_t is the exchange rate computed from the CADUS hedging instrument value at time t. We define simultaneous prices as prices from a unique time frame of observation which considers the information transportation and treatment time between trading venues (latencies).

We can write:

$$\gamma_t^{Short} = \frac{S_{US,t}^{Bid}/S_{CAD,t}^{Ask}}{r_t^{Ask}} \text{ and } \gamma_t^{Long} = \frac{S_{US,t}^{Ask}/S_{CAD,t}^{Bid}}{r_t^{Bid}}$$

as the time series of the short and long relative spreads, where the exponents Bid and Ask are the stock prices on the bid and ask side.

Using the Augmented Dickey-Fuller test for stationarity, we see that both the $\{\gamma_t^{Short}\}_{t=1}^{T}$ and $\{\gamma_t^{Long}\}_{t=1}^{T}$ time series from January 7th 2019 to June 28th 2019 are stationary for almost all stocks in all trading days where the three exchanges are open at the same time, at a *p*-value of 1%, with continuous observation time. Details are presented in Table A.1 of Appendix A. Given that the SPRD time series are stationary and exhibit strong mean-reversion, we define τ^i, $i \in \{Short, Long\}$ as the equilibrium level of the mean-reverting processes $\{\Gamma_t^i\}_{t=1}^{T}$ with observations $\{\gamma_t^i\}_{t=1}^{T}$.

Market order arbitrage strategy

A potential arbitrage opportunity arises when the synthetic is not in equilibrium with the observable exchange rate at time t, that is when:

$$\gamma_t^i \neq \tau^i,\ i \in \{Short, Long\},$$

where τ^i is the mean equilibrium value. The arbitrage opportunity ends when the equilibrium is restored at time $t' > t$ where t' is defined as:

$$t' \quad \equiv argmin_{s>t}\{s \mid \gamma_s^i = \tau^i\},\ i \in \{Short, Long\}.$$

The synthetic is overvalued at time t when:

$$\gamma_t^{Long} = \frac{S_{US,t}^{Ask}/S_{CAD,t}^{Bid}}{r_t^{Bid}} > \tau^{Long}.$$

In that case, since Γ_t^{Long} is mean-reverting, this mispricing can be exploited by shorting $1/\tau^{Long}$ shares of the American stock, taking a long position of one share in the Canadian counterpart (which means that we short the synthetic), and taking a long position in the CADUS of the same value as the American stock position in order to hedge our position, all transactions at time t. Then, we must revert the three positions at time t' using market orders to lock the profit per Canadian stock ($P_{t'}$) in Canadian dollars at time t':

$$P_{t'} = \frac{1}{\tau^{Long} r_{t'}^{Ask}} (S_{US,t}^{Bid} - S_{US,t'}^{Ask}) + (S_{CAD,t'}^{Bid} - S_{CAD,t}^{Ask}) + \frac{S_{US,t}^{Bid}}{\tau^{Long} r_{t'}^{Ask}} \left(\frac{r_{t'}^{Bid}}{r_t^{Ask}} - 1 \right) - c_{CAD,t'}^{Long}.$$

where $c_{CAD,t'}^{Long}$ measures the trading costs. However, given that the CADUS is very liquid, its bid-ask spread is minimal, so we can use the following approximation for the profitability of the positions:

$$P_{t'} \approx \frac{1}{\tau^{Long} r_{t'}^{Bid}} (S_{US,t}^{Bid} - S_{US,t'}^{Ask}) + (S_{CAD,t'}^{Bid} - S_{CAD,t}^{Ask}) + \frac{S_{US,t}^{Bid}}{\tau^{Long} r_{t'}^{Bid}} \left(\frac{r_{t'}^{Bid}}{r_t^{Ask}} - 1 \right) - c_{CAD,t'}^{Long} \qquad (1)$$

where we have substituted $r_{t'}^{Ask}$ with $r_{t'}^{Bid}$. On the other hand, supposing a perfect hedge, we only buy a fraction of the currency futures of nominal N_{CME} that equals the amount invested in the American stock at time t. So only a fraction of the constant futures' trading price is paid on this cost-per-share basis. The trading costs paid for opening and closing our positions in Canadian dollars at time t', $c_{CAD,t'}^{Long}$, are approximated by:

$$c_{CAD,t'}^{Long} \approx 2c_{TSX} + 2\frac{c_{NYSE}}{\tau^{Long} r_t^{Bid}} + 2\frac{c_{CME}}{N_{CME} r_t^{Bid}} \cdot \frac{S_{US,t}^{Bid}}{\tau^{Long} r_t^{Ask}}$$

where c_{TSX} and c_{NYSE} are the per-share trading fees on the TSX (in CAD) and the NYSE (in USD), respectively, which are constant through time in our period, and c_{CME} is the per-contract trading costs (in USD) with nominal $N_{CME} = \text{CAD }\$10{,}000$, which is the Micro CAD/USD nominal.

When the three instruments return to equilibrium, the definition of t' implies that:

$$\frac{S_{US,t'}^{Ask}/S_{CAD,t'}^{Bid}}{r_{t'}^{Bid}} = \tau^{Long} \Longrightarrow \frac{S_{US,t'}^{Ask}}{\tau^{Long} r_{t'}^{Bid}} = S_{CAD,t'}^{Bid}.$$

Using this last equality in the approximation (1), we get:

$$P_{t'} \quad \approx \frac{S_{US,t}^{Bid}}{\tau^{Long} r_t^{Ask}} - S_{CAD,t}^{Ask} - c_{CAD,t'}^{Long},$$

which means that to generate a positive profit at time t', we at least need to have:

$$P_{t'} > 0 \Leftrightarrow \quad \frac{S_{US,t}^{Bid}}{\tau^{Long} r_t^{Ask}} - S_{CAD,t}^{Ask} > c_{CAD,t'}^{Long}$$

We include a factor $\alpha \geq 0$ to introduce a potential mark-up under imperfect competition. The last profit inequality then becomes:

$$\begin{aligned} \frac{S_{US,t}^{Bid}}{\tau^{Long} r_t^{Ask}} - S_{CAD,t}^{Ask} &> c_{CAD,t'} + \alpha S_{CAD,t}^{Ask} \\ \gamma_t^{Long} \frac{r_t^{Bid} S_{CAD,t}^{Bid}}{S_{US,t}^{Ask}} \frac{S_{US,t}^{Bid}}{\tau^{Long} r_t^{Ask}} - S_{CAD,t}^{Ask} &> c_{CAD,t'} + \alpha S_{CAD,t}^{Ask} \\ \gamma_t^{Long} &> \tau^{Long} \underbrace{\frac{r_t^{Ask} S_{US,t}^{Ask}}{r_t^{Bid} S_{US,t}^{Bid}} \frac{(1+\alpha) S_{CAD,t}^{Ask} + c_{CAD,t'}^{Long}}{S_{CAD,t}^{Bid}}}_{>1, \text{in normal market conditions}} \equiv \kappa_t^{Over}. \end{aligned} \tag{2}$$

Equation (2) gives us a dynamic upper non-parametric threshold κ_t^{Over} indicating when a short position in our relative spread (SPRD) is profitable because it is overvalued considering trading costs and bid-ask spreads when only market orders are used. This profitability holds when there is a return to equilibrium to close the positions.

The same logic with opposite positions also holds when the synthetic is undervalued at time t, or when:

$$\gamma_t^{Short} = \frac{S_{US,t}^{Bid} / S_{CAD,t}^{Ask}}{r_t^{Ask}} < \tau^{Short}.$$

This results in a dynamic lower non-parametric threshold at which a long position in the synthetic is profitable considering trading costs and bid-ask spreads when market orders are used:

$$\gamma_t^{Short} < \tau^{Short} \underbrace{\frac{r_t^{Bid} S_{US,t}^{Bid}}{r_t^{Ask} S_{US,t}^{Ask}} \frac{(1-\alpha) S_{CAD,t}^{Bid} - c_{CAD,t'}^{Short}}{S_{CAD,t}^{Ask}}}_{<1, \text{in normal market conditions}} \equiv \kappa_t^{Under} \tag{3}$$

where $c_{CAD,t'}^{Short} \approx 2c_{TSX} + 2\frac{c_{NYSE}}{\tau^{Short} r_t^{Bid}} + 2\frac{c_{CME}}{N_{CME} r_t^{Bid}} \cdot \frac{S_{US,t}^{Ask}}{\tau^{Short} r_t^{Ask}}$.

Once again, the profitability of the strategy holds when there is a return to equilibrium to close the long position of SPRD.

From equations (2) and (3), we have a set of two signals, γ_t^{Long} and γ_t^{Short}, where $\gamma_t^{Long} > \gamma_t^{Short}\ \forall t$ (which implies that $\tau^{Long} > \tau^{Short}$) in normal market conditions and with their respective dynamic non-parametric thresholds, κ_t^{Over} and κ_t^{Under}, where $\kappa_t^{Over} > \tau^{Long} > \tau^{Short} > \kappa_t^{Under}\ \forall t$.

The arbitrage strategy can then be summarized as follows:

- When γ_t^{Long} crosses κ_t^{Over} from below: short $1/\tau^{Long}$ shares of $S_{US,t}$, long $S_{CAD,t}$ and long the CADUS for the same value as the one invested in the American stock,
- When γ_t^{Short} crosses κ_t^{Under} from above: long $1/\tau^{Short}$ shares $S_{US,t}$, short $S_{CAD,t}$ and short the CADUS for the same value as the one invested in the American stock,
- Close the positions when the equilibrium is restored at t'.
- Repatriate the profits generated at the NYSE and CME to the TSX whenever their sum crosses N_{CME}.

Finally, we add the per-share fixed colocation cost and proprietary data feed cost to compute net profit on a given period.

Limit order arbitrage strategy

We now switch to limit orders, as paying the bid-ask spread on the three instruments can be very costly. The strategy remains the same as with the market orders. The main difference is in the profitability equation used to find the entry thresholds. The relative spread is overvalued at time t when:

$$\gamma_t^{Short} = \frac{S_{US,t}^{Bid}/S_{CAD,t}^{Ask}}{r_t^{Ask}} > \tau^{Short}.$$

In that case, we short SPRD at time t and revert the three positions when the equilibrium of Γ_t^{Short} is restored at time t'. This results in a profit in Canadian dollars of:

$$P_{t'} = \frac{1}{\tau^{Short} r_{t'}^{Ask}} (S_{US,t}^{Ask} - S_{US,t'}^{Bid}) + (S_{CAD,t'}^{Ask} - S_{CAD,t}^{Bid}) + \frac{S_{US,t}^{Ask}}{\tau^{Short} r_{t'}^{Ask}} \left(\frac{r_{t'}^{Ask}}{r_t^{Bid}} - 1 \right) - \tilde{c}_{CAD,t'}^{Short} \tag{4}$$

per Canadian stock, where $\tilde{c}_{CAD,t'}^{Short}$ has the same formula as $c_{CAD,t'}^{Short}$, but instead of c_{TSX} and c_{NYSE} being the per-share trading-costs, they are now per-share trading-rebates because the limit orders provide liquidity to the markets as opposed to market orders. This means that both c_{TSX} and c_{NYSE} are negative values in the limit order case.

Employing the same logic as previously used to obtain the non-parametric entry thresholds κ_t^{Over} and κ_t^{Under}, we find that the dynamic upper threshold indicating a profitable short position in our relative synthetic spread using limit orders is given by:

$$\gamma_t^{Short} > \tau^{Short} \underbrace{\frac{r_t^{Bid} S_{US,t}^{Bid}}{r_t^{Ask} S_{US,t}^{Ask}} \frac{(1+\alpha) S_{CAD,t}^{Bid} + \tilde{c}_{CAD,t'}^{Short}}{S_{CAD,t}^{Ask}}}_{\text{multiplicative term}} \equiv \tilde{\kappa}_t^{Over}, \tag{5}$$

and the dynamic lower non-parametric threshold for long positions in our relative synthetic spread using limit orders is given by:

$$\gamma_t^{Long} < \tau^{Long} \underbrace{\frac{r_t^{Ask} S_{US,t}^{Ask}}{r_t^{Bid} S_{US,t}^{Bid}} \frac{(1-\alpha) S_{CAD,t}^{Ask} - \tilde{c}_{CAD,t'}^{Long}}{S_{CAD,t}^{Bid}}}_{\text{multiplicative term}} \equiv \tilde{\kappa}_t^{Under}. \tag{6}$$

Notice that the term multiplying the equilibrium level in equation (2) is always greater than the multiplicative term in equation (5). This means that arbitrage opportunities are available at a lower level of γ_t^{Short} with limit orders, and thus should be more frequent. This is true since limit orders greatly reduce the costs related to the strategy. The same observation can be made for the long position non-parametric thresholds of equations (3) and (6): limit orders push the entry thresholds to a more easily attainable level compared with market orders.

From equations (5) and (6), we have a set of two signals, γ_t^{Short} and γ_t^{Long} with their respective dynamic non-parametric thresholds, $\tilde{\kappa}_t^{Over}$ and $\tilde{\kappa}_t^{Under}$. The arbitrage strategy can then be summarized as follows:

- When γ_t^{Short} crosses $\tilde{\kappa}_t^{Over}$ from below: short $1/\tau^{Short}$ shares of $S_{US,t}$, long $S_{CAD,t}$ and long the CADUS for the same value as the one invested in the American stock,
- When γ_t^{Long} crosses $\tilde{\kappa}_t^{Under}$ from above: long $1/\tau^{Long}$ shares $S_{US,t}$, short $S_{CAD,t}$ and short the CADUS for the same value as the one invested in the American stock,
- Close the positions when the equilibrium is restored at t'.
- Repatriate the profits generated at the NYSE and CME to the TSX whenever their sum crosses N_{CME}.

Strategy at the portfolio level and aggregate hedging

Consider a universe Ω of N cross-listed stocks on the TSX and the NYSE, $|\Omega| = 2N$. We wish to execute the cross-listed stocks arbitrage strategy defined in the previous sections, on every pair contained in that universe. This extension is applicable with both market orders and limit orders and is important for the real application of the two previous models.

Due to the construction of our strategy, aggregating every position in a single portfolio offers a built-in hedging effect against movements of the CAD/USD exchange rate whenever positions are opened in both Γ_t^{Short} and Γ_t^{Long}, because the aggregated position in the American market is reduced compared to the sum of the absolute position of every independent portfolio for each pair. The hedge can be optimized with the use of currency futures. This section explores that extension.

Let us define $v_{TSX,t}^{(n)}, v_{NYSE,t}^{(n)} \in \mathbb{R}, n \in \{1, \ldots, N\}$ the size of the position in the cross-listed stock n in both markets at time t. A position is long when the size is positive, a position is short when the size is negative, and the size is zero when no position is opened in the stock. Let us also define the total non-repatriated profits, in USD, generated at NYSE and CME at time t respectively by $G_{NYSE,t}$, $G_{CME,t} \in \mathbb{R}$. Hence, the portfolio's exposures in Canadian dollars at the TSX, NYSE and CME at time t are respectively given by:

$$V_{TSX,t} = \sum_{n=1}^{\mathrm{N}} v_{TSX,t}^{(n)} S_{TSX,t}^{(n)},$$

$$V_{NYSE,t} = \sum_{n=1}^{\mathrm{N}} v_{NYSE,t}^{(n)} \frac{S_{NYSE,t}^{(n)}}{r_t} + \frac{G_{NYSE,t}}{r_t},$$

$$V_{CME,t} = \frac{v_{CME,t}^{*} N_{CME} + G_{CME,t}}{r_t},$$

where $v_{CME,t}^{*} \in \mathbb{R}$ is the optimal position size in the CADUS futures at time t that we are trying to obtain. The total value of the portfolio in Canadian dollars at time t, V_t, is given by:

$$V_t = V_{TSX,t} + V_{NYSE,t} + V_{CME,t}.$$

By taking a position in the CADUS that is the inverse of the position in the NYSE, we obtain:

$$V_{CME,t} = -V_{NYSE,t} \Leftrightarrow v_{CME,t}^{*} = -\frac{V_{NYSE,t} r_t + G_{CME,t}}{N_{CME}}, \tag{7}$$

which results in a neutral aggregated position in the American market; $V_{NYSE,t} + V_{CME,t} = 0$. The portfolio's value is now simply given by $V_t = V_{TSX,t} \Longrightarrow \frac{\mathrm{d}V_t}{\mathrm{d}r_t} = \frac{\mathrm{d}V_{TSX,t}}{\mathrm{d}r_t} = 0$. The last equality supposes the mathematical independence of the Canadian stocks' prices and the CAD/USD exchange rate. That is to say, in the universe Ω, a portfolio invested in cross-listed stock pairs that follows the proposed strategy for every pair achieves an optimal hedge against currency risk at any time t when that portfolio has a neutral aggregated position in the American market. If the aggregated position in the NYSE stocks is not neutral, we can take a position of $v_{CME,t}^{*}$ contracts in the currency futures to get a perfect hedge.

Thus, the optimal hedge against the currency risk for a portfolio of pairs at time t is achieved by having the position in the currency futures presented in equation (7). In other words, currency risk is only present when the aggregated position in the American market is not neutral.

The hedging of the portfolio is done by rebalancing our position in the CME to the optimal value, if necessary, whenever we open or close positions in pairs of cross-listed stocks, compared with the pair-wise strategy that requires taking the inverse position taken in the NYSE at every arbitrage opportunity.

4. Latencies, arbitrage costs, and arbitrage risks

Latencies and arbitrage costs

A factor of interest in this contribution is latency. In trading terms, latency refers to the time it takes for an agent to interact with the market. We follow closely Hasbrouck and Saar's (2013) measure of latency based on three components: the time it takes for a trader to learn about an event, generate a response, and have the exchange act on that response. We split that definition into two separate quantities so that we can have more granularity on the impact of latency on the high-frequency trading strategies.

The first quantity of importance is the latency of a message from an exchange to TSX, which includes the one-way transportation time of the information to TSX, and the information treatment time needed by the agent's servers collocated at TSX and having access to a proprietary data feed. The second quantity of importance is the latency of a message from the TSX to another exchange, which is comprised of the one-way transportation time of the information from TSX to the receiving exchange, and the matching engine delay of that last exchange.

Information treatment time refers to the timespan required to receive and analyze incoming information from the exchanges, followed by the decision to trade or not. Exchange server procedure considers information reception at the exchange gates, limit order book (LOB) positioning or matching of an incoming limit order (with the LOB) and issuing traders' confirmation to the server gates. Round-trip latency measures the total latency cost for a transaction between two exchanges.

Our raw data include all trades and all LOB level-one positions from TSX, NYSE and CME. These events are timestamped at the exchange gates. To synchronize information at our TSX colocation, we update the raw datasets by considering the above-mentioned latencies. We can therefore identify adequately synchronized arbitrage opportunities.

Table 1 documents the 2019 latency costs, trading costs, rebates, colocation costs, and proprietary data feed costs used in this study. Orders and positions are managed at TSX's colocation premises in Toronto (TSX, 2020). Information comes from the TSX, the NYSE, and the CME. We address asynchronicities by adjusting the TSX timestamps based on round-trip transportation time, arbitrageur information processing delays, and exchanges matching engine delays presented in Table 2. Following conversations with quant traders and data scientists, we apply a two-regime model associated with normal and extreme market conditions based on quote and trade message intensity. The regime shifts, from the normal state to the extreme one, are due mainly to bursts in the events stream, phenomena well documented in the literature (Friederich and Payne, 2015; Menkveld, 2016; Egginton, et al., 2016; Dixon et al., 2019).

Table 1. Arbitrage costs

Definition	Description	Measurement	In Deltix
Information transportation time between exchanges	Transportation time details: Toronto – Chicago: Fiber paths Toronto – New York: Microwave path (regular) Fiber path (extreme situations)	See Table 2	Adjusted raw dataset timestamp fed to Deltix
Information treatment time	Timespan required to receive and analyze incoming information from the exchanges, followed by the decision to trade or not.	See Table 2	Adjusted raw dataset timestamp fed to Deltix
Exchange trading fees	TSX member trading fees per share[1] NYSE Type A stocks per share[2] CME Globex CAD/USD FX futures per contract[3]	Removing: $0.0015 Providing: ($0.0011) Removing: $0.00275 Providing: ($0.00120) $100k notional value: $0.32 $10k notional value (e-micro): 0.04$	Applied to matched orders
Colocation cost	Colocation with exchange connectivity rates	Half cabinet (21U, 3 kw maximum): $5,250 monthly Initial set-up fee: $5,250 one-time	Included in monthly portfolios performance
Proprietary data feed	TSX & Venture level 1 Distribution Trading use case license	$4,000 monthly	Included in monthly portfolios performance

[1] *https://www.tsx.com/resource/en/1756/tsx-trading-fee-schedule-effective-june-4-2018-en.pdf*

[2] *https://www.nyse.com/markets/nyse/trading-info/fees*

[3] *https://www.cmegroup.com/company/clearing-fees.html*

Table 1 also documents the positive trading fees for the removers of liquidity and rebates for the providers. Colocation costs in Toronto are considered in our monthly portfolio performance estimations, as well as proprietary data feeds which enable some trading firms to receive updates from the exchange faster than other traders who do not pay for this service. We now document additional arbitrage risks considered in our study.

Arbitrage risks

Execution risk

The choice between limit and market orders relies, in part, on the difference between the non-execution risk and the execution risk (Mavroudis, 2019; Dugast, 2018; Liu 2009; Kozhan and Tham, 2012; Brolley, 2020). To empirically solve this trade-off, we first evaluated our algorithm's performance using market orders exclusively. As we will see, using only market orders leads to a negative economic value with our data in the sense that the cost of immediacy (conceding the bid-ask spread) cannot be borne by the arbitrageur in the vast majority of trades. This high cost also results in a very low number of potential arbitrage opportunities, since the divergence of SPRD is rarely large enough to compensate it. We then constrained our algorithm to the limit orders, except for the liquidation of the positions to avoid overnight exposures. We also use marketable limit orders to offset unexecuted legs. There remain two additional risks.

Non-execution risk

We evaluate the non-execution risk costs by managing the LOB queuing priorities. We mitigate the risk of non-execution by keeping, dynamically, our limit orders to the LOB's level one. This is implemented conditional on the persistence of an expected profitable arbitrage. Otherwise, we liquidate positions, if any, by issuing marketable limit orders (Dahlström and Nordén, 2018).

Table 2. Latencies[1]

	TRANSPORTATION ONE WAY TO TSX		ARBITRAGEUR	TRANSPORTATION ONE WAY FROM TSX		MATCHING ENGINE	TOTAL
Market condition	Exchanges from–to	Transportation time	Information treatment	Exchanges from–to	Transportation time	Exchange server	Round-trip latency
Normal	TSX–TSX	+ 5 µs	+ 10–70 µs	TSX–TSX	+ 5 µs	+ 100–300 µs	120–380 µs
	NYSE–TSX	+ 2.4 ms	+ 10–70 µs	TSX–NYSE	+ 2.4 ms	+ 100–300 µs	4.91–5.17 ms
	CME–TSX	+ 5 ms	+ 10–70 µs	TSX–CME	+ 5 ms	+ 1–5 ms	11.01–15.07ms
Extreme	TSX–TSX	+ 5–10 µs	+ 200–500 µs	TSX–TSX	+ 5–10 µs	5–10 ms	5.21–10.52 ms
	NYSE–TSX	+ 4.8–9.6 ms	+ 200–500 µs	TSX–NYSE	+ 4.8–9.6 ms	5–10 ms	14.80–29.7 ms
	CME–TSX	+ 5–10 ms	+ 200–500 µs	TSX–CME	+ 5–10 ms	50–100 ms	60.20–120.50 ms

[1] Latencies are obtained following discussions with a major Canadian financial institution trading actively in Canada and in the United-States. ms: millisecond; µs: microseconds.

Mean-reversion risk

Mean-reversion risk arises after the initial positions are taken. It materializes when the circuit breaker timer is triggered. All arbitrage legs are then liquidated via marketable limit orders. As we will see, this risk is very low in our data since the processes Γ_t^{Short} and Γ_t^{Long} are stationary for almost all stocks and trading days.

5. Practical considerations for strategy implementation

The equilibrium value of the relative spread, $\tau^i,\ i \in \{Short, Long\}$ can be easily computed *a posteriori* at the end of the day. However, in practice, these quantities need to be known in real time to find the arbitrage opportunities. To account for overnight basis adjustment, a simple approximation could be the sample average of the Γ_t^i process during the first minutes of a trading day before starting the strategy. We eliminated the first two minutes of each trading day, however, because their processes are initially far from their daily equilibrium level, and only reach it around two minutes after the market opens.

The approximation is then used as the first value of τ^i when the strategy starts. From that starting point, the approximation is following a running average of γ_t^i at every LOB level one event in one of the three exchanges for a given stock and currency futures. Note that the strategy needs a constant equilibrium value from the opening trades to the closing trades, meaning that the τ^is are not updated when positions are still opened for a given pair.

The strategy assumes that the synthetic spreads return exactly to equilibrium at their respective time t'. But in practice, these processes are not continuous in time, hence there is a null probability that they would converge exactly to τ^i at any time. To solve this issue, we add another parameter β that controls when the processes are near enough to their respective equilibrium to close the positions. The practical definition of t' becomes:

$$t'_{market,short} \equiv argmin_{s>t}\{s \mid \gamma_s^{short} \in [\tau^{short} - \beta(\tau^{short} - \kappa_s^{Under}), \tau^{short} + \beta(\tau^{short} - \kappa_s^{Under})]\}$$

for the process Γ_t^{Short} with the market order-based strategy,

$$t'_{market,long} \equiv argmin_{s>t}\{s \mid \gamma_s^{long} \in [\tau^{long} - \beta(\kappa_s^{Over} - \tau^{long}), \tau^{long} + \beta(\kappa_s^{Over} - \tau^{long})]\}$$

for the process Γ_t^{Long} with the market order-based strategy,

$$t'_{limit,short} \equiv argmin_{s>t}\{s \mid \gamma_s^{short} \in [\tau^{short} - \beta(\tau^{short} - \tilde{\kappa}_s^{Over}), \tau^{short} + \beta(\tau^{short} - \tilde{\kappa}_s^{Over})]\}$$

for the process Γ_t^{Short} with the limit order-based strategy, and

$$t'_{limit,long} \equiv argmin_{s>t}\{s \mid \gamma_s^{long} \in [\tau^{long} - \beta(\tilde{\kappa}_s^{Under} - \tau^{long}), \tau^{long} + \beta(\tilde{\kappa}_s^{Under} - \tau^{long})]\}$$

for the process Γ_t^{Long} with the limit order-based strategy.

The smaller β, the nearer the processes need to be to the equilibrium to close the positions.

The volumes sent to the market by the strategy are round lots because of the higher costs related to sending odd lot orders, meaning that the minimum volume that can be used in our strategy is 100 stocks on both the NYSE and the TSX. To capture as much of the arbitrage opportunities as possible without heavily impacting the price discovery processes, we dynamically determine the orders' volume following the first level volumes available in the LOB of the NYSE and the TSX for a given pair of stocks. The orders' volume sent on both markets is limited by the less active one, since for one stock in the Canadian market, we take a position of $1/\tau$ stocks in the American market. We have observed that τ^i does not deviate far enough from 1 to send a different number of lots in both markets for the same arbitrage opportunity. Therefore, the implemented strategy sends the same volumes to the TSX and NYSE.

Let us define $\tilde{v}^{side}_{market,t}$ as the median of the volume on the first LOB level on $side \in \{Bid, Ask\}$ in $market \in \{CAD, US\}$ based on the last 500 LOB level one updates preceding time t. Then, the volume sent to both markets at time t for any cross-listed stock, v_t, is computed as either:

$$v_t = 100\max\left(\min\left(\left\lfloor\frac{\tilde{v}^{Bid}_{CAD,t}}{100}\right\rfloor, \left\lfloor\frac{\tilde{v}^{Ask}_{US,t}}{100}\right\rfloor\right), 1\right)$$

or:

$$v_t = 100\max\left(\min\left(\left\lfloor\frac{\tilde{v}^{Ask}_{CAD,t}}{100}\right\rfloor, \left\lfloor\frac{\tilde{v}^{Bid}_{US,t}}{100}\right\rfloor\right), 1\right)$$

depending on whether market or limit orders are used, and whether a long or short position is opened or closed in SPDR.

As mentioned previously, we use currency futures to hedge our position from currency risk. We have also shown that the optimal position in that instrument is given by equation (7) at any time during the strategy's execution. To follow that position as closely as possible, we employ the Micro CAD/USD futures contract with a nominal of CAD$10,000, which we approximate by dividing the prices of our continuous futures by 10, because of its nominal of CAD$100,000.

Let $\hat{v}_{CME,t} \in \mathbb{Z}$ be the number of Micro CAD/USD futures contracts needed at time t which best approximates the position size theoretically needed at the CME at that time, $v^*_{CME,t}$, without under-hedging the aggregated position in the American markets. We compute its value as:

$$\hat{v}_{CME,t} = \begin{cases} \lfloor v^*_{CME,t} \rfloor \; if \; v^*_{CME,t} \leq 0 \\ \lceil v^*_{CME,t} \rceil \; if \; v^*_{CME,t} > 0 \end{cases}, \forall t.$$

Because of the high nominal value of the futures, we cannot perfectly hedge the American positions. In the market order strategy, only market orders are used to follow as much as possible $\hat{v}_{CME,t}$ during the strategy's execution. In the limit order strategy, limit orders are sent to the top-of-the book prices, or canceled, or updated at every market event modifying $\hat{v}_{CME,t}$ to achieve the same goal. Latency makes it more complicated to get exactly a CME volume of $\hat{v}_{CME,t}$ at all times.

To mitigate the mean-reversion risk and the non-execution risk, specifically for the limit order-based strategy, a timer of 15 minutes is used to cancel any orders and close any position resulting from opening a position in the synthetic spread (SPRD) using marketable limit orders. The timer starts when the orders are sent to the markets and ends only when the orders are filled, and the positions are closed. Along the same vein, stop-losses are also implemented so that if the prices, in the LOB level one, diverge drastically from pending limit order prices, these would be canceled, and any opened position would be closed with marketable limit orders.

Even though the strategy is built to be theoretically profitable for every pair, the cross-listed stocks' characteristics could lead to unprofitable trades. To determine how the underlying factors of a profitable pair differ from the ones of a non-profitable pair, we resort to supervised learning. The resulting machine learning algorithm allows us to predict the future profitability of our pairs, thus enabling dynamic pair selection and optimizing the strategy's performance by filtering out potentially non-profitable pairs.

Specifically, we utilize a decision tree algorithm, because of its interpretability. We apply this non-parametric model to predict if a given pair will be profitable in the next period based on the data in previous periods. We treat this problem as a dynamic binary classification task where the output of the model at each period is either profitable or unprofitable for each pair in the universe Ω during the next period. See Appendix B for more details on the pair selection method using the decision tree algorithm.

6. Data, data synchronization, trading and quoting emulator, and empirical latencies

Data

We use LOB level one data and trade data that we get from: the TAQ NYSE OpenBook and the TAQ NYSE Trades historical data timestamped to the microsecond, the CME Market Depth FIX Canadian Dollar Futures historical data timestamped to the nanosecond, and Trades and Quotes Daily historical data from TMX Group timestamped to the nanosecond. All the data was timestamped at the respective exchanges, and span from January 7th, 2019 to June 28th, 2019, inclusively. We only selected dates where the three exchanges were opened, meaning that we eliminated every holiday from our sample.[6]

Overall, there are 120 trading days in our data set. We have access to 74 pairs of cross-listed stocks that were listed on both the TSX and the NYSE during at least two weeks of that period. Pairs where one of the stocks got de-listed from an exchange at any point were kept in the sample, but the strategy was only applied to periods where both stocks of the pair were listed and active. All cross-listed S&P/TSX 60 stocks are present in our sample during the six months. Table A2 describes every available pair and Table A3 includes their aggregated statistics during the period of analysis.

The time series of daily number of trades and quotes in the two exchanges for some pairs of stocks of interest are presented in Figure C1 of Appendix C. The four rows of graphs in Figure C1 present the trades and quotes data of some of the most often selected stocks for arbitrage. We do

[6] TSX: February 18th: Family Day; April 19th: Good Friday; May 20th: Patriot's Day.
NYSE and CME: January 21st: Martin Luther King Jr. Day; February 18th: President's Day; April 19th: Good Friday; May 27th: Memorial Day.

not observe any pattern between the number of trades and the number of quotes. The main differences seem to be related to the type of industry.

We use the quarterly CAD/USD futures listed on CME: 6CH9 expiring March 19, 2019; 6CM9 expiring June 18, 2019; and 6CU9 expiring September 17, 2019. We do not use monthly futures because they have a lesser amount of float. The continuous futures contract is created by concatenating the three futures' data and by adjusting the LOB level one and trade prices of the consecutive contracts so that no jumps are artificially created. The concatenation dates are determined based on the daily transaction volume of consecutive futures. That is, whenever the futures contract with the farthest expiration date generates a significantly higher daily transaction volume than its predecessor and remains more actively traded, we switch to that futures' trades and quotes for the continuous futures that we use in the strategy.

Data synchronization

The strategy is launched each week, from Monday to Friday, starting at 9:30 am and ending at 4:00 pm Eastern Time when the three exchanges are all opened to continuous trading. Both the TSX and NYSE are in the Eastern Time zone, but the CME is in the Central Time zone, one hour behind. Hence, we add an hour to the time stamps of the CME data to synchronize the three exchanges' clocks.

Trading and quoting emulator

Our methodology relies on the implementation of the trading strategies in Deltix's QuantOffice. The Deltix trading suite allows us to replay the synchronized events of the three stock markets (level one LOB and trades) as they were obtained in streaming by traders. By handling these events and following our orders position in the queues, Deltix determines as realistically as possible the real-time performance that would have been obtained with our strategies. Note that a single ex-ante set of parameters was tested. This implementation makes it possible to consider trading fees and rebates, latency costs and other trading risks and costs presented in Section 4. It confirms the order status (creation, cancelation, or execution) just as it would have happened in streaming trading considering market frictions and ever-changing market states. Standard reports, such as a trade report and a performance report, are generated at the end of a strategy's execution and these are used to compute our results.

Moreover, Deltix manages the individual and aggregated positions, and calculates the respective Profit and Loss Reports (PnL) altogether with performance statistics. These PnLs represent the economic value of our arbitrage opportunities. Using our performance as a benchmark, we can evaluate the economic impact of latency risk by varying the aforementioned latency parameters. The general rules of the trading and quoting emulator on LOB level one data and information on how executions and non-executions occur are presented in Appendix D.

Empirical latencies

A major Canadian financial institution mentioned to us that latency is following two very different distributions, shifting between a normal latency and an extreme latency distribution at certain times, as documented in Table 2. To help us recreate this behavior, we use a latency regime variable that varies depending on the number of messages a certain exchange received in the last millisecond on a per-asset basis. This quantity is a good proxy of an exchange's server traffic, which has a positive relationship with computational delays occurring during the information treatment time and the matching engine time components of latency.

Our latency regime variable for a given asset remains in its normal state up to a certain static threshold for the number of messages in a single millisecond for that asset, which we set as the

95th percentile of its empirical distribution. Our estimation of that distribution used a random sample of six weeks, where each sampled week came from a different month contained in our data.

Let us define $q^i_{95\%}$ as the 95th percentile of the empirical distribution of the number of messages in one millisecond for asset $i \in \{1, \dots, 2n+1\}$ where $n = 74$ is the number of stock pairs and the additional asset is the CADUS futures and define q^i_j the number of messages during the millisecond preceding and ending at message $j \in [1, N^i]$ where N^i is the total number of messages for asset i during the full period. Let us also define $L^i_j \in \{normal, extreme\}$ the latency regime of asset i at message j. Then, its value is computed as follows:

$$L^i_j = \begin{cases} normal & if\ q^i_j < q^i_{95\%} \\ extreme & if\ q^i_j \geq q^i_{95\%} \end{cases} \forall i, j.$$

For both latency regimes, the latency to and from TSX is set as the sum of the intervals' center of each of their components found in Table 2, for the respective market condition. Deltix allows the latency to be only a natural number of milliseconds, so we round the latencies up to the closest integer. Table 3 details the empirical latencies used.

Table 3. Latencies used when testing the strategies, depending on the latency regime, the origin of the message and the exchange where the message is sent.

Latency regime	Exchanges from–to	Latency	Exchanges from–to	Latency
Normal	TSX–TSX	1 ms	TSX–TSX	1 ms
	NYSE–TSX	3 ms	TSX–NYSE	3 ms
	CME–TSX	6 ms	TSX–CME	8 ms
Extreme	TSX–TSX	1 ms	TSX–TSX	8 ms
	NYSE–TSX	8 ms	TSX–NYSE	15 ms
	CME–TSX	8 ms	TSX–CME	83 ms

By adding the corresponding latency to the original exchange timestamp of every message in our data, we can approximately synchronize the data feeds of the three geographically distant exchanges into a single point of observation as they would be in practice because of the natural and technological limits of information propagation. Our methodology emulates that relativistic effect so that what is observed by the algorithm at any point is a past state of the markets. The same idea applies when the algorithm sends an order to a given exchange. We add the corresponding latency so that the agent does not interact immediately with that exchange. This makes it possible to study the influence of latency on the performance of high-frequency trading strategies.

7. Empirical results

We now present the statistical results of our study in four steps. We first estimate the performance of the trading strategy of Wah (2016) applied to our data. The goal of this exercise is to isolate the importance of considering latencies, execution risk, and trading costs when evaluating the benefits of HFT arbitrage. It also serves as a benchmark to compare our trading strategies and test if previously proposed arbitrage strategies are profitable in our more recent data. This contribution focuses on latency arbitrage where stock price disparities are due to market fragmentation and delays in updating the public price quote. Latency costs, trading fees and execution risk are not considered in this study. As we will see, the contribution overestimates the profits of HFT trading. Then we analyze the model of Budish et al (2015). We obtain a similar conclusion about the profitability of arbitrage trading, as we did with Wah (2016), but for different reasons that we discuss below.

We then present the results from our strategies. We show that arbitrage with market orders is not profitable, while arbitrage with limit orders provides reasonable profits when latencies, rebates, exchange fees, and non-execution risk are considered. Other conclusions are discussed.

Wah (2016) and Budish et al (2015) chose not to close the accumulated positions at the end of the day, which exposes the trader to multiple financial risks: interest on positions, dividends to deliver on short positions, overnight shifts in microstructure, margin calls, among others. Opened positions in different exchanges that last multiple days, combined with a change in microstructure on one exchange, can easily generate large losses that could only be partially compensated by the counterpart. This is especially true in the case of Budish et al (2015), where one asset of the pair is in the futures market and the other in the equity market; a change in basis or a switch in front month contract may represent significant risks that should be mitigated. In that sense, we think that an appropriate arbitrage strategy should close intraday positions before market close, as we do in our approach.

The strategies of Wah (2016) and Budish et al (2015) are first implemented with their theoretical settings with minor modifications to adapt them to our data. In that sense, the prices observed at the NYSE are continuously transferred to CAD following the CADUS futures observed at the CME. In addition, we used two hypotheses employed in their model. There is an absence of latency, and opened positions at an exchange can be immediately closed at another exchange, resulting in a trade. The second columns of Tables 4 and 5 present the results that are obtained with our data for these two strategies, following as closely as possible their respective theoretical framework. In the next three columns of Table 4 and 5, positions are closed at the exchange where they were opened and when the identified arbitrage opportunities are over. This occurs whenever the CAD prices of the cross-listed stocks at the TSX and NYSE uncross following Wah's (2016) and Budish et al's (2015) respective definition of crossed markets. Latency is also considered in these columns. The practical approaches in columns 3, 4 and 5 are implemented in Deltix QuantOffice.

Wah (2016) contribution

Wah (2016) utilizes direct-feed data from MIDAS, a platform at the U.S. Securities and Exchange Commission (SEC) that provides access to order and quote messages on all U.S. stock exchanges. Cross-market arbitrage opportunities are analyzed from 11 U.S. equities exchanges. The author assumes there is a single infinitely fast latency arbitrageur. When the arbitrageur detects a latency arbitrage opportunity, the strategy is to submit market orders to the exchanges involved in the cross-market arbitrage opportunity. The data used by Wah (2016) includes market orders for the 495 tickers of the S&P 500 from January 1, 2014 to December 31, 2014. Latency arbitrage opportunities across these exchanges were observed to happen very frequently during that year and they generated a profit exceeding USD $3.03 billion to the infinitely fast latency arbitrageur.[7]

When we look at column 2 of Table 4, the results of the original model with our data generate a gross profit of CAD $10.5 million for 74 stocks in two exchanges for six months.[8] If we extend these results to eleven exchanges with 495 stocks over one year, this generates about CAD $1.7 billion (USD $1.3) in the year 2019.[9]

The main difference with Wah's original study can be explained by the characteristics of the stocks in the two studies and by the relative sizes of the exchanges. To have a comparable market environment to Wah (2016), when generating the CAD $1.7 billion result we assumed there is only one very fast arbitrageur in colocation in only one exchange and trading in the 11 exchanges. If we extend the possibility that the trading activities are generated by the very fast arbitrageur in colocation in the eleven markets, we obtain about $8.5 billion (USD $6.4) in annual gross profits

[7] 46 tickers from the Russell 2000 were also studied but their profits are not included in the $3.03 billion result.

[8] In this section we do not use the futures contracts for hedging the exchange rate. We do however use the exchange rate updates continuously to obtain pure variations in stock prices between Toronto and NY exchanges.

[9] ($10,465,046.40 × (495/74) × 11 × (252/114) = $1,702,170,313.80).

($1.7*5).[10] We also observe, in column 2 of Table 4, that the trading strategy is no longer profitable in the original model as well as in the three other extended models, when we introduce the actual trading fees for market orders corresponding to those of our data.

Assuming an infinitely fast arbitrageur cannot correspond to any known trading application in the real world. In the extended columns, the average trade time passes from zero to about 8 minutes. In column 3 (Wah extended 1) we observe worse results explained by additional practical trading considerations, which generate even more losses than expected for arbitrage activities with market orders.

The main difference between columns 2 and 3 is explained by the fact that, in column 3, positions are closed in the exchange where they were opened, a trading rule not followed by Wah (2016).

Table 4. Replication of Wah (2016) study with our 2019 data

1 Model	2 Wah original	3 Wah extended 1	4 Wah extended 2	5 Wah extended 3
Latency multiplier	0[1]	0	1	3
Pair selection	No	No	No	No
Gross profit	$10,465,046.40	$10,410,162.62	$5,414,332.92	$5,740,168.72
Loss	$0.00	-$23,736,850.67	-$21,823,992.78	-$21,187,392.42
Trading fees	-$11,589,089.82	-$6,417,798.70	-$6,101,152.47	-$5,613,317.48
Trading rebates	$0.00	$0.00	$0.00	$0.00
Total net profit	-$1,124,043.42	-$19,744,486.75	-$22,510,812.33	-$21,060,541.18
Mean daily net profit	-$9,860.03	-$173,197.25	-$197,463.27	-$184,741.59
Median daily net profit	-$4,912.22	-$157,572.47	-$178,977.89	-$162,136.32
Mean daily net profit per pair, per day	-$133.24	-$2,340.50	-$2,668.42	-$2,496.51

[10] Here we assume the arbitrageur exploits 55 links between the exchanges even if they receive information at one single observation point. A better approximation should consider the real volumes of arbitrage between the exchanges.

1 Model	2 Wah original	3 Wah extended 1	4 Wah extended 2	5 Wah extended 3
p-value Kolmogorov-Smirnov test[2]			2.07E-02	0.9913
1st most profitable day (date - profit)	2019/03/15 - $121,868.66	2019/04/05 - $108,222.86	2019/01/28 - -$39,115.83	2019/04/08 - -$9,772.76
5th most profitable day (date - profit)	2019/06/11 - $22,490.93	2019/01/28 - -$17,322.76	2019/04/03 - -$61,734.24	2019/03/26 - -$48,984.20
1st most unprofitable day (date - profit)	2019/03/01 - -$72,404.63	2019/06/25 - -$579,907.07	2019/06/25 - -$558,091.12	2019/06/25 - -$515,415.61
5th most unprofitable day (date - profit)	2019/03/27 - -$62,178.69	2019/06/20 - -$415,400.31	2019/06/04 - -$399,199.39	2019/06/04 - -$374,359.74
Average time in trade (excl. futures contracts)[3]	00:00:00.00	00:08:28.55	00:08:52.60	00:08:52.60
Average time in trade (incl. futures contracts)	n/a	n/a	n/a	n/a
# net profitable trades	396,287.00	32,642.00	28,902.00	30,829.00
# net unprofitable trades	280,634.00	567,788.00	543,695.00	541,301.00
# trades	676,921.00	600,430.00	572,597.00	572,130.00
% net profitable trades	58.54%	5.44%	5.05%	5.39%
Average volume per trade	3,457.73	2,072.57	2,066.17	1,902.58
Average net profit per trade	-$1.66	-$32.88	-$39.31	-$36.81
Average profit per net profitable trades	$14.02	$313.17	$178.73	$177.44
Average profit per net unprofitable trades	-$23.81	-$52.78	-$50.90	-$49.01

[1] Latency for receiving information from MIDAS is included. Other latencies are not considered by the author.

[2] H0: F(x) = G(x), H1: F(x) ≤ G(x). F(x), G(x) = CDF of daily net profits without and with latency, respectively: 2.07E-02 is for extended 2 vs extended 1 and 0.9913 is for extended 3 vs extended 2.

[3] HH: MM: SS. U: hours: minutes: seconds: fractions of a second.

In the last two columns of Table 4 we add latency in trading activities. In column 4 (Wah extended 2), we use the practical latency parameters of Table 3 in this article, and in the last column we multiply that parameter by 3 to see how sensible the results are to higher latencies. The results show clearly that considering latencies significantly reduces the number of trades and their profitability (significant *p*-value). However, even more latency (last column) does not significantly affect the results between the last two columns which seems to confirm there is a limit in the profitability of competing by simply increasing speed.

Table 5. Budish et al (2015) model with our 2019 data

1 Model	2 Budish original	3 Budish extended 1	4 Budish extended 2	5 Budish extended 3
Latency multiplier	0	0	1	3
Pair selection	No	No	No	No
Gross profit	$984,002.00	$20,548.90	$17,807.05	$17,325.31
Loss	$0.00	-$508,311.18	-$426,499.14	-$488,113.93
Trading fees	-$31,840.47	-$150,448.97	-$113,983.51	-$130,049.47
Trading rebates	$0.00	$0.00	$0.00	$0.00
Total net profit	$952,161.53	-$638,211.25	-$522,675.60	-$600,838.09
Mean daily net profit	$8,352.29	-$5,598.34	-$4,584.87	-$5,270.51
Median daily net profit	$7,678.35	-$3,754.85	-$3,275.18	-$3,463.51
Mean daily net profit per pair, per day	$112.87	-$75.65	-$61.96	-$71.22
p-value Kolmogorov-Smirnov test[1]			9.91E-01	0.1798
1st most profitable day (date - profit)	2019/01/28 - $22,152.41	2019/05/10 - -$110.95	2019/06/17 - -$94.94	2019/06/17 - -$210.58
5th most profitable day (date - profit)	2019/03/20 - $15,680.08	2019/06/28 - -$286.40	2019/06/28 - -$282.70	2019/06/04 - -$488.29
1st most unprofitable day (date - profit)	2019/06/14 - $1,709.61	2019/01/28 - -$43,411.00	2019/01/28 - -$30,094.50	2019/01/28 - -$33,768.10
5th most unprofitable day (date - profit)	2019/06/10 - $3,152.40	2019/01/15 - -$17,126.79	2019/01/16 - -$14,028.32	2019/01/29 - -$18,145.97
Average time in trade (excl. futures contracts)[2]	00:00:00.00	00:01:29.68	00:01:46.38	00:01:17.63

1 Model	2 Budish original	3 Budish extended 1	4 Budish extended 2	5 Budish extended 3
Average time in trade (incl. futures contracts)	n/a	n/a	n/a	n/a
# net profitable trades	22,185.00	498.00	629.00	636.00
# net unprofitable trades	2,278.00	50,778.00	41,097.00	53,226.00
# trades	24,463.00	51,276.00	41,726.00	53,862.00
% net profitable trades	90.69%	0.97%	1.51%	1.18%
Average volume per trade	252.53	570.17	530.75	469.17
Average net profit per trade	$38.92	-$12.45	-$12.53	-$11.16
Average profit per net profitable trades	$39.09	$37.60	$25.20	$24.50
Average profit per net unprofitable trades	-$2.31	-$12.94	-$13.10	-$11.58

[1] H0: F(x) = G(x), H1: F(x) ≤ G(x). F(x), G(x) = CDF of daily net profits without and with latency, respectively: 9.91E-01is for extended 2 vs extended 1 and 0.1798 is for extended 3 vs extended 2.

[2] HH: MM: SS. U: hours: minutes: seconds: fractions of a second.

Budish et al (2015) contribution

This contribution examines arbitrage opportunities between the two largest financial instruments that track the S&P 500 index, the SPDR S&P 500 exchange traded fund (ticker SPY) and the S&P 500 E-mini futures contract (ticker ES), using millisecond-level direct feed data from different stock exchanges and the Chicago Mercantile Exchange. The application is consequently very different from arbitrage trading of the same stock in two different exchanges but some comparisons with our research are important given that this article suggests strong modifications to the functioning of continuous HFT trading. The authors first demonstrate that the high correlation between the two securities observed from the bid-ask midpoints breaks down at very high-frequency time. This correlation breakdown creates technical arbitrage opportunities estimated at approximately USD $75 million of gross profit per year for the two securities alone on all markets where the SPY is traded (not only at the NYSE) and when there is no entry in the market. Their period of analysis includes many high volatility periods such as the 2007-2009 financial crisis. For a more regular year like 2005, the total gross profit is USD $35 million.[11] Verifying from Bloomberg that the share of the NYSE for this market is 25%, the annual gross profit for 2005 is USD $8.75 million for the NYSE alone. These numbers represent gross profits because trading fees are not considered, nor are latencies and exchange fees. Only bid-ask spread costs are computed.

The above numbers come from the following market environment: There is no arbitrageur entry and trading firms observe variations in the signal (perfectly correlated with the fundamental value of the stock) on the stock price with zero-time delay. There is zero latency in sending orders to the exchange and receiving updates from the exchange. This is a pure continuous trading environment with no asymmetric information and inventory costs.

Budish et al's (2015) trading strategy involves paying half the bid-ask spread in both markets. Most of the time the instantaneous profits are negative. When there is correlation breakdown, one instrument's price can jump significantly while the other's price has not changed. At such moments, buying the cheaper instrument and selling the more expensive instrument can be sufficiently profitable to overcome the bid-ask spread costs.

Table 5 present the results obtained from our data and using the arbitrage strategy presented in Appendix A.2 of their article. We observe, in column 2 of Table 5, that gross profit is limited to CAD $1 million for six months of continuous trading or about CAD $2 million for a year, which is below the CAD $10.60 million (USD $8.75) for the low volatility year of 2005 with their data. Many factors can explain the difference. The main difference is mostly related to the average trade

[11] The CBOE Volatility Index (VIX) of the average closing price was equal to 12.81 in 2005, 32.69 in 2008, and 15.39 in 2019.

volume of the assets. They document 800 daily arbitrage opportunities in their data, while in our data we have 184 daily arbitrage opportunities with their strategy for the 74 stocks.

We also observe that introducing trading fees does not significantly affect the profitability in the second column. The main difference in profitability is obtained when we consider trading time. This effect is observed in the next three columns where the average trade time increases from 0 to a time bracket between one and two minutes when we introduce practical trading rules for a better representation of market trading functioning. The percentage of profitable trades goes from more than 90% to less than 2% and the strategy becomes unprofitable in these three columns, mainly because the positions must be closed in the exchange where they are opened, a practical rule not applied in their article.

Our contribution with market orders

The main results from our strategy when using market orders are presented in Table 6. This strategy is not profitable because it is too expensive to obtain enough liquidity and orders are subject to execution risk (Loss). Trading fees affect the profitability of this strategy because the arbitrageur consumes liquidity with market orders. Thus, following our theoretical strategy with market orders is hazardous, especially when latency is considered. Indeed, we also observe, in columns three and four, that increasing latency reduces the net profitability even more and this effect is largely significant in both columns (significant *p*-values). Finally, the utilization of future contracts increases the average trading time, an aspect that is specific to international arbitrage. Our small number of arbitrage opportunities, explained by the use of market orders, implies that intraday values of our realized profits do not vary sufficiently to modify our positions in the futures contracts that hedge these quantities. This results in positions in the futures that are only closed hours, or even days, after being opened.

Our contribution with limit orders

The most interesting results from our contributions are from limit orders where arbitrageurs mainly provide liquidity to the markets. In Table 7, we observe a gross profit of CAD $ 9.6 million with selected pairs of cross-listed stocks obtained with supervised machine learning from our universe of 74 possible pairs[12] (see Appendix B), and for six months of trading. This result can be compared to the CAD $1 million obtained when trading with the Budish et al (2015) model (column 2, Table 5). Adding latency in the next columns affects the profitability of our strategy by reducing the net profits by about 25%. However, the percentage of net profitable trades is rather constant between the three columns. The profitability (unprofitability) between days of trading is also quite stable and using futures contracts for hedging the exchange risk does not increase the average time of trade very much because of the constant movement of our realized profits that are repatriated at every CAD $10,000 of gain or loss. The average volume per trade is quite low and stable but is similar to that in Budish et al (2015), as can be seen in the second column of Table 5. We could have used larger volumes with higher probability of non-execution risk. We chose to be conservative to minimize the impact on the price discovery process. The annual colocation cost and proprietary data feed total cost in Toronto is $116,250. Consequently, international arbitrage of cross-listed stock is profitable with our proposed limit order strategy even when all latencies, costs and risks are considered.

Therefore, the main question is the following: does a net annual profit of about CAD $8 million (USD $6 million, column 3 Table 7, with real latencies and all costs) seem reasonable for this international arbitrage activity that can be easily managed by one trader in a large trading firm? Note that Budish et al's (2015) original model generated a gross annual profit of USD $8.75

[12] This method of pair selection was also applied to market orders.

million from the NYSE in 2005 (CAD $10.60), in a year where the VIX was comparable to that of 2019. But their model made only about CAD $2 million of gross annual profit with our data in 2019 because the market activity is much less intense with our selected cross-listed stocks than with their two very liquid financial assets. Moreover, as they claimed, their trading model was quite simple and they predicted that a more sophisticated one should generate higher profits, which we demonstrated here in an international context.

To eliminate the probability of back test overfitting (Bailey et al, 2014), we only tested one set of parameters for our strategies, which we deemed reasonable beforehand: $\alpha = 0, \beta = 0.05$ (See Section 5). It is applied to every pair and every day of our data. Of course, the probability that this set of parameters is the optimal one for any pair and any day is close to zero, and if we had back tested the strategies multiple times, we could have selected the set that generated the greatest profitability and performance metrics of our portfolio. However, by using a single set of parameters fixed before any testing, and reporting the results generated by it, we ensure that our findings are generalizable. Hence, the metrics that were shown in this section could be improved and our results thus offer a conservative, but reasonable, measure of the profitability of international arbitrage of cross-listed stocks between Canada and the US.

Table 6. Results with market orders

1 Model	2 Market orders	3 Market orders 1	4 Market orders 2
Latency multiplier	0	1	3
Pair selection	No	No	No
Gross profit	$38,660.35	$41,508.69	$41,620.24
Loss	-$58,361.15	-$96,751.29	-$128,442.17
Trading fees	-$17,890.26	-$22,121.43	-$31,985.04
Trading rebates	$0.00	$0.00	$0.00
Total net profit	-$37,591.06	-$77,364.03	-$118,806.97
Mean daily net profit	-$329.75	-$678.63	-$1,042.17

1 Model	2 Market orders	3 Market orders 1	4 Market orders 2
Median daily net profit	-$18,24	-$207.53	-$595.92
Mean daily net profit per pair, per day	-$4.46	-$9.17	-$14.08
p-value Kolmogorov-Smirnov test[1]		6.42E-14	1.28E-8
1st most profitable day (date - profit)	2019/03/06 - $354.30	2019/05/31 - $21.63	2019/05/31 - $51.54
5th most profitable day (date - profit)	2019/06/21 - $196.92	2019/06/17 - -$2.54	2019/04/29 - -$94.49
1st most unprofitable day (date - profit)	2019/01/30 - -$4,053.94	2019/01/16 - -$4,682.15	2019/05/16 - -$4,692.79
5th most unprofitable day (date - profit)	2019/03/26 - -$2,095.20	2019/01/29 - -$3,504.39	2019/01/28 - -$3,785.02
Average time in trade (excl. futures contracts)[2]	00:06:34.41	00:06:37.83	00:04:42.15
Average time in trade (incl. futures contracts)[2]	02:12:28.60	00:59:30.36	00:57:59.53
# net profitable trades	1,284.00	1,092.00	1,590.00
# net unprofitable trades	2,130.00	2,927.00	4,814.00
# trades	3,414.00	4,019.00	6,404.00
% net profitable trades	37,61%	27.17%	24.83%
Average volume per trade	1,529.78	1,592.15	1,449.57
Average net profit per trade	-$11.01	-$19.25	-$18.55
Average profit per net profitable trades	$26.46	$32.92	$21.99
Average profit per net unprofitable trades	-$33.60	-$38.71	-$31.94

[1] H0: F(x) = G(x), H1: F(x) ≤ G(x). F(x), G(x) = CDF of daily net profits without, and with, latency, respectively: 6.42E-14 is for market orders 1 vs market orders and 1.28E-8 is for market orders 2 vs market orders 1.

[2] HH: MM: SS. U: hours: minutes: seconds: fractions of a second.

Table 7. Results with limit orders

1 Model	2 Limit orders	3 Limit orders 1	4 Limit orders 2
Latency multiplier	0	1	3
Pair selection	Yes	Yes	Yes
Gross profit	$9,608,178.87	$8,641,338.63	$8,363,528.28
Loss	-$4,757,168.60	-$5,041,665.26	-$5,168,902.58

1 Model	2 Limit orders	3 Limit orders 1	4 Limit orders 2
Trading fees	-$78,132.64	-$82,067.16	-$83,537.87
Trading rebates	$553,201.20	$476,071.01	$458,542.50
Total net profit	$5,326,078.83	$3,993,677.22	$3,569,630.33
Mean daily net profit	$46,719.99	$35,032.26	$31,312.55
Median daily net profit	$44,453.98	$33,756.44	$29,610.42
Mean daily net profit per pair, per day	$2,273.19	$1,704.51	$1,523.53
p-value Kolmogorov-Smirnov test[1]		1.30E-06	0.04211
1st most profitable day (date - profit)	2019/05/09 - $100,142.51	2019/05/09 - $82,330.71	2019/05/09 - $77,292.31
5th most profitable day (date - profit)	2019/05/13 - $78,509.62	2019/06/20 - $58,157.95	2019/05/07 - $53,633.28
1st most unprofitable day (date - profit)	2019/06/04 - $15,061.17	2019/03/13 - $12,210.91	2019/03/13 - $9,130.81
5th most unprofitable day (date - profit)	2019/03/18 - $22,810.62	2019/03/18 - $15,997.46	2019/03/18 - $13,349.39
Average time in trade (excl. futures contracts)[2]	00:01:29.51	00:01:39:10	00:01:41.22
Average time in trade (incl. futures contracts)	00:01:46.55	00:01:56.61	00:01:58.19
# net profitable trades	1,063,897.00	930,388.00	892,772.00
# net unprofitable trades	325,351.00	322,230.00	327,096.00
# trades	1,389,248.00	1,252,618.00	1,219,868.00
% net profitable trades	76.58%	74.28%	73.19%
Average volume per trade	188.10	187.99	188.36
Average net profit per trade	$3.83	$3.19	$2.93
Average profit per net profitable trades	$9.51	$9.76	$9.84
Average profit per net unprofitable trades	-$14.71	-$15.78	-$15.94
% trade using marketable orders	16.42%	19.56%	20.50%

[1] H0: F(x) = G(x), H1: F(x) ≤ G(x). F(x), G(x) = CDF of daily net profits without and with latency, respectively: 1.30E-06 is for limit orders 1 vs limit orders and 0.04211 is for limit orders 2 vs limit orders 1.

[2] HH: MM: SS. U: hours: minutes: seconds: fractions of a second.

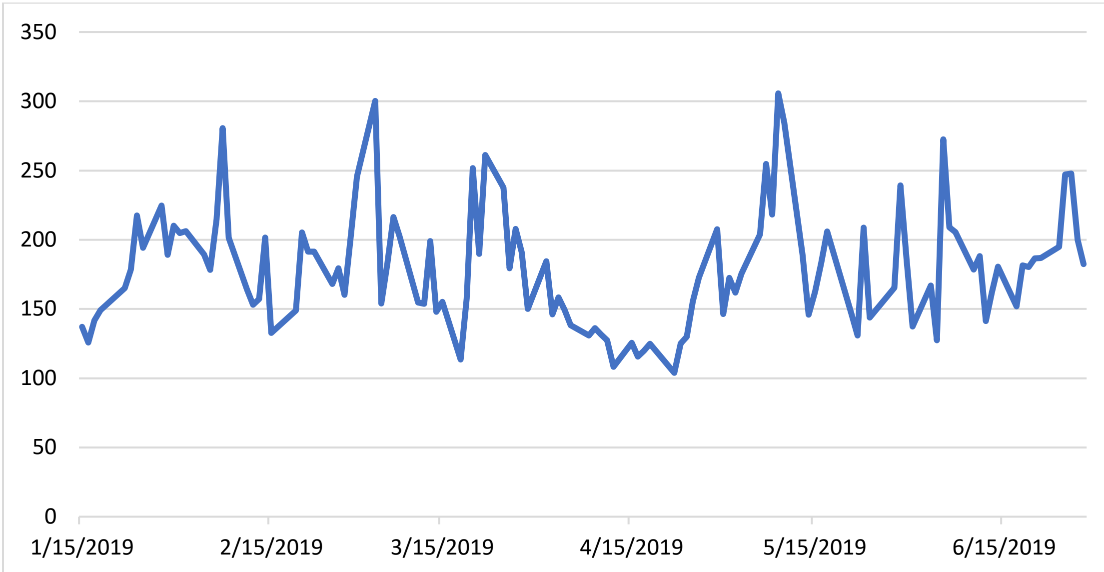
8. Trading strategy performance

Statistics

In this section, we present a more detailed view of the performance of the limit order strategy in the real latency setting, presented in column 3 of Table 7. We define a captured arbitrage opportunity as an opportunity where the positions in a pair at TSX and NYSE are both opened and closed with limit orders following the arbitrage strategy described in Section 3. This excludes arbitrage opportunities where a least one leg had to be closed by the stop-loss or the chronometer circuit breakers implemented for risk management.

Figure 1 shows the mean daily number of captured arbitrage opportunities per ticker and the mean duration of the positions behind these opportunities. The number of captured arbitrage opportunities (Panel 1a) exhibits some daily fluctuations, but the quantity remains stationary over the period. On average, there are 180 captured arbitrage opportunities per ticker per day. The mean duration, computed as the mean of the daily mean of captured opportunity pairs, is about 122 seconds (Panel 1b), and is also stationary during our period of analysis. Note that both quantities are anticorrelated (Pearson correlation coefficient: -0.923). This is because the strategy does not enter a new position when the previous one is still opened. Thus, a longer time to close both legs of the strategy directly leads to a lesser number of potential arbitrage opportunities to be captured.

Panel 1a: Mean daily number of captured arbitrage opportunities per selected pair



Panel 1b: Mean duration in seconds per captured arbitrage opportunity over all selected pairs

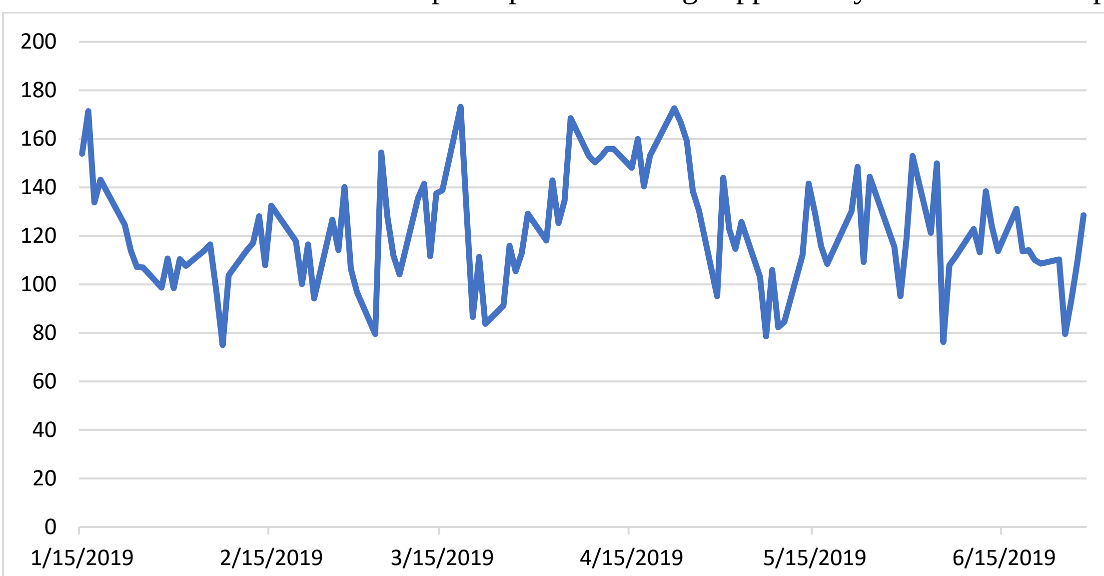
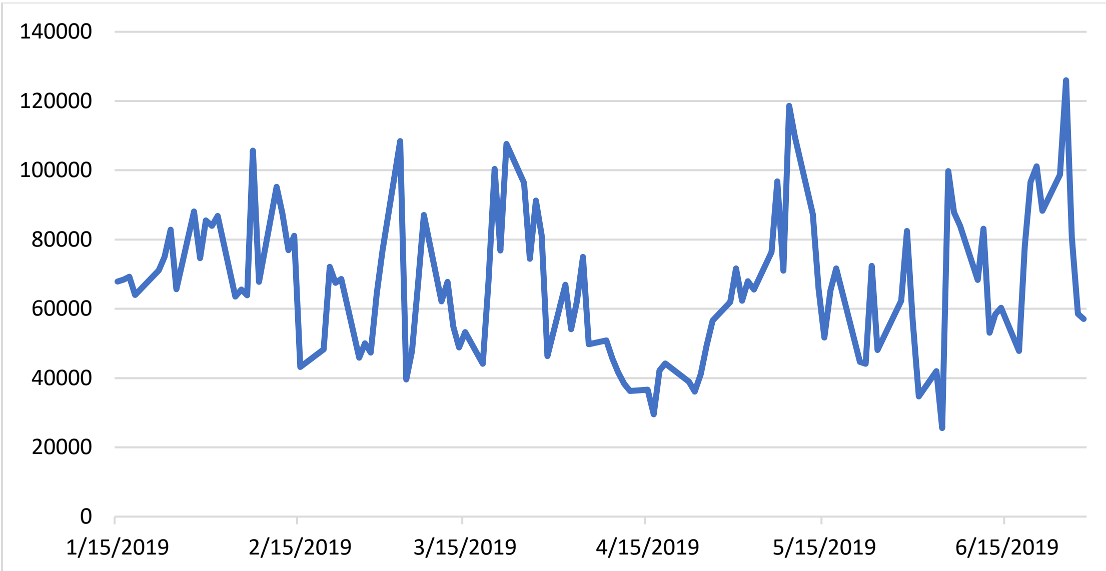


Figure 1. Captured arbitrage opportunities during the period of analysis

Figure 2 shows the daily net profit measured as the average per captured arbitrage opportunity as well as the total realized net profit per day over the selected assets in the first six months of 2019. The mean total daily realized net profit is CAD $67,369 (Panel 2a) and the mean net profit per captured arbitrage opportunity is around CAD $19 (Panel 2b), in line with the expected high-frequency quoting activities. Per ticker, the daily mean is equal to CAD $3,411.

Panel 2a: Total daily net profit from captured arbitrage opportunity over all selected pairs (CAD)



Panel 2b: Daily net mean profit per captured arbitrage opportunity (CAD)

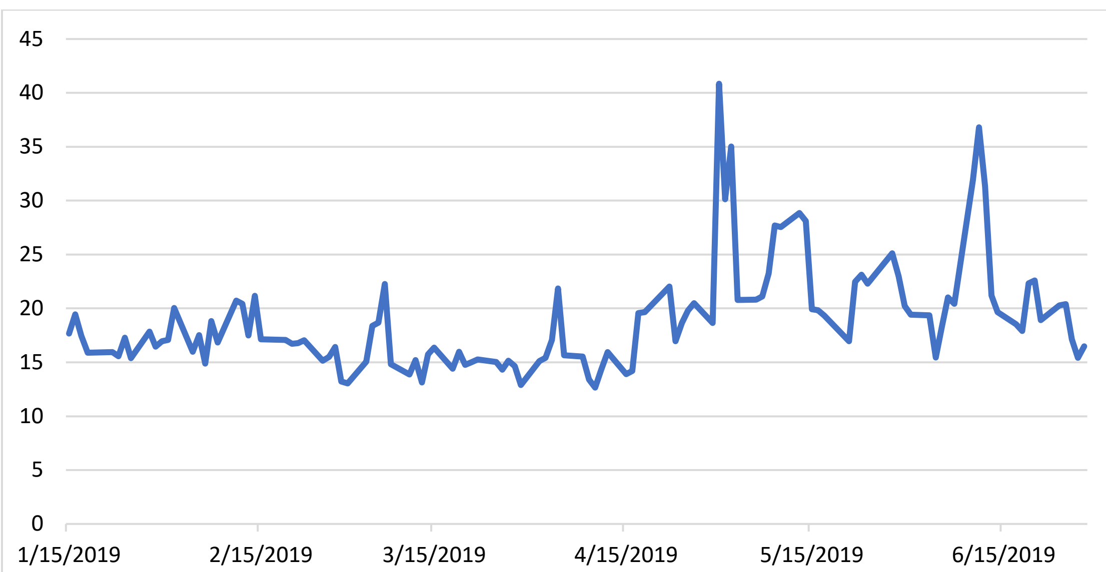


Figure 2. Profitability of captured arbitrage opportunities during the period of analysis

Figure 3 shows the empirical cumulative distribution function (CDF) of the net profit per captured arbitrage opportunity in CAD. Based on this CDF, 99.7% of the captured arbitrage opportunities are profitable. The median is around CAD \$11, and the 99 percentile is around CAD \$110. This confirms the theoretical validity of the strategy, meaning that when an arbitrage opportunity is perfectly captured with limit orders, it is almost guaranteed to be profitable. The remaining 0.3% of unprofitable captured arbitrage opportunities are obtained because we cannot always close the positions at the exact equilibrium value, as explained in Section 4.

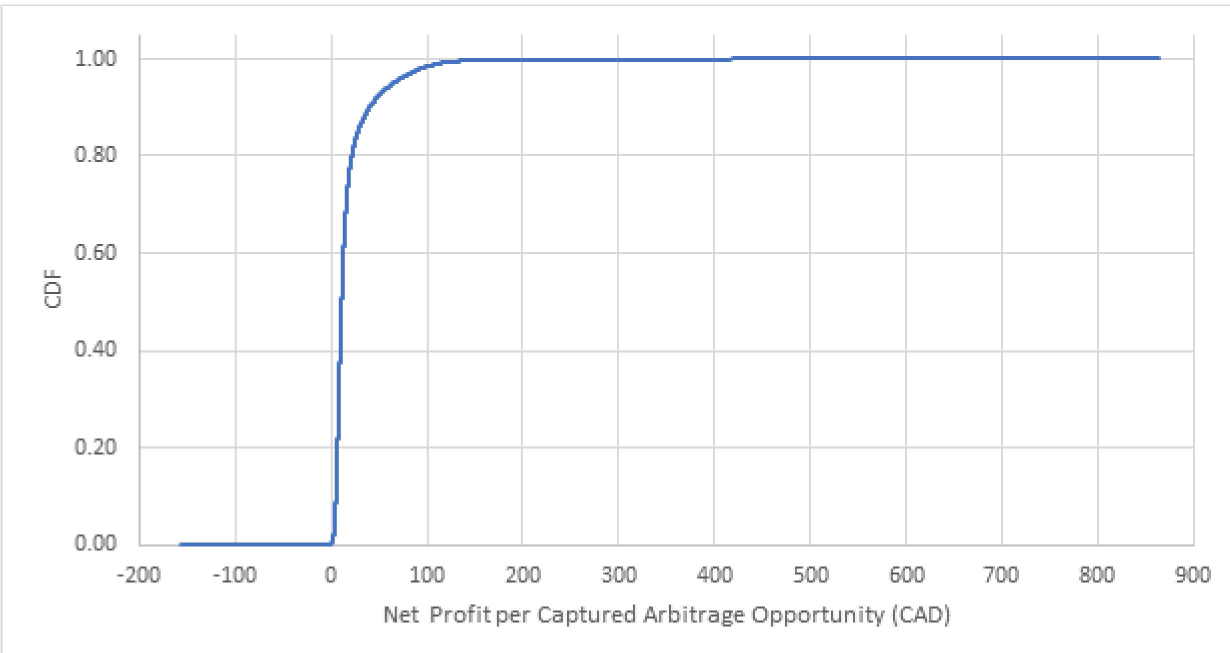


Figure 3. Empirical CDF of net profit per captured arbitrage opportunity (CAD)

Regression analysis

To better understand the stylized facts affecting the daily net profitability of the strategy, we employ a regression analysis. Using standard variables such as the intraday volatility of the assets' mid-price traded at exchange $i \in \{TSX, NYSE, CME\}$ on day t ($vol_{i,t}$), the average bid-ask spreads ($spread_{i,t}$), the total trading volumes ($trade_{i,t}$), and the total quantity of messages resulting from the LOB level one updates ($messages_{i,t}$), all in their respective currency, we want to explain the average net profitability of the selected pairs on day t ($\overline{profits_t}$). We compute every variable with

$i \in \{TSX, NYSE\}$ as the weighted mean of the stock-level variable in the selected pair on day t, where the weight accorded to a specific stock is the proportion of its daily traded value compared with the total traded value for every stock of the same exchange in our portfolio on that day (all in CAD). Table 8 reports the descriptive statistics of these variables, and Table A5 reports their Pearson correlation coefficients. All variables are described in Table A4.

Table 8. Descriptive statistics of variables used in the regression analysis to explain the daily net profit of the strategy with limit orders.

Variable	Mean	Std. Dev.	Min.	Q1	Median	Q3	Max.
$\overline{profits}$	3,411	1237	1,636	2,543	3,201	4,002	8,471
vol_{TSX}	0.458	0.143	0.259	0.361	0.412	0.524	0.974
vol_{NYSE}	0.467	0.151	0.269	0.357	0.420	0.548	1.007
vol_{CME}	0.086	0.047	0.024	0.054	0.074	0.114	0.244
$spread_{TSX}$	5.791	1.267	3.567	4.916	5.688	6.213	1.097
$spread_{NYSE}$	6.854	1.279	4.800	5.835	6.732	7.385	1.079
$spread_{CME}$	0.576	0.020	0.542	0.566	0.576	0.584	0.715
$trade_{TSX}$	775,033	361,920	349,538	506,020	686,478	949,768	2,288,334
$trade_{NYSE}$	280,935	153,312	118,714	183,374	226,571	296,655	973,461
$trade_{CME}$	64,664	75,053	4,195	27,383	34,537	46,817	297,363
$messages_{TSX}$	59,136	13,474	35,229	48,619	58,494	67,034	94,736
$messages_{NYSE}$	53,719	13,253	32,036	43,001	51,791	62,492	101,549
$messages_{CME}$	192,958	55,922	66,246	150,944	186,874	223,187	346,698
Count			114				

The volatilities of the mid-price of cross-listed stocks have similar distributions on both stock exchanges. The same applies for the spread and the number of messages from LOB level one. On the other hand, the volume of trades at the TSX is almost three times greater than at the NYSE, which is expected from a portfolio composed entirely of Canadian instruments.

As in Wah (2016) and Budish et al (2015), from Table A5, we observe a significant and positive relationship between the strategy's profitability and the volatility of the markets. The bid-ask spread of the stocks is the variable that is the most highly and positively correlated with the profitability of the strategy, which is expected since the strategy uses limit orders. Finally, the numbers of updates of LOB level one are all statistically and positively correlated to the strategy's profitability, which will be explained later in this section.

As expected, the pairs of same variables on the TSX and NYSE exchanges are highly correlated. To reduce potential multicollinearity, we combine each pair of equity variables into one variable by using the mean of the respective TSX and NYSE variable values, thus creating the variables $\overline{vol}_{stocks,t}$, $\overline{spread}_{stocks,t}$, $\overline{trade}_{stocks,t}$ and $\overline{messages}_{stocks,t}$. The linear regression model is written as follows, for day $t \in \{1,2,\dots,114\}$:

$$\begin{aligned}\overline{profits}_t = \; & b_0 + b_1 vol_{CME,t} + b_2 \overline{vol}_{stocks,t} + b_3 spread_{CME,t} + b_4 \overline{spread}_{stocks,t} \\ & + b_5 trade_{CME,t} + b_6 \overline{trade}_{stocks,t} + b_7 messages_{CME,t} \\ & + b_8 \overline{messages}_{stocks,t} + \varepsilon_t,\end{aligned}$$

where $\varepsilon_t \sim N(0,\sigma^2), \forall t$. The regression coefficients are obtained by ordinary least squares, and the covariance matrix is estimated with the heteroskedasticity and autocorrelation consistent approach of Newey-West (1987). Table 9 summarizes the regression results.

Table 9. OLS linear regression for the average daily net profitability of the limit order strategy with Newey-West covariance matrix estimation

Variable	Coefficient	p-value
$intercept$	-3,823.011	0.202
vol_{CME}	-2,533.717	0.103
$\overline{vol}_{stocks}$	695.229	0.284
$spread_{CME}$	-1,817.241	0.723
$\overline{spread}_{stocks}$	791.142	0.000
$trade_{CME}$	0.001	0.518
$\overline{trade}_{stocks}$	-0.002	0.009
$messages_{CME}$	0.003	0.139
$\overline{messages}_{stocks}$	0.069	0.000
$Adj.\ R^2$	0.662	
$F\ stat$	22.570	

As the regression suggests, only the number of LOB level one update messages, the size of the spread and the trading volume of the stocks contribute significantly to the daily net profits generated for our portfolio of cross-listed stock pairs. These results are consistent with our machine learning pair selection methodology (See appendix B for more details). A larger spread for the stocks is directly beneficial to our limit order strategy, which can be explained by equations (4) and (5). Together, these equations tell us that a larger spread lead to a higher profit for any given arbitrage opportunity and that the profitable arbitrage opportunities are thus more frequent for days with larger spreads. As for the number of messages, the result is intuitive because a higher level one activity generally increases the likelihood of our active limit orders to be filled, considering the execution rules used in the professional trading software (see Appendix D for more details), or cancelled because of our risk management circuit breakers in the case where the prices deviate from our limit orders' prices. Hence, the more messages we observe, the faster our orders can be executed or canceled and the faster the strategy can move on to the next opportunity (which was observed in Figure 1), as opposed to days when markets are quieter and limit orders can remain in the LOB for longer periods of time. Lastly, a larger trading volume contributes negatively to our profitability, especially at the NYSE. The higher latency to that exchange prevents us from reacting very rapidly compared to other participants collocated at the NYSE. Thus trades occurring before our limit orders included in the LOB (or even before the information was analyzed by our algorithm) can cause the mispricing to dissipate.

Profitability

Figure 4 shows the net cumulated profits over the entire period on a trade basis. There is minimal intraday drawdown, and as was shown in Figure 2 (Panel a), the net daily profits are stationary, which explains the quasi linearity of the function in Figure 4.

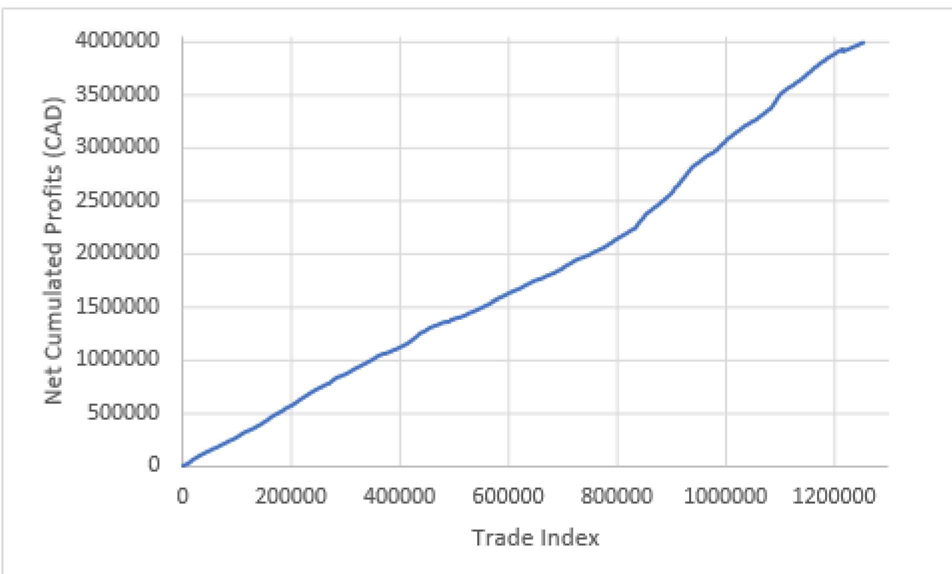


Figure 4. Net cumulated profits (CAD) on a trade basis over the entire period

Figure 5 presents the daily maximum of net aggregated positions taken at each exchange for our portfolio of selected pairs. The maximum net open position in absolute value is around CAD $453,000 at the TSX, CAD $465,000 at the NYSE, and CAD $590,000 at the CME, meaning that an investment of CAD $1,000,000 to cover the margins is more than enough (note that only a margin of USD $1,100 per CAD/USD futures contract is needed at the CME). Given the annual net profit of CAD $8 million generated by the strategy in 2019, this results in an annual net return of 700%. When considering management fees of 5%, the annual net return is 660%.

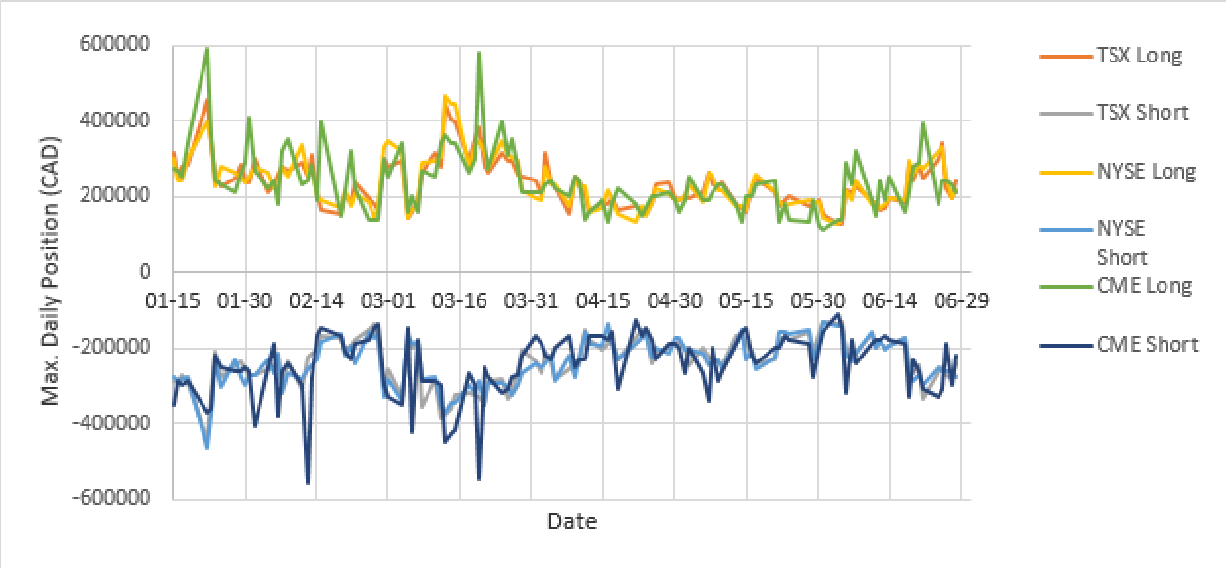


Figure 5. Maximum daily net aggregated long and short positions of the selected pairs portfolio at the three exchanges

Figure 6 shows the empirical CDF of the needed aggregated net margin in CAD. This margin at time t can be expressed as follows:

$$M_t = \left|V_{TSX,t}\right| + \left|V_{NYSE,t} - \frac{G_{NYSE,t}}{r_t}\right| + \frac{1{,}100}{r_t}\left|V_{CME,t} - \frac{G_{CME,t}}{r_t}\right| / 100{,}000 \, .$$

Once again, we can see that a capital of CAD $1,000,000 always covers the margins in the three exchanges, while CAD $185,000 covers 80% of the needed margins at any time, meaning that the high levels of aggregated positions are transitory.

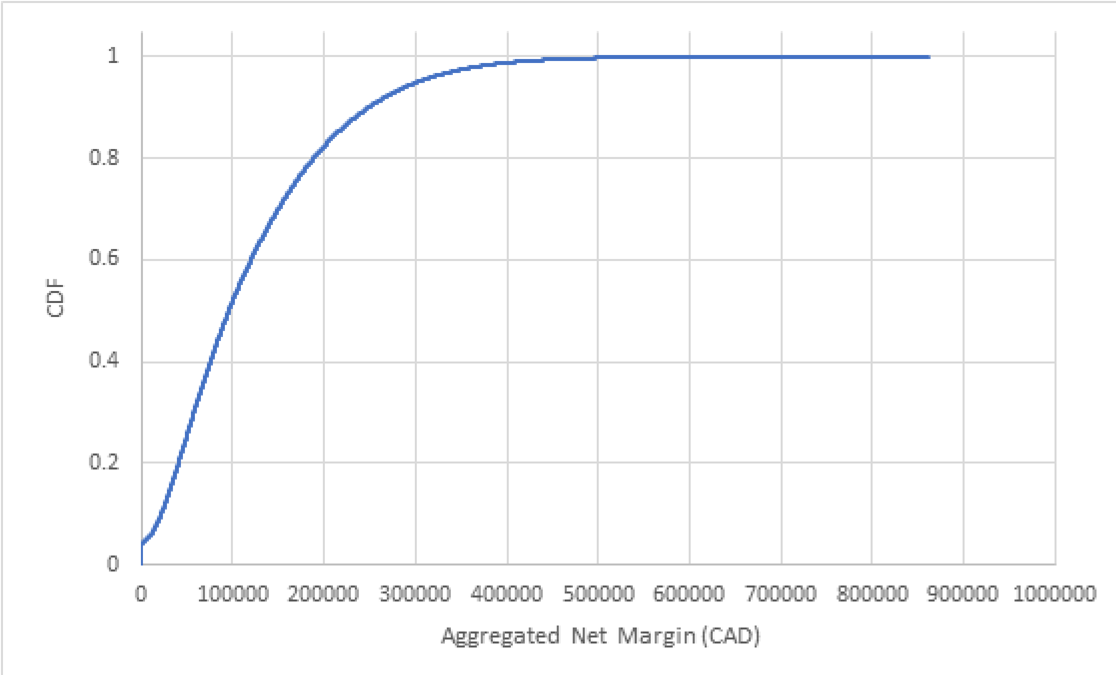


Figure 6. Empirical CDF of the needed aggregated net margin in CAD

The annualized Sharpe ratio computed from the daily returns and the margin of CAD $1 million is 51.04 (48.5 when considering management fees). It is very high, but our daily profits are perfectly comparable to the trading profits of HFTs found in Baron et al (2014). Our result is mainly explained by the very low volatility of the profits as seen in Figures 3 and 5. Also the Deflated Sharpe Ratio proposed by Bailey and López de Prado (2014) is approximately equal to 1. This very high value is mainly explained by the fact that we did not resort to multiple back testing trials, generating an absence of variance across the trials and a quasi null likelihood of a false discovery.

9. Conclusion

We study the profitability of mean-reverting international arbitrage activities of cross-listed stocks which includes hedging the FX risk, and involves three exchanges in Canada and the United

States. We explore mean-reverting arbitrage activities with a novel arbitrage strategy that we test across three North American exchanges during the first six months of 2019: the New York Stock Exchange (NYSE) and the Chicago Mercantile Exchange (CME) in the United States, and the Toronto Stock Exchange (TSX) in Canada.

This is the first contribution that examines stocks' cross-country latency arbitrage. We work with a unique temporal frame of reference, meaning that we synchronize the data feeds from the three exchange venues taking into account explicitly the latency that comes from the transportation of information between the exchanges and its treatment time. We also consider all potential arbitrage costs. We verify that latency arbitrage is profitable with order book transactions and queuing priorities. We consider the obtained profits as reasonable when compared with previous contributions in the literature. International latency arbitrage with market orders is never profitable.

Our original goal was not to contribute to the normative discussion about the effect of continuous HFT on the general welfare of financial markets. It was to replicate the precise behavior of a trading firm to provide a better estimate of the arbitrage market functioning with high-frequency trading. Our results could be useful to improve the understanding of the complex nature of high-frequency trading.

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Appendices

Appendix A. Additional tables

Table A1. Number of days where the Augmented Dickey-Fuller test for non-stationarity is rejected at p =1% for Γ^{Short} and Γ^{Long}. The test was applied on the daily time series of Γ^{Short} and Γ^{Long} between 9:32 am and 4:00 pm ET.

TSX Ticker | NYSE Ticker	Short	Long	TSX Ticker | NYSE Ticker	Short	Long
ABX | GOLD	0	0	IMG | IAG	0	0
AEM | AEM	0	0	JE | JE	1	0
AGI | AGI	0	0	K | KGC	0	0
AQN | AQN	0	0	KL | KL	0	0
ATP | AT	3	5	LAC | LAC	0	2
BAM.A | BAM	0	0	MFC | MFC	0	0
BB | BB	0	0	MG | MGA	0	0
BCB | COT	0	0	NEXA | NEXA	4	2
BCE | BCE	0	0	NOA | NOA	1	0
BMO | BMO	0	0	OR | OR	0	0
BNS | BNS	0	0	OSB | OSB	0	0
BTE | BTE	0	0	PD | PDS	0	0
BXE | BXE	6	12	PPL | PBA	0	0
CAE | CAE	0	0	PVG | PVG	0	0
CCO | CCJ	0	0	QSR | QSR	0	0
CLS | CLS	0	0	RBA | RBA	0	0
CM | CM	0	0	RCI.B | RCI	0	0
CNQ | CNQ	0	0	RFP | RFP	0	0
CNR | CNI	0	0	SEA | SA	0	0
CNU | CEO	4	19	SHOP | SHOP	0	0
CP | CP	0	0	SJR.B | SJR	0	0
CPG | CPG	0	0	SLF | SLF	0	0
CVE | CVE	0	0	STN | STN	1	2
ECA | ECA	0	0	SU | SU	0	0
EDR | EXK	0	0	T | TU	0	0
ELD | EGO	0	0	TA | TAC	0	0
ENB | ENB	0	0	TD | TD	0	0
ERF | ERF	0	0	TECK.B | TECK	0	0
FNV | FNV	0	0	THO | TAHO	0	0

TSX Ticker | NYSE Ticker	Short	Long	TSX Ticker | NYSE Ticker	Short	Long
FR | AG	0	0	TRI | TRI	0	0
FTS | FTS	0	0	TRP | TRP	0	0
FVI | FSM	0	0	TRQ | TRQ	1	1
G | GG	0	0	UFS | UFS	0	0
GIB.A | GIB	0	0	VET | VET	0	0
GIL | GIL	0	0	WEED | CGC	0	0
GOOS | GOOS	0	0	WPM | WPM	0	0
HBM | HBM	0	0	YRI | AUY	0	0

Note: Since we observe a low number of days where Γ^{Short} and Γ^{Long} are not stationary, the mean-reversion risk is minimal in our strategy for almost all pairs. Even though that risk is very low, we still implement circuit breakers with a timer and a stop-loss to capture the most arbitrage opportunities as possible.

Table A2. List of available cross-listed stocks with the TSX ticker and NYSE ticker counterpart. Also included are the company's name, economic sector and S&P/TSX 60 membership status.

TSX Ticker	NYSE Ticker	Company	Sector	S&P/TSX 60
ABX	GOLD	Barrick Gold Corp.	Materials	Yes
AEM	AEM	Agnico Eagle Mines Ltd.	Materials	Yes
AGI	AGI	Alamos Gold Inc.	Mining	No
AQN	AQN	Algonquin Power & Utilities Corp.	Clean Technology	No
ATP	AT	Atlantic Power Corp.	Clean Technology	No
BAM.A	BAM	Brookfield Asset Management Inc.	Financials	Yes
BB	BB	Blackberry Ltd.	Information Technology	Yes
BCB	COT	Cott Corp.	Consumer Products & Services	No
BCE	BCE	BCE Inc.	Telecommunication Services	Yes
BMO	BMO	Bank of Montreal	Financials	Yes
BNS	BNS	Bank of Nova Scotia	Financials	Yes
BTE	BTE	Baytex Energy Corp.	Oil & Gas	No
BXE	BXE	Bellatrix Exploration Ltd.	Oil & Gas	No
CAE	CAE	CAE Inc.	Technology	No
CCO	CCJ	Cameco Corp.	Energy	Yes
CLS	CLS	Celestia Inc.	Technology	No

TSX Ticker	NYSE Ticker	Company	Sector	S&P/TSX 60
CM	CM	Canadian Imperial Bank of Commerce	Financials	Yes
CNQ	CNQ	Canadian Natural Resources Ltd.	Energy	Yes
CNR	CNI	Canadian National Railway Company	Industrials	Yes
CNU	CEO	CNOOC Ltd.	Oil & Gas	No
CP	CP	Canadian Pacific Railway Ltd.	Industrials	Yes
CPG	CPG	Crescent Point Energy Corp.	Energy	Yes
CVE	CVE	Cenovus Energy Inc.	Energy	Yes
ECA	ECA	Encana Corp.	Energy	Yes
EDR	EXK	Endeavour Silver Corp.	Mining	No
ELD	EGO	Eldorado Gold Corp.	Mining	No
ENB	ENB	Enbridge Inc.	Energy	Yes
ERF	ERF	Enerplus Corp.	Oil & Gas	No
FNV	FNV	Franco-Nevada Corp.	Materials	Yes
FR	AG	First Majestic Silver Corp.	Mining	No
FTS	FTS	Fortis Inc.	Utilities	Yes
FVI	FSM	Fortuna Silver Mines Inc.	Mining	No
G	GG	Goldcorp Inc.	Materials	Yes
GIB.A	GIB	CGI Group Inc.	Information Technology	Yes
GIL	GIL	Gildan Activewear Inc.	Consumer Discretionary	Yes
GOOS	GOOS	Canada Goose Holdings Inc.	Consumer Products & Services	No
HBM	HBM	HudBay Minerals Inc.	Mining	No
IMG	IAG	IAMGold Corp.	Mining	No
JE	JE	Just Energy Group Inc.	Utilities & Pipelines	No
K	KGC	Kinross Gold Corp.	Materials	Yes
KL	KL	Kirkland Lake Gold Ltd.	Mining	No
LAC	LAC	Lithium Americas Corp.	Mining	No

TSX Ticker	NYSE Ticker	Company	Sector	S&P/TSX 60
MFC	MFC	Manulife Financial Corp.	Financials	Yes
MG	MGA	Magna International Inc.	Consumer Discretionary	Yes
NEXA	NEXA	Nexa Resources S.A.	Mining	No
NOA	NOA	North American Construction Group	Industrial Products & Services	No
OR	OR	Osisko Gold Royalties Ltd.	Mining	No
OSB	OSB	Nordbord Inc.	Industrial Products & Services	No
PD	PDS	Precision Drilling Corp.	Industrial Products & Services	No
PPL	PBA	Pembina Pipeline Corp.	Utilities	Yes
PVG	PVG	Pretium Resources Inc.	Mining	No
QSR	QSR	Restaurant Brands International Inc.	Consumer Discretionary	Yes
RBA	RBA	Ritchies Bros. Auctioneers Inc.	Industrial Products & Services	No
RCI.B	RCI	Rogers Communication Inc.	Telecommunication Services	Yes
RFP	RFP	Resolute Forest Products Inc.	Industrial Products & Services	No
SEA	SA	Seabridge Gold Inc.	Mining	No
SHOP	SHOP	Shopify Inc.	Technology	No
SJR.B	SJR	Shaw Communications Inc.	Telecommunication Services	Yes
SLF	SLF	Sun Life Financials Inc.	Financials	Yes
STN	STN	Stantec Inc.	Industrial Products & Services	No
SU	SU	Suncor Energy Inc.	Energy	Yes
T	TU	Telus Corp.	Telecommunication Services	Yes
TA	TAC	TransAlta Corp.	Utilities & Pipelines	Yes
TD	TD	Toronto-Dominion Bank	Financials	Yes
TECK.B	TECK	Teck Resources Ltd.	Materials	Yes

TSX Ticker	NYSE Ticker	Company	Sector	S&P/TSX 60
THO	TAHO	Tahoe Resources Inc.	Mining	No
TRI	TRI	Thomson Reuters Corp.	Consumer Discretionary	Yes
TRP	TRP	TransCanada Corp.	Energy	Yes
TRQ	TRQ	Turquoise Hill Resources Ltd.	Mining	No
UFS	UFS	Domtar Corp.	Consumer Products & Services	No
VET	VET	Vermilion Energy Inc.	Oil & Gas	No
WEED	CGC	Canopy Growth Corp.	Life Sciences	No
WPM	WPM	Wheaton Precious Metals Corp.	Mining	No
YRI	AUY	Yamana Gold Inc.	Mining	No

Table A3. Aggregated statistics of every pair from January 7th, 2019 to June 28th, 2019. # Trades is the total number of trades. # Quotes is the total number of quote messages. Volatility is the annualized standard deviation of the daily returns computed from close prices. Min., Mean, Median and Max. prices are respectively the minimum, empirical mean, empirical median and maximum trade prices during that period. All prices are in local currency.

TSX Ticker | NYSE Ticker	# Trades	# Quotes	Volatility	Min. Price	Mean Price	Median Price	Max. Price
ABX | GOLD	1355455 | 411552	12609016 | 16754385	0.3 | 0.31	15.37 | 11.52	20.63 | 15.77	20.64 | 15.77	21.67 | 16.44
AEM | AEM	712143 | 245409	11968871 | 9549364	0.23 | 0.24	51.39 | 38.72	66.73 | 50.99	66.73 | 50.99	69.13 | 52.5
AGI | AGI	342922 | 114855	3003641 | 7242356	0.4 | 0.42	4.89 | 3.68	7.82 | 5.98	7.83 | 5.99	8.25 | 6.27
AQN | AQN	454837 | 46109	3530170 | 2506458	0.12 | 0.12	13.5 | 10.13	15.87 | 12.13	15.88 | 12.13	16.6 | 12.54
ATP | AT	33558 | 23008	373750 | 653362	0.34 | 0.35	2.97 | 2.23	3.13 | 2.39	3.13 | 2.39	4.01 | 3.01
BAM.A | BAM	690629 | 252633	12662244 | 15994647	0.12 | 0.13	52.49 | 39.35	62.41 | 47.68	62.4 | 47.68	65.06 | 48.72
BB | BB	482209 | 154435	4813106 | 6724215	0.36 | 0.37	9.31 | 7.1	9.74 | 7.44	9.75 | 7.45	13.74 | 10.29
BCB | COT	171235 | 115407	3257388 | 5107512	0.28 | 0.26	16.9 | 12.73	17.35 | 13.27	17.33 | 13.27	21.06 | 15.92
BCE | BCE	829443 | 142454	7381944 | 12112698	0.1 | 0.11	53.05 | 39.75	59.47 | 45.43	59.5 | 45.44	62.75 | 47.14
BMO | BMO	927212 | 141737	8251253 | 9050862	0.11 | 0.13	88.92 | 66.65	98.76 | 75.45	98.76 | 75.45	106.51 | 79.34
BNS | BNS	1170563 | 151151	11897780 | 16543643	0.1 | 0.12	68.29 | 50.58	70.23 | 54.28	70.22 | 54.27	75.93 | 57.61
BTE | BTE	351485 | 45578	2114183 | 3694628	0.58 | 0.59	1.9 | 1.42	2.01 | 1.54	2.02 | 1.54	3.13 | 2.32
BXE | BXE	4760 | 724	18630 | 59042	0.87 | 0.74	0.48 | 0.35	0.62 | 0.47	0.62 | 0.46	0.74 | 0.55
CAE | CAE	357585 | 42689	2499568 | 2888853	0.27 | 0.27	24.99 | 18.74	35.07 | 26.79	35.1 | 26.81	36.86 | 27.42
CCO | CCJ	457592 | 139670	4065374 | 13566637	0.24 | 0.25	13.42 | 9.92	13.86 | 10.6	13.9 | 10.62	17.12 | 13.03
CLS | CLS	163847 | 61588	1530011 | 2468489	0.42 | 0.42	8.26 | 6.17	8.93 | 6.82	8.93 | 6.83	13.08 | 9.96
CM | CM	898170 | 112553	9389778 | 8047186	0.14 | 0.16	12.85 | 74.37	102.67 | 78.44	102.67 | 78.45	115.07 | 87.35
CNQ | CNQ	1679058 | 376524	17383323 | 37749573	0.27 | 0.29	33.76 | 25.33	35.13 | 26.85	35.15 | 26.86	42.56 | 31.76
CNR | CNI	885761 | 207348	7133075 | 6098839	0.13 | 0.14	100.34 | 75.18	120.61 | 92.12	120.64 | 92.11	127.96 | 95.08
CNU | CEO	156 | 41310	1936032 | 1056232	0.23 | 0.25	203.96 | 154.96	223.75 | 170.96	223.7 | 170.85	255.18 | 193.52
CP | CP	294566 | 130121	3407525 | 4345765	0.16 | 0.17	239.65 | 179.68	306.01 | 233.77	305.76 | 233.56	318.75 | 241.2
CPG | CPG	726046 | 101013	4085865 | 8280375	0.6 | 0.62	3.24 | 2.44	4.31 | 3.3	4.32 | 3.3	5.98 | 4.45
CVE | CVE	1125554 | 186950	8954509 | 17429240	0.33 | 0.35	9.62 | 7.24	11.46 | 8.75	11.46 | 8.75	14.26 | 10.6
ECA | ECA	1349297 | 395043	10526262 | 20763317	0.43 | 0.45	6.12 | 4.56	6.61 | 5.05	6.61 | 5.05	10.35 | 7.7

TSX Ticker | NYSE Ticker	# Trades	# Quotes	Volatility	Min. Price	Mean Price	Median Price	Max. Price
EDR | EXK	77642 | 47413	785259 | 2389713	0.46 | 0.47	2.27 | 1.69	2.69 | 2.06	2.7 | 2.06	3.84 | 2.85
ELD | EGO	376622 | 117803	3826242 | 6586161	0.74 | 0.76	3.36 | 2.52	7.55 | 5.77	7.55 | 5.77	7.65 | 5.82
ENB | ENB	1838436 | 310091	15384861 | 21529655	0.18 | 0.18	43.74 | 32.76	46.92 | 35.85	46.92 | 35.85	51.22 | 38.04
ERF | ERF	458784 | 111965	4590223 | 10576424	0.38 | 0.4	8.76 | 6.54	9.88 | 7.54	9.86 | 7.54	13.1 | 9.73
FNV | FNV	440300 | 136221	4644852 | 4540514	0.2 | 0.2	90.2 | 67.97	110.48 | 84.38	110.49 | 84.41	114.36 | 86.87
FR | AG	267277 | 133209	3895094 | 10620884	0.43 | 0.44	6.67 | 5.02	10.19 | 7.79	10.19 | 7.79	10.7 | 8.12
FTS | FTS	632431 | 89824	5855287 | 6557030	0.08 | 0.09	44 | 33.03	51.61 | 39.44	51.62 | 39.43	52.95 | 40.09
FVI | FSM	192403 | 72897	1263078 | 2316467	0.43 | 0.44	3.22 | 2.39	3.74 | 2.85	3.74 | 2.86	5.55 | 4.18
G | GG	559098 | 489352	5158992 | 13485393	0.28 | 0.28	12.46 | 9.38	15.08 | 11.31	15.08 | 11.31	15.74 | 11.8
GIB.A | GIB	407242 | 80116	3157513 | 3061503	0.12 | 0.12	80.5 | 60.41	100.18 | 76.55	100.16 | 76.56	104.24 | 78.05
GIL | GIL	388478 | 127055	4230021 | 5849583	0.16 | 0.16	40.38 | 30.42	50.4 | 38.52	50.4 | 38.51	52.95 | 39.55
GOOS | GOOS	504696 | 370437	7196994 | 5321814	0.64 | 0.65	42.38 | 31.67	50.17 | 38.37	50.12 | 38.36	79.89 | 59.96
HBM | HBM	468047 | 93531	3035588 | 4150247	0.41 | 0.43	6.1 | 4.52	7.07 | 5.4	7.08 | 5.41	10.42 | 7.83
IMG | IAG	330380 | 136225	2828060 | 7338150	0.59 | 0.59	3.08 | 2.28	4.38 | 3.35	4.38 | 3.35	5.24 | 3.96
JE | JE	126472 | 26532	725152 | 664006	0.41 | 0.42	4.16 | 3.1	5.57 | 4.26	5.59 | 4.28	5.76 | 4.34
K | KGC	459455 | 204849	3423667 | 5137030	0.36 | 0.37	4.04 | 3.01	5.09 | 3.89	5.1 | 3.89	5.28 | 4
KL | KL	809303 | 265465	5883847 | 8194945	0.38 | 0.38	32.75 | 24.78	55.72 | 42.57	55.67 | 42.54	57.99 | 44.04
LAC | LAC	79010 | 19548	554294 | 409145	0.55 | 0.56	3.98 | 3	5.17 | 3.95	5.19 | 3.96	6.43 | 4.89
MFC | MFC	1055980 | 146708	9309901 | 20560263	0.18 | 0.2	19.65 | 14.73	23.83 | 18.2	23.82 | 18.2	25.18 | 18.7
MG | MGA	707903 | 225565	8590471 | 8690419	0.25 | 0.27	57.34 | 42.51	65.24 | 49.85	65.29 | 49.87	76.11 | 56.92
NEXA | NEXA	1965 | 33607	876459 | 421816	0.41 | 0.37	11 | 8.24	12.85 | 9.76	12.93 | 9.74	17.05 | 12.77
NOA | NOA	65976 | 27325	698430 | 698192	0.33 | 0.35	11.99 | 8.98	14.13 | 10.79	14.14 | 10.8	18.37 | 13.63
OR | OR	301144 | 108305	3337488 | 3325920	0.33 | 0.33	11.29 | 8.51	13.55 | 10.35	13.56 | 10.36	16.08 | 12.08
OSB | OSB	298421 | 48770	2157883 | 2327255	0.37 | 0.37	26.31 | 19.46	32.33 | 24.7	32.34 | 24.71	39.96 | 30.45
PD | PDS	275432 | 65425	1716199 | 7079878	0.58 | 0.6	2.2 | 1.65	2.43 | 1.86	2.43 | 1.87	4.05 | 3
PPL | PBA	725753 | 164168	8752195 | 16892003	0.14 | 0.15	41.9 | 31.46	48.38 | 36.95	48.32 | 36.92	50.65 | 37.93
PVG | PVG	357774 | 145260	4959405 | 12922001	0.48 | 0.49	8.85 | 6.65	13.1 | 10.01	13.11 | 10.02	13.69 | 10.4

TSX Ticker | NYSE Ticker	# Trades	# Quotes	Volatility	Min. Price	Mean Price	Median Price	Max. Price
QSR | QSR	470727 | 255245	6007156 | 5078923	0.22 | 0.21	71.83 | 53.84	90.68 | 69.29	90.67 | 69.28	93.28 | 70.46
RBA | RBA	110642 | 79826	2151942 | 2558254	0.18 | 0.18	42.64 | 31.87	43.74 | 33.41	43.74 | 33.41	49.85 | 37.90
RCI.B | RCI	705107 | 191170	5570214 | 5263925	0.16 | 0.16	65.4 | 48.68	70.04 | 53.54	70.05 | 53.54	73.82 | 55.91
RFP | RFP	8157 | 52024	1004058 | 1093796	0.44 | 0.44	8.04 | 6.03	9.13 | 6.99	9.25 | 7.08	12.80 | 9.66
SEA | SA	65739 | 57074	1567849 | 1442803	0.41 | 0.41	14.74 | 10.95	17.66 | 13.50	17.66 | 13.5	20.10 | 15.24
SHOP | SHOP	318277 | 405448	5354211 | 4998081	0.37 | 0.38	185.2 | 138.74	389.59 | 297.37	389.96 | 297.74	446.40 | 338.91
SJR.B | SJR	472788 | 111248	4124435 | 6200442	0.13 | 0.14	25.29 | 18.97	26.61 | 20.33	26.61 | 20.33	28.10 | 21.07
SLF | SLF	793187 | 175893	7900086 | 14513600	0.16 | 0.17	44.74 | 33.51	54.11 | 41.34	54.1 | 41.34	55.97 | 41.76
STN | STN	127841 | 4530	774649 | 689458	0.14 | 0.16	29.97 | 22.51	31.35 | 23.95	31.36 | 23.96	33.68 | 25.12
SU | SU	1535134 | 353457	17683883 | 57905722	0.20 | 0.21	38.64 | 28.96	40.79 | 31.17	40.8 | 31.17	46.50 | 34.86
T | TU	619142 | 111636	5611749 | 7265649	0.09 | 0.11	44.51 | 33.37	48.33 | 36.92	48.33 | 36.94	51.22 | 38.28
TA | TAC	290829 | 34737	1611785 | 1531971	0.30 | 0.30	5.78 | 4.35	8.40 | 6.43	8.41 | 6.44	10.14 | 7.61
TD | TD	1479643 | 216189	14119272 | 36534277	0.11 | 0.13	67.33 | 50.57	76.34 | 58.33	76.35 | 58.33	77.58 | 58.86
TECK.B | TECK	928365 | 326145	12283636 | 19162216	0.32 | 0.34	26.15 | 19.41	29.97 | 22.9	29.96 | 22.90	34.31 | 25.74
THO | TAHO	54648 | 34374	402457 | 1300237	0.30 | 0.32	4.54 | 3.43	4.94 | 3.75	4.94 | 3.75	5.18 | 3.93
TRI | TRI	402214 | 143245	4255310 | 5270140	0.14 | 0.14	62.92 | 47.15	84.11 | 64.25	84.1 | 64.25	88.97 | 67.26
TRP | TRP	1107841 | 245069	9958583 | 20378966	0.11 | 0.12	51.23 | 38.4	64.64 | 49.38	64.63 | 49.39	66.93 | 50.46
TRQ | TRQ	235612 | 91315	1223615 | 2755322	0.51 | 0.52	1.51 | 1.12	1.61 | 1.23	1.61 | 1.24	2.84 | 2.17
UFS | UFS	52684 | 144294	4057849 | 2763647	0.31 | 0.31	48.04 | 36.11	57.54 | 44.46	57.54 | 44.46	70.88 | 53.89
VET | VET	604493 | 110319	4984108 | 5249017	0.28 | 0.30	26.54 | 19.79	28.29 | 21.61	28.28 | 21.61	36.83 | 27.48
WEED | CGC	1828075 | 606702	13002523 | 8134021	0.54 | 0.56	37.25 | 28.01	52.97 | 40.44	52.84 | 40.37	70.98 | 52.73
WPM | WPM	622933 | 258698	9502407 | 20797947	0.25 | 0.26	24.75 | 18.55	31.38 | 23.98	31.37 | 23.98	33.85 | 25.24
YRI | AUY	337374 | 159634	2226240 | 4512850	0.43 | 0.44	2.41 | 1.79	3.33 | 2.54	3.33 | 2.55	3.78 | 2.88

Table A4. Variable definitions and symbols

Variable name (units)	Symbol	Variable definition and construction*
Daily average net profits per selected pair (CAD)	$\overline{profits}_t$	$\frac{\sum_{n=1}^{N} profits_t^{(n)}}{\lvert P_t \rvert}$, where $profits_t^{(n)}$ is the net profits in CAD generated by the pair of cross-listed stocks n on day t, and $\lvert P_t \rvert$ is the cardinality of the set of selected pairs on that day from our machine learning methodology, i.e. the number of selected pairs. Non-selected pairs have $profits_t^{(\cdot)} = 0$.
Intraday mid-price volatility of TSX stock (%)	$vol_{TSX,t}$	$\frac{\sum_{n=1}^{N} w_{TSX,t}^{(n)} vol_{TSX,t}^{(n)}}{\sum_{n=1}^{N} w_{TSX,t}^{(n)}}$, where $vol_{TSX,t}^{(n)}$ is the percentage ratio of standard deviation to the average of the mid-price series of cross-listed stock n at the TSX on day t.
Intraday mid-price volatility of NYSE stock (%)	$vol_{NYSE,t}$	$\frac{\sum_{n=1}^{N} w_{NYSE,t}^{(n)} vol_{NYSE,t}^{(n)}}{\sum_{n=1}^{N} w_{NYSE,t}^{(n)}}$, where $vol_{NYSE,t}^{(n)}$ is the percentage ratio of standard deviation to the average of the mid-price series of cross-listed stock n at the NYSE on day t.
Average intraday mid-price volatility for TSX-NYSE (%)	$\overline{vol}_{stocks,t}$	$\frac{vol_{TSX,t} + vol_{NYSE,t}}{2}$
Intraday mid-price volatility of CME futures (%)	$vol_{CME,t}$	Percentage ratio of standard deviation to the average of the mid-price series of the CADUS futures on day t.
Daily average bid-ask spread of TSX stock (bps)	$spread_{TSX,t}$	$\frac{\sum_{n=1}^{N} w_{TSX,t}^{(n)} spread_{TSX,t}^{(n)}}{\sum_{n=1}^{N} w_{TSX,t}^{(n)}}$, where $spread_{TSX,t}^{(n)}$ is the arithmetic mean of the bid-ask spread series in bps of the cross-listed stock n at the TSX on day t.
Daily average bid-ask spread of NYSE stock (bps)	$spread_{NYSE,t}$	$\frac{\sum_{n=1}^{N} w_{NYSE,t}^{(n)} spread_{NYSE,t}^{(n)}}{\sum_{n=1}^{N} w_{NYSE,t}^{(n)}}$, where $spread_{NYSE,t}^{(n)}$ is the arithmetic mean of the bid-ask spread series in bps of the cross-listed stock n at the NYSE on day t.
Average bid-ask spread for TSX-NYSE (bps)	$\overline{spread}_{stocks,t}$	$\frac{spread_{TSX,t} + spread_{NYSE,t}}{2}$

Variable name (units)	Symbol	Variable definition and construction*
Daily average bid-ask spread of CME futures (bps)	$spread_{CME,t}$	Arithmetic mean of the bid-ask spread series of the CADUS futures on day t.
Daily trading volume of TSX stock (shares)	$trade_{TSX,t}$	$\frac{\sum_{n=1}^{N} w_{TSX,t}^{(n)} trade_{TSX,t}^{(n)}}{\sum_{n=1}^{N} w_{TSX,t}^{(n)}}$, where $trade_{TSX,t}^{(n)}$ is the trading volume of the cross-listed stock n at the TSX on day t.
Daily trading volume of NYSE stock (shares)	$trade_{NYSE,t}$,	$\frac{\sum_{n=1}^{N} w_{NYSE,t}^{(n)} trade_{NYSE,t}^{(n)}}{\sum_{n=1}^{N} w_{NYSE,t}^{(n)}}$, where $trade_{NYSE,t}^{(n)}$ is the trading volume of the cross-listed stock n at the NYSE on day t.
Average daily trading volume for TSX-NYSE (shares)	$\overline{trade}_{stocks,t}$	$\frac{trade_{TSX,t} + trade_{NYSE,t}}{2}$
Daily trading volume of CME futures (contracts)	$trade_{CME,t}$	Total volume of CADUS futures contracts traded on day t.
Daily number of level one messages for TSX stock (scalar)	$messages_{TSX,t}$	$\frac{\sum_{n=1}^{N} w_{TSX,t}^{(n)} messages_{TSX,t}^{(n)}}{\sum_{n=1}^{N} w_{TSX,t}^{(n)}}$, where $messages_{TSX,t}^{(n)}$ is the number of level one messages for the cross-listed stock n at the TSX on day t.
Daily number of level one messages for NYSE stock (scalar)	$messages_{NYSE,t}$	$\frac{\sum_{n=1}^{N} w_{NYSE,t}^{(n)} messages_{NYSE,t}^{(n)}}{\sum_{n=1}^{N} w_{NYSE,t}^{(n)}}$, where $messages_{NYSE,t}^{(n)}$ is the number of level one messages for the cross-listed stock n at the NYSE on day t.
Average daily number of level one messages for TSX-NYSE (scalar)	$\overline{messages}_{stocks,t}$	$\frac{messages_{TSX,t} + messages_{NYSE,t}}{2}$
Daily number of level one messages for CME futures (scalar)	$messages_{CME,t}$	Total number of level one messages for the CADUS futures on day t

*A cross-listed stock n listed at exchange i has a daily traded value on day t of $w_{i,t}^{(n)} = \sum_{q=1}^{Q_{i,t}^{(n)}} v_{i,q}^{(n)} S_{i,q}^{(n)}$ for $Q_{i,t}^{(n)}$ the number of trades at that exchange for that stock on that day resulting from the limit order strategy, $v_{i,q}^{(n)}$ the volume of the trade and $S_{i,q}^{(n)}$ the value of the stock when the trade ended. Note that $Q_{\cdot,t}^{(\cdot)} = 0$ for every non-selected pair on day t.

Table A5. Pearson correlation matrix of the variables used in the regression analysis. Bold values are statistically different from 0.

	$\overline{profits}$	vol_{TSX}	vol_{NYSE}	vol_{CME}	$spread_{TSX}$	$spread_{NYSE}$	$spread_{CME}$	$trade_{TSX}$	$trade_{NYSE}$	$trade_{CME}$	$messages_{TSX}$	$messages_{NYSE}$	$messages_{CME}$
$\overline{profits}$	**1.000**	**0.474**	**0.490**	0.134	**0.613**	**0.669**	-0.114	0.031	**0.228**	0.118	**0.344**	**0.228**	**0.394**
vol_{TSX}		**1.000**	**0.983**	**0.290**	**0.462**	**0.471**	0.007	**0.242**	**0.607**	0.025	**0.354**	**0.249**	0.150
vol_{NYSE}			**1.000**	**0.322**	**0.432**	**0.447**	0.010	**0.263**	**0.609**	0.019	**0.390**	**0.287**	0.180
vol_{CME}				**1.000**	0.117	0.140	**0.244**	0.021	0.133	0.167	0.157	0.125	**0.258**
$spread_{TSX}$					**1.000**	**0.932**	-0.137	**-0.315**	0.160	0.126	**-0.197**	**-0.304**	0.180
$spread_{NYSE}$						**1.000**	-0.092	-0.165	**0.191**	0.117	-0.097	**-0.190**	**0.237**
$spread_{CME}$							**1.000**	0.017	-0.052	-0.090	0.066	0.101	**-0.351**
$trade_{TSX}$								**1.000**	**0.742**	-0.054	**0.702**	**0.688**	-0.033
$trade_{NYSE}$									**1.000**	0.026	**0.630**	**0.516**	-0.005
$trade_{CME}$										**1.000**	-0.014	-0.060	**0.209**
$messages_{TSX}$											**1.000**	**0.845**	0.170
$messages_{NYSE}$												**1.000**	**0.192**
$messages_{CME}$													**1.000**

Appendix B. Decision tree learning for recurrent pair selection

For T the number of days in our data, $\mathcal{T} = \{1, 2, \ldots, T\}$, the daily indices, and $k \in \mathbb{Z}_{<T/3}$ the period length at which we recurrently select pairs in Ω, we do pair selection every k days throughout our data, beginning after two periods, because the first two periods are needed for the first decision tree to be trained. Set $M = \left\lfloor \frac{T}{k} \right\rfloor$, with the model training day indices $\mathcal{M} = \{2k, 3k, \ldots, Mk\} \subset \mathcal{T}$. Define $\{\boldsymbol{X}_{p,t}\}_{t\in\mathcal{T}}$, $\boldsymbol{X}_{p,t} \in \boldsymbol{\mathcal{X}} \subseteq \mathbb{R}^d$ the multivariate stochastic process of the d daily features with time series $\{\boldsymbol{x}_{p,t}\}_{t\in\mathcal{T}}$ and $\{\Pi_{p,t}\}_{t\in\mathcal{T}}$. $\Pi_{p,t} \in \mathbb{R}$ is the net daily profit process of pair $p \in \{1,2,\ldots,n\}$ with time series $\{\pi_{p,t}\}_{t\in\mathcal{T}}$, where $\pi_{p,t}$ includes the gross profit, loss, and trading rebates and fees in Canadian dollars resulting from the strategy's execution on pair p during day t.

Let us also define $\{Y_{p,m}\}_{m\in\mathcal{M}}$, $Y_{p,m} \in \mathcal{Y} \equiv \{-1, 1\}$, the profitability class process of pair p, with time series $\{y_{p,m}\}_{m\in\mathcal{M}}$ where $y_{p,m} = -1$ when the pair p is unprofitable and $y_{p,m} = 1$ when that same pair is profitable during $\{t \mid m - k + 1 \leq t \leq m\}$. The time series is computed as follows:

$$y_{p,m} = \begin{cases} -1 \; if \; \sum_{t=0}^{k-1} \pi_{p,m-t} \leq 0 \\ \;\; 1 \; if \; \sum_{t=0}^{k-1} \pi_{p,m-t} > 0 \end{cases}, \forall p, m.$$

The decision tree's goal is to try to learn a series of $M - 1$ functions $H_m: \boldsymbol{\mathcal{X}} \to \mathcal{Y}$, where each one maps the features of all the pairs in $\{t \mid m - 2k + 1 \leq t \leq m - k\}$ to their respective profitability class during $\{t \mid m - k + 1 \leq t \leq m\}$ for all $m \in \mathcal{M}$. To do so, we use the arithmetic mean of the daily features' time series as inputs to the model. Hence, the set of training examples for the decision tree at time m is given by:

$$\left\{\left(\frac{\sum_{t=0}^{k-1} x_{1,m-k-t}}{k}, y_{1,m}\right), \left(\frac{\sum_{t=0}^{k-1} x_{2,m-k-t}}{k}, y_{2,m}\right), \ldots, \left(\frac{\sum_{t=0}^{k-1} x_{n,m-k-t}}{k}, y_{n,m}\right)\right\}, \forall m.$$

Based on that set, the decision tree tries to find H_m using Gini's impurity criterion and information gain. To avoid overfitting issues, we limit the algorithm to a maximum depth of 2. The resulting classification function, $\widehat{H}_m$, is then used to predict the profitability of each pair in the next interval $\{t \mid m+1 \leq t \leq \ m+k\}$ from the most recent features:

$$\widehat{H}_m\left(\frac{\sum_{t=0}^{k-1} x_{p,m-t}}{k}\right) = \hat{y}_{p,m+k}, \forall p, m.$$

Hence, we can select the set of pairs that will be traded throughout the next interval, which are given by:

$$P_m \equiv \ \left\{j \mid \hat{y}_{j,m+k} = 1, 1 \leq j \leq n\right\},\ \forall m.$$

The decision tree is completely retrained only on the corresponding training examples at each $m \in \mathcal{M}$, so that only local patterns are used in the prediction of the pairs' profitability. Note that we cannot launch the pair selection method until $t = 2k$, because the first k days are used to generate the features, and the following k days are used to compute the profitability of the pairs. Together, these features and profitability values form the first set of training examples on which we train the first decision tree at $t = 2k$.

The daily features computed at the TSX and NYSE are the:

- bid-ask spread's arithmetic mean,
- total trading volume,
- ratio of the number of trades per quotes,
- coefficient of variation of the mid-price,
- the total number of trades and quotes,
- and a measure of the previous period's profitability,

with $k = 3$ days.

The profitability prediction accuracy of the decision tree at time $m \in \mathcal{M}, A_m$, is computed as follows:

$$A_m = \frac{\sum_{p=1}^{n} \boldsymbol{I}_{\{\hat{y}_{p,m+k}=y_{p,m+k}\}}}{n}, \forall m$$

where $\boldsymbol{I}_{\{\hat{y}_{p,m+k}=y_{p,m+k}\}} = \begin{cases} 1 & if\ \hat{y}_{p,m+k} = y_{p,m+k} \\ 0 & if\ \hat{y}_{p,m+k} \neq y_{p,m+k} \end{cases}$, the indicator function.

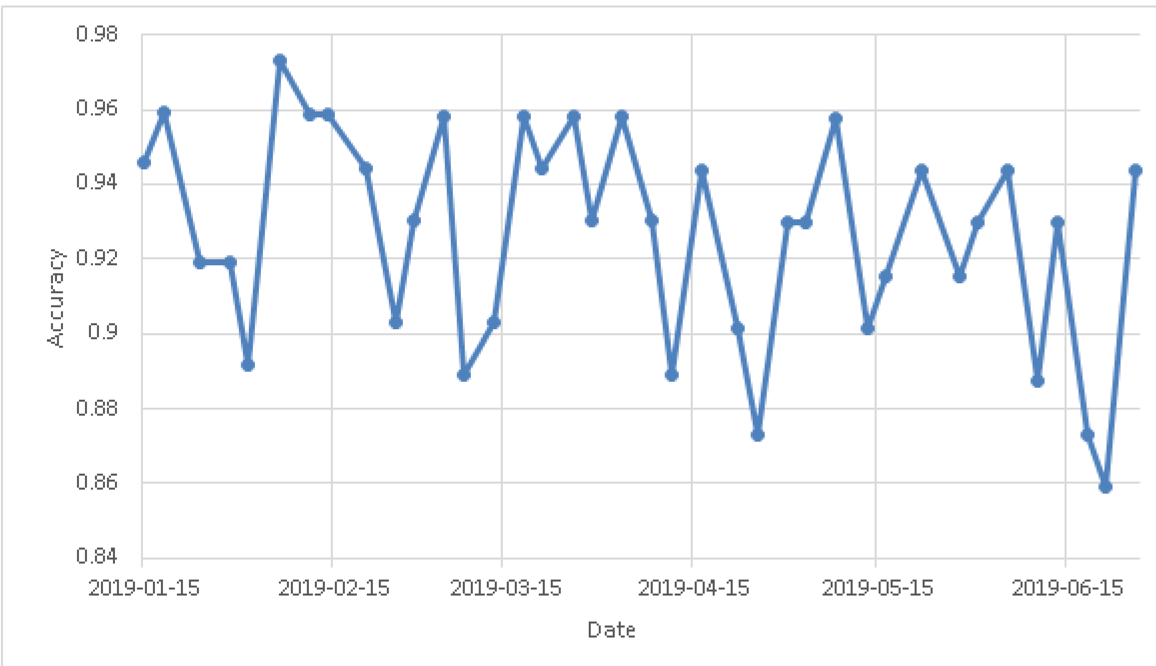


Figure B1. Profitability of prediction accuracy $A_m, \forall m \in \mathcal{M}$ of the dynamic decision tree approach from January 15th to June 20th, 2019, computed every $k = 3$ days.

From Figure B1, we observe that our methodology predicts that the next three days of each pair will be profitable at an average of 92% accuracy. We also observe that the predicted accuracy does not vary very much in our period of analysis. This process is repeated until the end of our data. Figure B2 presents the selected pairs in time for our portfolio. We observe that only 36 pairs pairs (in green) were selected at least one time.

Figure B2. Predicted profitability of each pairs in time generated by our dynamic decision tree-based approach from January 15th to June 28th, 2019: $\hat{y}_{p,m+k}, \forall p \in \{1,2,\dots,74\}, \forall m \in \mathcal{M}$. The selected pairs in time, P_m, are in green, non-selected pairs are in red, and pairs where at least one stock is de-listed are in yellow. About 36 pairs were selected by the model at least one time.

Figures B3 to B5 represent the decision trees learned for pair selection at the beginning of our period of analysis at 2019-01-15, in the middle period at 2019-03-29, and towards the end at 2019-06-13, respectively. Each rectangle in a tree is a node with the best rule that minimizes the Gini impurity of the corresponding child nodes. A rule is a criterion that splits the feature space into distinct subspaces. Feature vectors that fall within one of the resulting subspaces are then passed to the corresponding child node. Feature vectors that respect the interval specified by the rule in a node continue to the bottom left, and if they do not, they continue to the right until they arrive to a leaf where the prediction takes place. The learned rules are the first line of each non-leaf node. Leaves have no rule and are located at the bottom of the trees. The prediction made at the leaves is the most predominant class in the node's sub data set, where the number of instances of each class is given by the vector "value."

To determine the pair selection variables, well-established stylized facts are dynamically fed ex-ante into a decision tree using three days of high-frequency data. We restrict the information set to variables from tick trades and limit order level one and the target is our strategy's daily profitability class for each pair. The tree learning is done after markets close and is used during the three following intraday activities. Most of the time, two conceptually appealing stylized facts drive the pair selections: the bid-ask spread, an important component of endogenous liquidity providers profitability (Brogaard et al, 2018; Ait-Sahalia, 2017) and the number of messages, tightly linked to liquidity (Hendershott et al, 2011; Hasbrouck and Saar, 2013). The three decision trees below exemplify a recurring decision tree structure. The pair selection methodology based on them generates more than satisfactory results, given their out-of-sample high profitability prediction accuracy and the excellent stability in the performance through time (See Figure B1).

This confirms that the features selected by the decision trees are reliable predictors of the profitability of each pair traded.

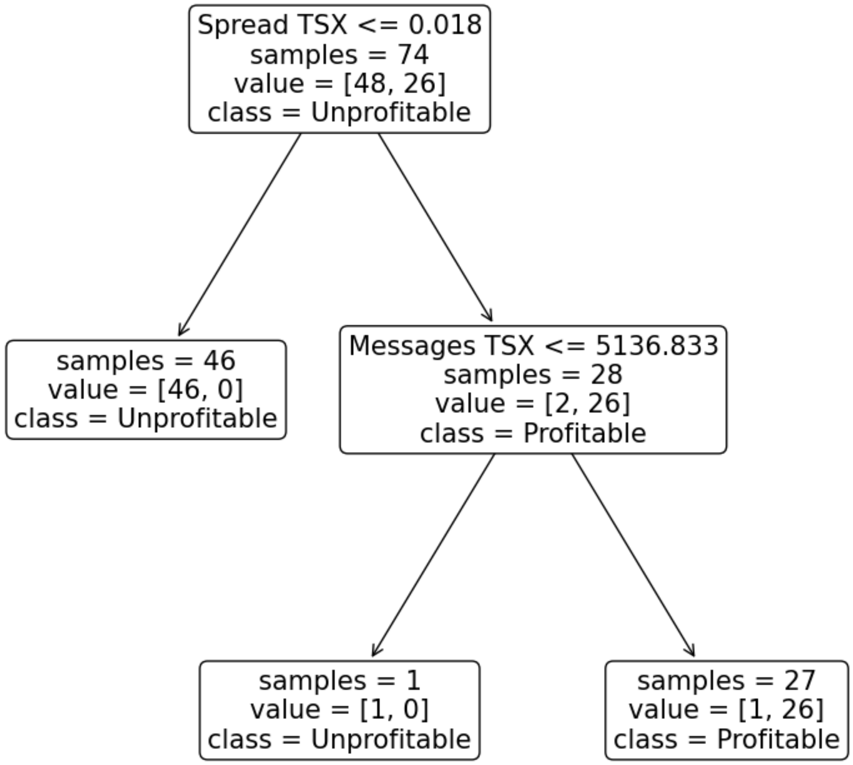


Figure B3. Tree for portfolios profitability on 2019-01-15

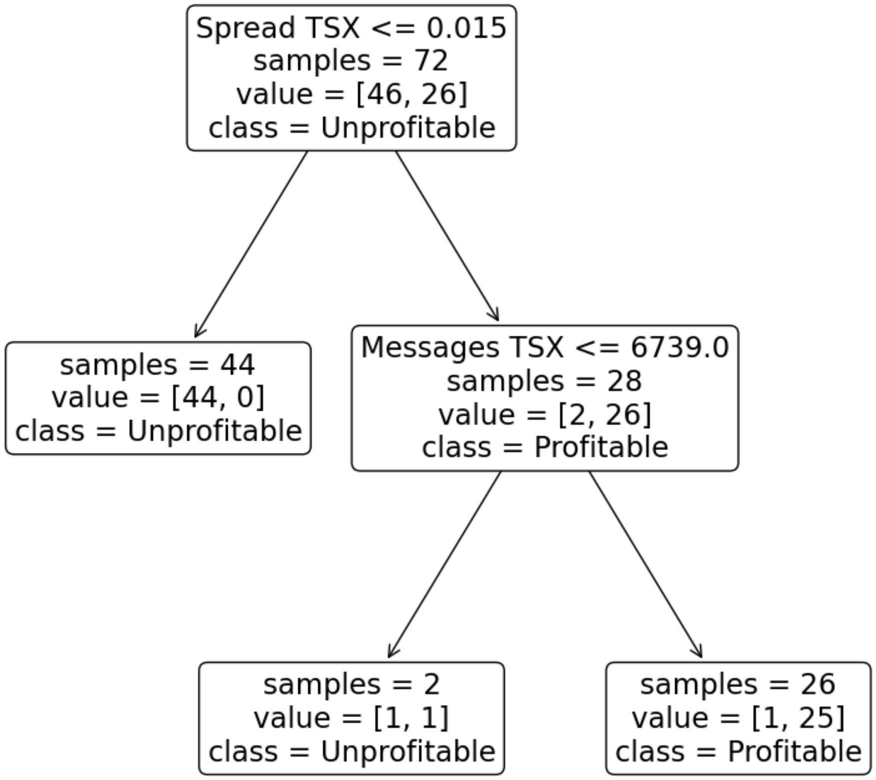


Figure B4. Tree for portfolios profitability on 2019-03-29

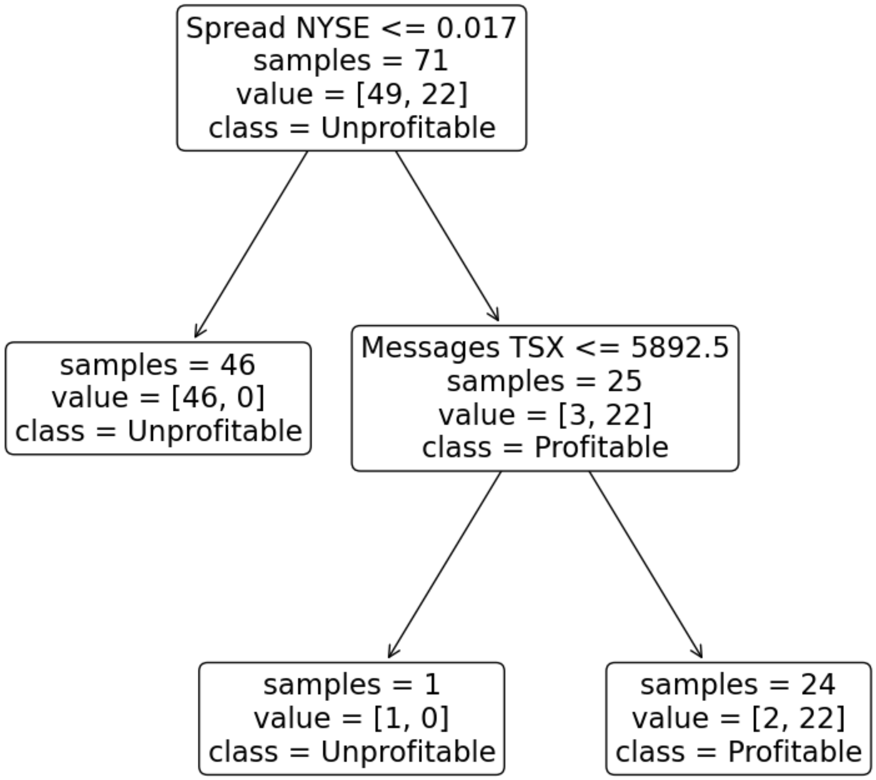
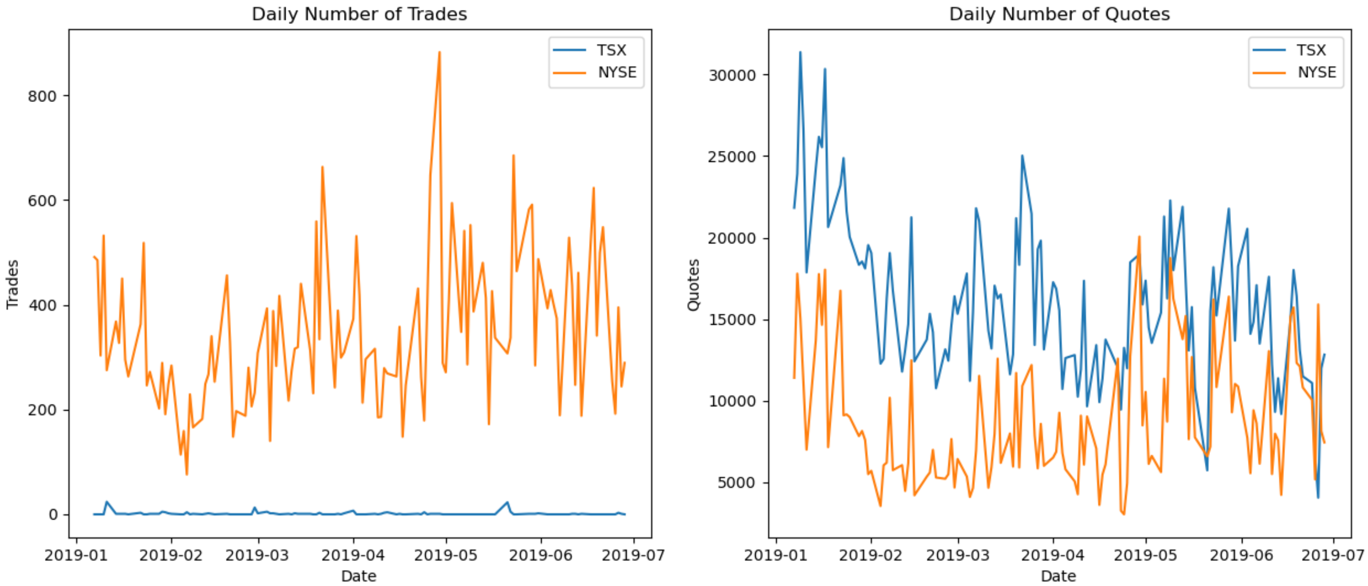


Figure B5. Tree for portfolios profitability on 2019-06-13

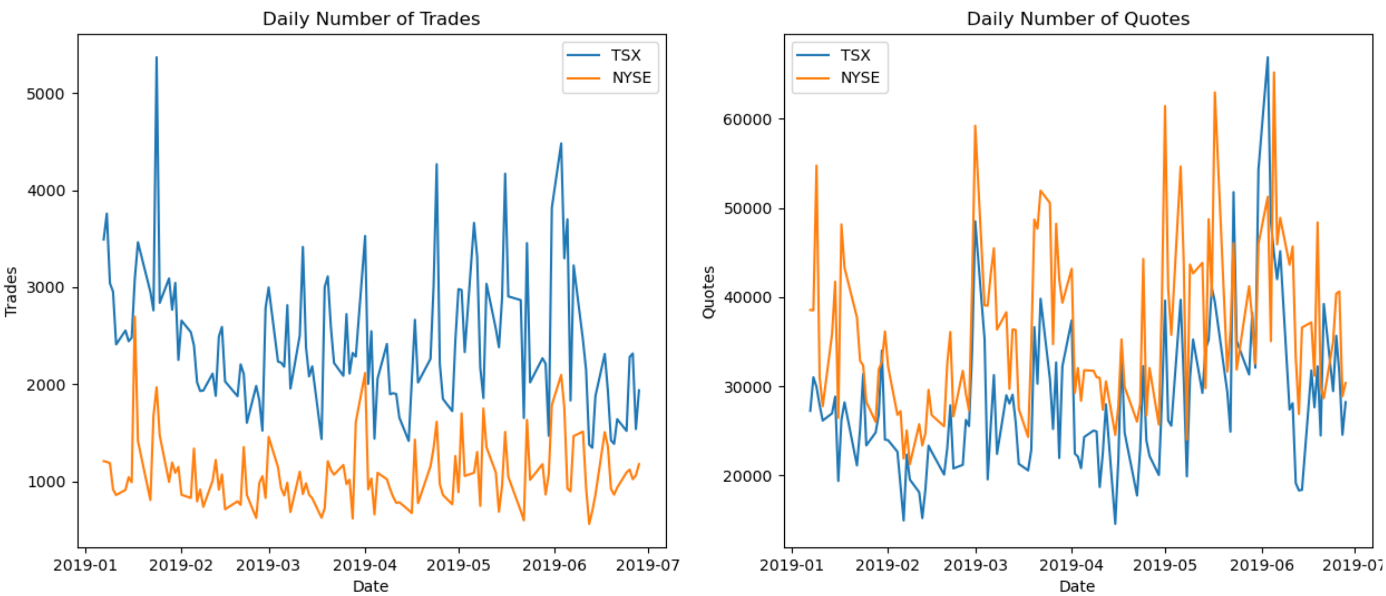
Appendix C. Daily number of trades and quotes for four pairs of cross-listed stocks.

The first company in Panel A, CNOOC, focuses on the exploitation, exploration and development of crude oil and natural gas in offshore China. It has barely any trades over the period, especially on the TSX. However, it received a decent amount of first level quotes on both exchanges with a bit more activity on the TSX. The second company, the Canadian Pacific Railway (CP), gets a fair amount of trading activity on both exchanges with a slight edge on the TSX. Both exchanges receive a comparable number of first level quotes for that Canadian company, which are about the double of CNOOC. The third company, Shopify, a Canadian e-commerce firm, has a similar trades and quotes profile on both exchanges. Both are higher than those of CP. The final company, Canopy Growth, a Canadian cannabis producer, has the highest number of trades and quotes on both exchanges and TSX leads in both quantities.

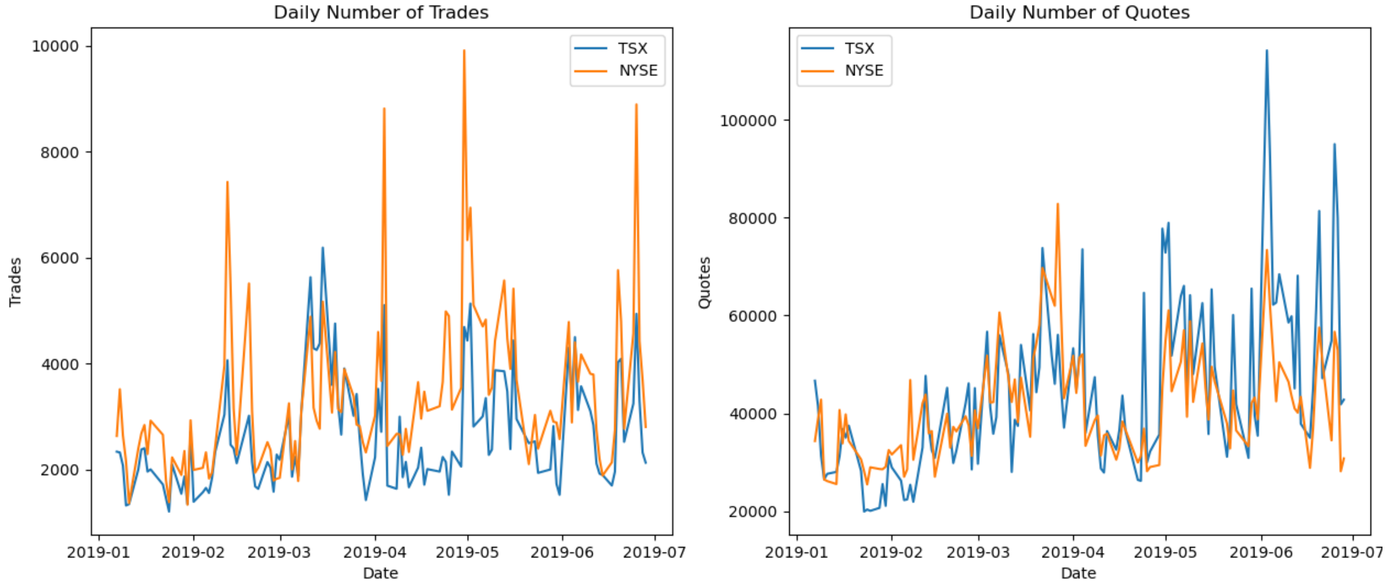
Panel A: Daily number of trades and quotes for TSX:CNU and NYSE:CEO



Panel B: Daily number of trades and quotes for TSX:CP and NYSE:CP



Panel C: Daily number of trades and quotes for TSX:SHOP and NYSE:SHOP



Panel D: Daily number of trades and quotes for TSX:WEED and NYSE:CGC

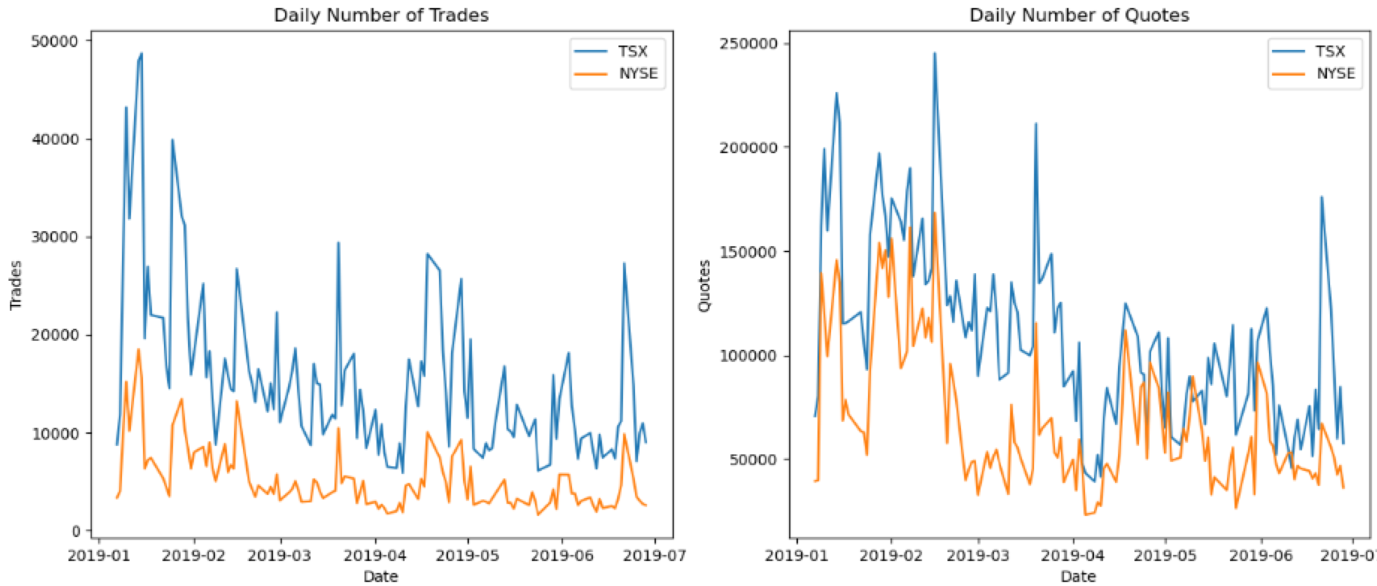


Figure C1. Daily number of trades and quotes for four pairs of cross-listed stocks

Appendix D. Server rules and execution occurrences

1. Each limit order has a standing quantity that must be executed before the order is executed.
2. That standing quantity is computed from the following steps:
   - 2 a. If the limit order's price of a buy/sell order is equal to the best bid/ask price, the order's standing quantity becomes the current best bid/ask volume.
   - 2.b If the limit price of a buy/sell order is below/above the best bid/ask price, the order's standing quantity is undefined. In that instance, the trading and quoting emulator waits for the limit order's price to be equal to the best bid/ask price and it sets the standing quantity according to 2.a.
   - 2.c If the limit order's buy/sell price is above/below the best bid/ask price, the order is filled.
   - 2.d If the standing quantity has been defined for a limit order, it can only be changed for a future execution.
3. A limit order can be executed by a trade occurring at the limit order's price. The standing quantity must be executed first. If it has been executed completely, then the limit order can be executed. If the remaining trade size is not large enough to fill the limit order's size, then a partial filling occurs. Limit orders with an undefined standing quantity cannot be executed by a trade.
4. A limit order can be executed when the best bid/ask price becomes lower/greater than the buy/sell limit order's price. This also holds for limit orders with undefined standing quantities.
5. A limit order is filled when the best bid/ask price becomes greater/lower than the sell/buy limit order's price, regardless of its standing quantity. This also holds for limit orders with undefined standing quantities.

The the trading and quoting emulator is conservative in some regards, especially considering the static standing quantity that must be executed before the corresponding limit order, because it ignores cancelations decreasing that quantity after the order has been placed, which follows from rules 1 and 2.a. Also, whenever a limit order is placed deeper than LOB level 1 and its price becomes the top of the book after some time, the limit order is put at the end of the queue of all the orders also at the new level 1 regardless of its actual position in that queue, which follows from rules 2.a and 2.b.