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A Taxonomy of Multilayer Network Design and a Survey of Transportation and Telecommunication Applications[ǂ]

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Abstract. Multilayer network design represents an important problem class when interwoven design decisions must be simultaneously considered. Examples of such cases are the selection of trains and blocks in freight rail transportation and the selection of physical and logical paths in telecommunications. Each set of those design variables are then defined on a particular network making up a layer with its own nodes, which can representor not the same physical or conceptual locations, potential arcs, with fixed selection cost and with or without limited capacities, and, possibly, multicommodity demands, which need to be routed within the layer by selecting/opening the appropriate arcs. The particular characteristic and challenge of multilayer network design consists in the various design and flow-connectivity requirements linking the decisions on different layers. Thus, for example, to select a light path, all the links making up the supporting physical path must be installed. Similarly, to select a transportation service, all the supporting resources must be selected together with their feasible working paths in the corresponding layers. We propose the first classification and a state-of-the-art survey of multilayer network design problems. The survey focuses on applications in transportation and telecommunications, as well as on solution methods. We also propose a general modeling framework which encompasses the models in the literature.

Keywords. Networks, multilayer network design, transportation, telecommunications, combinatorial optimization.

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1. Introduction

Multilayer network design (*MLND*) represents an important class within the well-known network design combinatorial optimization field (Magnanti and Wong, 1984; Crainic et al., 2021b), with major applications in transportation (see, e.g., Cordeau et al., 2001; Zhu et al., 2014; Crainic et al., 2018; Crainic and Hewitt, 2021) and telecommunications (see, e.g., Dahl et al., 1999; Knippel and Lardeux, 2007) system planning.

Several interwoven design decisions are simultaneously considered in a multilayer network design problem and formulation, each set of those design variables being defined on a particular network making up a layer. Each layer network has its own nodes, which can represent or not the same physical or conceptual locations, potential arcs, with fixed selection cost and with or without limited capacities, and, possibly, multicommodity demands, which need to be routed within the layer by selecting/opening the appropriate arcs. At least one layer has multicommodity demand to satisfy.

A particular layer might not involve any commodity demand, but still has to be designed to support the design of the network and the multicommodity-flow distribution within the other layers. Indeed, different from the *multi-echelon* (Cordeau et al., 2006), the *multi-tier* (Crainic et al., 2009, 2021c), the *multilevel* (Balakrishnan et al., 1994; Costa et al., 2011), and the *hierarchical* (Obreque et al., 2010; Lin, 2010) network design problem settings, multilayer network design is characterized by inter-layer coupling constraints enforcing the property that an arc in a given layer is related to a subset of arcs, typically forming a path or a cycle, in another layer. When each arc in layer l' is related to a subset of arcs in layer l, we say that l' is *supported* by l and that l is *supporting* l'.

Two types of inter-layer coupling requirements are identified, namely *flow* and *design* connectivity, respectively. *Design connectivity* arises when an arc in layer l' can be selected only if a given set of arcs are opened in the supporting layer l. *Flow connectivity*, on the other hand, signals that flows in a layer are related to the flows of another layer, e.g., the amount of flow on each arc in layer l may be computed as combinations, the sum, generally, of the flows on the connected arcs in the supported layer l'. Connectivity requirements between layers may be either one-to-one or one-to-many. The former arises when each layer is supporting or is supported by one other layer only, while a one-to-many connectivity exists when at least one of the layers is supporting or is supported by more than one layer. Note that, only the one-to-one connectivity is possible for two-layer network design In general, the objective is to find a minimum cost design and commodity-flow distribution for all layers, while satisfying typical network design constraints in each layer, as well as coupling constraints between layers.

Multilayer network design is often used to integrate decisions at a given planning level or at different planning levels: strategic, tactical, and operational. Solving the multilayer network design problem typically generates an optimal solution that cannot be obtained by solving sequentially each of the single-layer network design problems, thus yielding significant cost savings. An example of such an integration can be found in railway freight planning (Chouman and Crainic, 2021), where cars have to be sorted and grouped ("classified") into *blocks*. Blocks are then grouped into train *services* moving blocks between terminals. Grouping cars into blocks avoids performing operations on each car individually at terminals, reducing the number and cost of operations to be performed in each terminal. Zhu et al. (2014) modeled the problems of determining blocks (which block to be built) and selecting services in a single *integrated freight rail service network design* formulation. The authors represent the problem using a three-layer network including car, block, and service layers. Each layer consists of a *time-space network*, where the terminals (physical nodes) are duplicated over the time horizon to represent the time dependency. A node in such a network represents a terminal at a specific time, and each arc represents a transfer from a terminal at a given time to either the same terminal at another time or another terminal at another time. The service layer includes moving and stopping arcs. The block layer includes moving arcs, each corresponding to a path of moving and stopping arcs in the service layer, and transfer arcs to move blocks between service sections. The car layer consists of block links, each corresponds to a chain of block-transfer arcs and service sections in the block layer, and car arcs on which cars are moved in each terminal. To open a service-section arc in the block layer, a chain of moving and stopping links must be open in the service layer. To select a projected block link in the car layer, a chain of block and projected service-section links needs to be open in the block layer. Another example can be found in telecommunication-system planning, where one layer might be an internet (virtual) network whose arcs are supported by arcs in an optical fiber (physical) layer. A chain of supporting arcs has to be opened in the physical layer to open an arc in the internet network. In this example, there is an integration of a strategic decision (physical network design) with a tactical one (virtual network design).

Network design models, solution techniques, and applications have been surveyed in, e.g., Magnanti and Wong (1984), Minoux (1989), Crainic (2000), and Crainic et al. (2021b), without addressing the multilayer field, however. To the best of our knowledge, Kivelä et al. (2014) aims to do so, but it still does not cover MLND problems in any extensive way, citing a few references on telecommunication applications only. A formal definition of multilayer network design problem setting and modeling is still missing, as is an integrative view of the field. Our objective is to address this gap, in particular with respect to applications in transport and telecommunications.

The contribution of this paper is threefold. First, we propose a classification of multilayer network design problems, which emphasizes the multilayer features, such as the number of layers and the type of coupling requirements between layers. Second, we synthesize the applications in transportation and telecommunications, as well as the methods used to address multilayer network design problems. Third, we propose a general modeling framework that encompasses most multilayer network design problems found in the transportation and telecommunications literature.

The paper is organized as follows. We propose a detailed definition and a general MLND modeling framework in Section 2. Section 3 is dedicated to the presentation of the MLND classification we propose. The literature is then surveyed based on this taxonomy, showing each time how the proposed general modeling framework encompasses the problems presented in the literature. In particular, Sections 4, 5, and 6 address two-layer MLND with design connectivity, two- and three-layer problems with design and flow connectivity, and L-layer problems with design connectivity, respectively Section 7 syntheses the exact and metaheuristic solution methods proposed in the literature to address MLND problems. We conclude in Section 8, discussing future research directions.

2. Multilayer Network Design, Definition and Formulation

First, a definition, followed by a general MLND modeling framework.

2.1. Definition

In network design, given a potential network (for simplicity, we assume all arcs are potential), which may might have capacitated arcs, several commodities, such as goods, data or people, have to be routed between different origin and destination points. A network has to be constructed by opening appropriate arcs in order to provide the possibility to move the commodity flows. *Design* and unit *flow* costs are associated to each arc. The design cost is incurred when opening (or selecting) the arc, while flow costs are incurred when moving commodity flows on the arc. Network design aims to select the arcs such that the multicommodity demand can be satisfied (flows can be distributed) on the constructed network, arc capacities are respected, and the total cost of designing the network and routing the flows is minimized (Crainic et al., 2021a).

There are several networks in multilayer network design, instead of only one. Each network corresponds to a layer and consists of nodes and (generally) potential arcs. Several commodities might need to be routed in each layer to satisfy demands between origin and destination nodes. A network has to be designed in most layers to satisfy the demands for transportation. Note that, some layers might not have a network to design, but need to be present to support the movements of flows, e.g., commodity entries and exists at origins and destinations, respectively, as well as waiting for operations during their journeys. Note also that, some layers might not have any commodity to move, but their arcs have to be opened to support the distribution of flows in other layers. When there is only one layer that has commodities to route, we have a *multilayer single flow-type network design problem.* When we have commodities on more than one layer, we obtain a *multilayer multiple flow-type network design problem.*

In addition to the design and flow costs, as well as flow capacities that might be associated with arcs on the various layers, *coupling requirements* exist between the layers of a potential multilayer network. Coupling requirements are based on the property that an arc in a given layer is related to a subset of arcs in another layer. Thus, when arc a of layer l' is related to arcs b and c in layer l, we say that l' is *supported* by l; l is *supporting* l'; a is *supported* by b and c; b and c are *supporting* a. Coupling requirements translate into *coupling constraints* in multilayer network design formulations.

The first type of coupling requirements and constraints, *design connectivity*, means that an arc opened in a given layer requires some arcs to be opened in another layer. An illustration is given in Figure 1, where arcs a and b in layer l' are supported, respectively, by paths (c, d) and (e, f, emphd) in layer l. Therefore, l' is supported by l, and l is supporting l'. In this particular example, to use arc a in layer l', all its supporting arcs in layer l, including arcs c and d have to be opened. Thus, for instance, the design-connectivity constraints in the integrated freight rail service network design problem consist in opening a chain of supporting services in the service layer to select the corresponding block in the block layer. In telecommunications, design-connectivity constraints force each arc opened in the virtual network (the network supported by the physical layer) to be supported by a chain of physical arcs in the physical layer (the supporting network of the virtual layer). Note that, design-connectivity requirements are not limited to the above examples. Another type arises when an arc in a layer requires at least one of the supporting arcs to be opened in another layer.

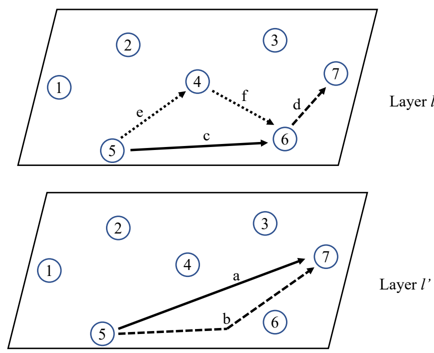

Figure 1: Multilayer network design with design connectivity: arc a in layer l' is supported by the path made of arcs $c$and d in layer l, and arc b in layer l' is supported by the path made of arcs e, f, and d

Based on the design-connectivity requirements, we define the *design-capacity* constraints, a new concept proper to multilayer network design. The design-capacity constraint of arc b in supporting layer l limits the number of selected arcs in supported layer l' for which arc b is the supporting arc. To illustrate, let the design-capacity of arc d in layer l in Figure 1 be equal to 1, which implies that at most one of the arcs a or $b$in layer l' may be opened in a feasible solution. To further illustrate, a design-capacity constraint is defined for each potential service in the integrated freight rail service network design problem, limiting the number of selected blocks for the service may carry, i.e., for which it may be the supporting service. In telecommunications, a design-capacity constraint may be defined for each physical arc to limit the number of opened virtual arcs for which it serves as a supporting arc.

Flow connectivity, the second type of connectivity requirements and constraints, relates the flows between different layers. The simplest such constraint arises when the flow on arc b is equal to the summation of the flows on all arcs for which b is a supporting arc. In Figure 1, for example, the flow on arc d would be equal to the summation of the flows on arcs a and b. Note that, with this particular type of flow-connectivity requirements, when only one layer has commodities to route, the flows on other layers can be deduced from the flows on that single layer. Such a problem would be considered as a multilayer single flow-type network design problem, even though there are flows on several layers.

It is noteworthy that, in some applications of multilayer network design, certain arcs of a layer can be independent of other layers. For example, in the integrated freight rail service nIetwork design problem, there are some arcs to move cars in each terminal that are not related to any arc of any other layer.

We complete this problem-setting description noticing that, some problems in the literature appear at first sight to be similar to the multilayer network design problem. These problems include the *multi-echelon/tier network design problem* (Cordeau et al., 2006; Crainic et al., 2009), the *multilevel network design problem* (Balakrishnan et al., 1994; Costa et al., 2011), and the *hierarchical network design problem* (Obreque et al., 2010; Lin, 2010). This similitude is only apparent, however. Indeed, in multilayer network design, connectivity constraints between layers are based on the property that an arc in a given layer is related to a subset of arcs (often a path) in a supporting layer. By contrast, in multi-echelon network design, two echelons represent different sets of arcs, while in multilevel or hierarchical network design, two levels share the same arcs, but with specific level-dependent costs and constraints.

We describe flow and design-connectivity constraints in the next subsection and discuss connectivity requirements in L-layer network design problems in Section 6

2.2. MLND formulation

Let $\mathcal{L}$ be the set of layers and $\mathcal{G}_l = (\mathcal{N}_l, \mathcal{A}_l)$ the network in each layer $l \in \mathcal{L}$, where N_l and A_l are the sets of nodes and arcs of layer $l \in \mathcal{L}$, respectively. Let u_{al} and v_{al} be the flow and the design capacity, respectively, of arc $a \in \mathcal{A}_l$ in layer $l \in \mathcal{L}$. Let $\mathcal{A}_l^+(n)$ and $\mathcal{A}_l^-(n)$ represent the sets of outgoing and incoming arcs of node $n \in \mathcal{N}_l$, and $\mathcal{K}_l$ the set of commodities to be routed through the network of layer $l \in \mathcal{L}$. There is no commodity to be routed in layer l when$\mathcal{K}_l = \infty$. The amount of each commodity $k \in \mathcal{K}_l$ that must flow from its origin $O(k) \in \mathcal{N}_l$ to its destination $D(k) \in \mathcal{N}_l$ is d^k.

Let $\mathcal{C}$ be the *connectivity-requirement* set composed of the ordered pairs (l, l') such that layer $l' \in \mathcal{L}$ is supported by $l \in \mathcal{L}$. In other words, $\mathcal{C}$ contains the pairs of layers having a design or flow connectivity requirement among them. Let $\mathcal{B}_{l'}^{al}$ be the set of arcs in layer l' supported by arc $a \in \mathcal{A}_l$. For example, in Figure 1, this set for arc 4 in layer l is $\{1, 2\}$. Let $\mathcal{D}_{bl'}^{l}$ be the set of arcs in layer l supporting arc $b \in \mathcal{A}_{l'}$. In Figure 1, this set for arc 1 in layer l' is $\{3, 4\}$.

Two sets of decision variables are defined, *design* and *flow* variables. The former may be binary or integer, depending on the particular application. When the decision is to open (select) or close (not to select) arc $a \in \mathcal{A}_l$ of layer $l \in \mathcal{L}$, then the design variable y_{al} assumes binary values. When the goal is to determine the number of capacity units on each arc $a \in \mathcal{A}_l$ of layer $l \in \mathcal{L}$, then the design variable y_{al} has integer values. The flow variables could take binary, continuous, or integer values depending on the problem. When the flow of each commodity has to be routed through a single path from its origin to its destination (non-bifurcated flows), then the flow variables take binary values. The variable x_{al}^k then indicates if commodity $k \in \mathcal{K}_l$ of layer $l \in \mathcal{L}$ uses arc $a \in \mathcal{A}_l$ or not. When the flow of each commodity can be distributed through several paths, then the flow variable x_{al}^k is continuous or integer (more rarely), representing the fraction of the demand of commodity $k \in \mathcal{K}_l$ of layer $l \in \mathcal{L}$ on arc $a \in \mathcal{A}_l$. Sets $\mathcal{X}$ and $\mathcal{Y}$ define required side constraints, as well as the domains of the flow and design variables, respectively. Set $(\mathcal{X}, \mathcal{Y})_{ll'}$ defines the coupling constraints for each pair of layers $(l, l') \in \mathcal{C}$, capturing application-specific connectivity requirements. We present several possible coupling constraints later in this section.

Let $\Psi(x)$ and $\Phi(y)$ represent the total flow-distribution and design cost functions, respectively. The proposed general *multilayer network design* formulation (*MLND*) can be stated as follows:

$$\min \quad \Psi(x) + \Phi(y) \tag{1}$$

subject to

$$\sum_{a \in \mathcal{A}_l^+(n)} x_{al}^k - \sum_{a \in \mathcal{A}_l^-(n)} x_{al}^k = w_n^k \qquad \forall l \in \mathcal{L}, \quad \forall n \in \mathcal{N}_l, \quad \forall k \in \mathcal{K}_l, \tag{2}$$

$$\sum_{k \in \mathcal{K}_l} d^k x_{al}^k \leq u_{al} y_{al} \qquad \forall l \in \mathcal{L}, \quad \forall a \in \mathcal{A}_l, \tag{3}$$

$$(x, y) \in (\mathcal{X}, \mathcal{Y})_{ll'} \qquad \forall (l, l') \in \mathcal{C}, \tag{4}$$

$$x \in \mathcal{X}, \tag{5}$$

$$y \in \mathcal{Y}. \tag{6}$$

The objective function, (1), minimizes the total cost. Constraints (2) are the usual *flow-conservation equations* ensuring that the demand flows are routed from their origins to their destinations in each layer, where $w_n^k = 1$ if $n = O(k)$, $w_n^k = -1$ if $n = D(k)$, and 0 otherwise. The *flow-capacity constraints* (3) ensure that the sum of the flows on each arc $a \in \mathcal{A}_l$ in layer $l \in \mathcal{L}$ does not exceed its flow capacity u_{al}. Constraints (5) and (6) define side constraints and the domains of the decision variables. Similarly to other network design problem settings, several side constraints may be added to a network design problem, among which *design balance* and *budget constraints* appear as the most important ones. Design-balance constraints arise when, at some nodes, the number of incoming opened arcs (representing, for example, resources or vehicles) must be equal to the number of outgoing opened arcs. A budget constraint limits the cost for building the network to a total budget.

The *connectivity constraints* (4) enforce the connectivity requirement (l, l'), that is, link the domains of the decision variables (x, y) of layer l' to those of layer l. Several types of connectivity constraints are encountered in the literature, depending on the application, as illustrated by the following three of design-connectivity types.

Design-capacity constraints (7) ensure that the number of selected arcs in layer l' supported by arc $a \in \mathcal{A}_l$, collected in set $\mathcal{B}_{l'}^{al}$, does not exceed its design capacity v_{al}. The left part of the inequality ensures that, when arc a in supporting layer l is opened, then at least one of its supported arcs is opened in the supported layer l'.

$$y_{al} \leq \sum_{b \in \mathcal{B}_{l'}^{al}} y_{bl'} \leq v_{al} y_{al} \qquad \forall (l, l') \in \mathcal{C}, \quad \forall a \in \mathcal{A}_l. \tag{7}$$

Multilayer all-design linking constraints (8) enforce the requirement that, to open arc $b \in \mathcal{A}_{l'}$, all its supporting arcs have to be opened in all supporting layers l. Such constraints arise, fe.g., in the integrated freight rail service network design problem, where to open a block, all its supporting services have to be opened in the service layer.

$$y_{bl'} \leq y_{al} \qquad \forall (l, l') \in \mathcal{C}, \quad \forall a \in \mathcal{A}_l, \quad \forall b \in \mathcal{B}_{l'}^{al}, \tag{8}$$

Multilayer min-design linking constraints (9) ensure that, for each arc b in a supported layer l', at least one arc has to be opened in the supporting layer $l \in \mathcal{L}$. Note that, constraints (8) imply (9), therefore, one of them in found only in most formulations. Such constraints appear, e.g., in the service network design with resource management problem where opening an arc in the service layer imposes opening one of the links in the resource layer (a resource activity cycle).

$$y_{bl'} \leq \sum_{a \in \mathcal{D}_{bl'}^{l}} y_{al} \qquad \forall (l, l') \in \mathcal{C}, \quad \forall b \in \mathcal{A}_{l'}, \tag{9}$$

Flow-connectivity requirements between layers may be added to the formulation as well. *Flow-accumulation constraints* (10) belong to this group, enforcing the requirement that the flow on each arc a in layer l be equal to the flow on all the arcs in layer l' supported by arc a.

$$x_{al}^{k} = \sum_{b \in \mathcal{B}_{l'}^{al}} x_{bl'}^{k} \qquad \forall (l, l') \in \mathcal{C}, \quad \forall a \in \mathcal{A}_l, \quad \forall k \in \mathcal{K}, \tag{10}$$

Note that, in some particular cases, constraints (10) might contradict flow conservation constraints (2). An example is when an arc in layer l supports two or more *reachable arcs* in layer l'. Two arcs (x_1, y_1) and (x_2, y_2) are said to be reachable if there is a path from x_1 to y_2 or from x_2 to y_1. Consider Figure 2 where arcs a and b are supported, respectively, by paths $(1, 2)$ and $(4, 6, 2, 7)$ in layer l. Suppose that there is a path between arcs a and b (the dashed arc in layer l'), i.e., arcs a and b are reachable. Arc 2 in layer l supports both arcs a and b in layer l'. Suppose that a commodity with demand d has to be routed from node A to node B using arcs a and b, as well as the path between these two arcs (dashed arc). Based on equation (10), the flow on arc 2 is equal to the summation of the flows on arcs a and b, which is $2d$. If we consider the destination node of arc 2, its total incoming flow is $2d$, but its total outgoing flow (on arc 7) is d. So equations (10) contradict flow conservation constraints (2). This issue does not arise in most time-space networks (e.g., Zhu et al., 2014), however, where the arcs in all layers point in the (same) time direction, which makes impossible for an arc in a layer to support two reachable arcs in another layer.

A different form of flow connectivity requirements exist when flows are non-bifurcated, which means the flow of each commodity has to be routed through a single path from its origin to its destination (flows cannot be split). The *non-bifurcated flow connectivity constraints* (11) are then added to the model. For each pair of arcs $a \in \mathcal{A}_l$ and $b \in \mathcal{B}_{l'}^{al}$, these constraints state that the flow of commodities $\mathcal{K}_{l'}$ can move on arc b in layer l' only if there is a flow of commodities $\mathcal{K}_l$ on arc a in layer l.

$$\sum_{k \in \mathcal{K}_{l'}} x_{bl'}^{k} \leq \sum_{k \in \mathcal{K}_l} x_{al}^{k} \qquad \forall (l, l') \in \mathcal{C}, \quad \forall a \in \mathcal{A}_l, \quad \forall b \in \mathcal{B}_{l'}^{al}. \tag{11}$$

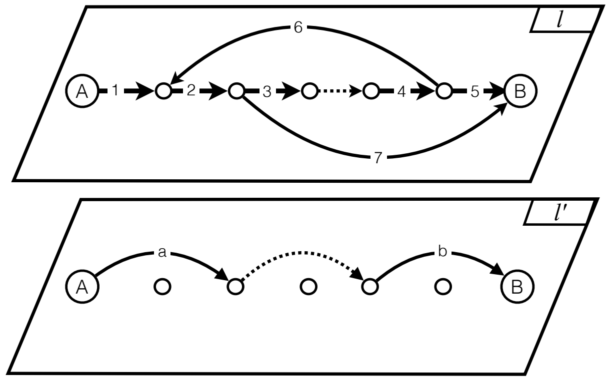

Figure 2: Flow-connectivity constraints contradicting flow-conservation constraints

3. Multilayer Network Design Taxonomy

We now introduce the taxonomy of multilayer network design problems we propose, and use it to classify the relevant literature in transportation and telecommunication system planning. The taxonomy is then used in the next sections to survey this literature.

Multilayer network design problems can be categorized into different classes based on three main dimensions (illustrated in Figure 3). The first dimension is the *number of layers*, from 2 to L. The *degree of connectivity* between layers makes up the second dimension, which includes *one-to-one connectivity* and *one-to-many connectivity*. The former exists when each layer is supporting or is supported by only one other layer, while the latter means that at least one of the layers is supporting or is supported by more than one layer. Note that, one-to-one connectivity is the only possible degree of connectivity for two-layer network design problems.

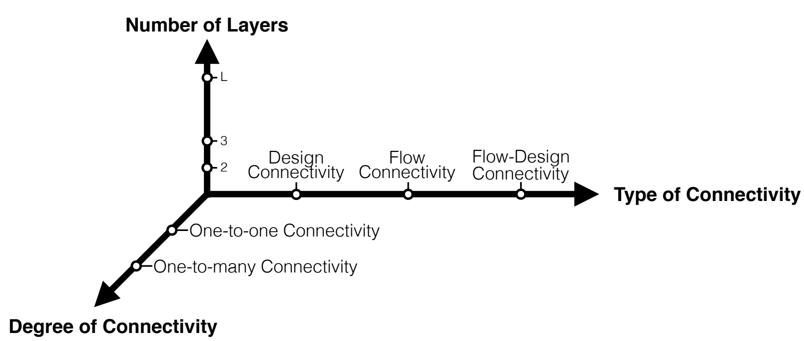

Figure 3: Taxonomy dimensions of multilayer network design problems

The third dimension is the *type of connectivity*, and it is made up of *design connectivity*, *flow connectivity*, which may be *single* or *multi-flow*, and *design-flow connectivity*. The last term indicates that both connectivity types are present at the same time. In the integrated freight rail service network design problem, for example, not only the designs of the layers are connected, but also the flow of each service is equal to the summation of the flows on its supported blocks.

Table 1 summarizes the multilayer network design models currently found in the transportation and telecommunications literature. The type of connectivity, **Design Connect**, **Design Connect**, and **Design Connect**, and the number of **Layers**, 2 and $L > 2$, make up the horizontal and vertical dimensions of the table, respectively. The **Degree** of connectivity, one-to-one ($\mathbf{D = 1}$) and one-to-many ($\mathbf{D = M}$) is indicated for each connectivity type when $L > 2$.

L	**Design Connect**	**Flow Connect**	**Design-Flow Connect**	
2	Dahl et al. (1999) Capone et al. (2007) Knippel and Lardeux (2007) Belotti et al. (2008) Koster et al. (2008) Fortz and Poss (2009) Orlowski (2009) Orlowski et al. (2010) Raack and Koster (2009) Mattia (2012) Mattia (2013) Andersen et al. (2009a) Andersen et al. (2011) Crainic et al. (2014) Taktak (2015)	Cordeau et al. (2001) Cohn and Barnhart (2003) Mercier et al. (2005) Mercier and Soumis (2007) Shao et al. (2015) Salazar-González (2014) Cacchiani and Salazar-González (2016)		
> 2	$\mathbf{D = 1}$	$\mathbf{D = M}$	$\mathbf{D = 1}$	$\mathbf{D = M}$
	Orlowski and Wessäly (2004) Knippel and Lardeux (2007)	Zeighami and Soumis (2017)	Zhu et al. (2014)	Crainic et al. (2018) Hewitt et al. (2019) Bilegan et al. (2021) Kienzle et al. (2021)

Table 1: Classification of multilayer network design problems in the literature

The table emphasizes that, historically, two-layer networks made up the main research area, each contribution focusing on one type of connectivity only. It is also noteworthy that most contributions in the literature address problem settings with one-to-one connectivity degree only. The field is evolving, however, toward more comprehensive problem settings with higher numbers of layers and more complex types and degrees of connectivity, as illustrated by the more recent entries displayed in the table. It is also interesting to notice that this evolution is fostered by Operations Research addressing more advanced and integrated planning challenges raised by contemporaneous transportation systems.

The next sections present a comprehensive survey on the multilayer network design models proposed for transportation and telecommunication system planning, according to the taxonomy we propose. In each case, we first provide a problem definition and literature review, followed by a discussion on how the modeling framework we propose in Section 2.2 addresses these applications and encompasses the formulations.

4. Two-Layer Network Design with Design Connectivity

Most contributions in transportation and telecommunication system planning are found in this class and are reviewed in Sections 4.1 and 4.2, respectively.

4.1. Service network design with single resource management

Service network design models are broadly used to address tactical planning issues for consolidation-based transportation systems Crainic and Hewitt (2021). Traditionally, most of these tactical-level planning models assumed the necessary resources (crews, power units, specific vehicles, etc.) are available at terminals when needed, their allocation, circulation, and management being addressed at more operational planning levels (e.g., crew scheduling and fleet management). Addressing problem settings with expensive assets, e.g. within airlines, railroads, and containership liners, lead to explicitly introducing resource-management concerns into tactical-planning formulations.

Two-layer network design formulations are proposed when considering a single resource type, predefined assignment of resources to the nodes (terminals) of the network, and the need of a single resource unit to support each selected service. Most contributions consider the time-dependency of demand and the resulting service definition and selection, yielding *scheduled service network design* (*SSND*) formulations. This is the problem setting we address in the following.

4.1.1. Literature review

Several initial contributions to this problem setting (e.g., Kim et al., 1999; Pedersen et al., 2009; Andersen et al., 2009b) addressed the resource-management issue by proposing arc-based formulations with design-balance constraints (Pedersen et al., 2009), which force the conservation, the *balance*, of resources (services) at the nodes of the potential service network. The problem encompasses two layers, one for selecting services and moving the demand, and a second for circulating the resource flows required to support the selected services. The layers are not explicit in these early contributions, however, presenting design-balanced SSND models where services, resources, and demand flows are all on the same layer.

The next step in methodological development was based on the observation that resource management and design-balance constraints imply that resources move in cycles on the time-space network of potential scheduled services. The cycles start and end at the node they are initially assigned to, and may include holding arcs for stops, waiting, and transferring at terminals, as well as repositioning arcs to move resources without supporting a service in order to get them the next task/service in the cycle. To model this problem, Andersen et al. (2009a) proposed a two-layer time-space service network. Potential services, with their departure and arrival times, make up the arcs of the service layer, while an arc in the resource layer is defined as a *resource cycle*. The design connectivity constraints then specify that each arc in the resource layer corresponds to a path of supported services, holding arcs, and repositioning arcs in the service layer, from a terminal at time t to the same terminal at time $t + T_{max}$, where T_{max} is the schedule length of the tactical plan. Figure 4 illustrates these notions, where cycles $r1$ and $r2$ in the resource layer support the sets of services $\{s1, s2, s3\}$ and $\{s4, s5, s6\}$ in the service layer, respectively.

Andersen et al. (2009a) showed the computational superiority of the cycle-based formulation compared to the design-balanced one. Crainic et al. (2014) enlarged the problem setting by explicitly considering selection costs and decisions for services and resource cycles. The latter encompass the management rules of the application; cycles may return several times to their assigned terminal node. A limited number of resources are allocated to each terminal, a constraint enforcing this limit in the model. Services are selected on

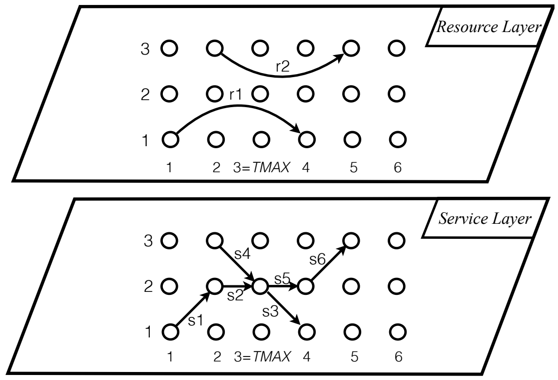

Figure 4: Resource cycles and their supported services in SSND-RM

the service layer, where the demand flows are also considered, while cycles are defined, based on the service arcs in the service layer, and selected on the resource layer.

4.1.2. MLND formulation

We follow the description of Crainic et al. (2014) to formulate the service network design problem with resource management within the model framework proposed in Section 2.2.

Let $\mathcal{K}$ be the set of commodities (single flow-type) and d^k be the demand for commodity $k \in \mathcal{K}$. In this problem, we have a service layer ($l = 1$) and a resource layer ($l = 2$). The connectivity set $\mathcal{C}$ is defined as $\{(2,1)\}$, meaning that the service layer is supported by the resource layer. To open an arc $b \in \mathcal{A}_1$ in the service layer, one of the supporting resource arcs in set $\mathcal{D}^2_{b1}$ should be opened in the resource layer. Let $\mathcal{V}$ be the set of terminals; θ_v, the set of resource arcs of layer 2 that depart from terminal $v \in \mathcal{V}$ during the scheduling length; h_v, the limit on the number of resources that may be used out of terminal $v \in \mathcal{V}$; f_{al}, the fixed cost of each arc $a \in \mathcal{A}_l, l \in \mathcal{L}$ to select the respective service or resource; c^k_{a1}, the unit transportation cost of commodity $k \in \mathcal{K}$ on arc $a \in \mathcal{A}_1$.

Define y_{a1} and y_{a2}, the design decision variables for service $a \in \mathcal{A}_1$ and resource $a \in \mathcal{A}_2$, respectively, and x^k_{a1}, the flow variable for commodity $k \in \mathcal{K}$ on arc $a \in \mathcal{A}_1$ of the service layer. The MLND model:

$$\min \quad \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}_1} c^k_{a1} x^k_{a1} + \sum_{l\in\mathcal{L}} \sum_{a\in\mathcal{A}_l} f_{al} y_{al} \tag{12}$$

subject to

$$\sum_{a\in A_1^+(n)} x^k_{a1} - \sum_{a\in A_1^-(n)} x^k_{a1} = w^k_n \qquad \forall n \in \mathcal{N}_1, \quad \forall k \in \mathcal{K}, \tag{13}$$

$$\sum_{k\in\mathcal{K}} d^k x^k_{a1} \le u_{a1} y_{a1} \qquad \forall l \in \mathcal{L}, \quad \forall a \in \mathcal{A}_1, \tag{14}$$

$$y_{b1} \leq \sum_{a \in \mathcal{D}^2_{b1}} y_{a2} \qquad \forall b \in \mathcal{A}_1, \tag{15}$$

$$\sum_{a \in \theta_v} y_{a2} \leq h_v \qquad \forall v \in \mathcal{V}, \tag{16}$$

$$x^k_{a1} \geq 0 \qquad \forall a \in \mathcal{A}_1, \quad \forall k \in \mathcal{K}, \tag{17}$$

$$y_{al} \in \{0, 1\} \qquad \forall l \in \mathcal{L}, \quad a \in \mathcal{A}_l. \tag{18}$$

The objective function, (12), minimizes the sum of the selection costs on the service and resource layers, plus the commodity transportation costs on the service layer. Constraints (13) are the flow conservation equations ensuring demand is satisfied in the service layer. Capacity-linking constraints (14) ensure that the total flow on each service arc is less than or equal to the capacity of service and that the service must be open in order to route the commodities. Service-resource coupling constraints (15) enforce that at least one of the resource arcs must be open in the resource layer in order to open a service arc. Constraints (13), (14), and (15) are equivalent to constraints (2), (3), and (9) of the general modeling framework, respectively. Terminal resource-capacity constraints (16) impose a limit on the number of resources of layer 2 that may be used out of terminal $v \in \mathcal{V}$ during the schedule length.

4.2. Telecommunication network design

Two layers are generally encountered in telecommunications networks, a *virtual* or *logical layer*) and a *physical layer*, or optical transport network. A typical example of such a problem setting is the design of an internet backbone network and of the physical fiber network supporting it. A path of links has to be opened in the physical layer to establish a connection in the internet network.

The same nodes are found in both layers, representing points where single-flow-type demand originates and terminates, as well as traffic-switching facilities. A set of origin-to-destination demands must be routed in the logical layer, supported by the physical layer. The nodes and links display several features: design cost, flow cost, *virtual flow capacity*, *physical design capacity*, and *node capacity*. The virtual flow capacity limits the commodity flow on logical links. The physical design capacity limits the number of logical links which may be supported by a physical link. The node capacity limits the number of virtual or physical links that can originate from or end to a particular node (see Appendix A for an illustration).

4.2.1. Literature review

Several contributions proposing two-layer network design models with design connectivity are found in the literature, where the selection of each link in the virtual layer corresponding to the design of all links of the associated path in the physical layer. To the best of our knowledge, the concept of layered networks in telecommunications goes backs to Balakrishnan et al. (1991). Dahl et al. (1999) proposed a two-layer network for a *PIPE* telecommunications application, where the objective is to find a minimum cost pipe (virtual links) selection and routing, while considering the design capacity of the physical links. Each demand has to be routed on a single virtual path. Therefore, the routing and the design variables are binary in the proposed formulation.

Capone et al. (2007) addressed a two-layer network design problem with node capacity and *multicast traffic* demand, where instead of point-to-point commodities, each commodity has an origin and multiple destinations. Therefore, a flow solution for each commodity is a tree, not a path. Knippel and Lardeux (2007) proposed a two-layer network design formulation with fixed costs for the virtual and physical arcs, while no flow costs are considered (the authors also briefly mentioned a model with L arbitrary layers). The model minimizes the total design cost of both layers. Parallel arcs are not used in the virtual layer; instead, the authors assumed each virtual flow could be routed on several physical paths. Two types of continuous variables are thus introduced to determine the amount of each commodity on each logical path, and the amount of each installed logical traffic routed on each physical path. *Metric inequalities* are developed from the dual of the path-based formulation to represent the feasible space of capacity vectors.

Koster et al. (2008) proposed a formulation for a problem setting with a predefined set of logical links. The problem includes the selection of nodes and *survivability requirements* against physical node and link failures, which mean that demand must be satisfied in the event of a single physical node or link failure. The survivability requirements are modeled as *survivability constraints*, where the demands are doubled, and the flow through an intermediate node is restricted to half of the demand value. Mattia (2012) addressed the same problem but, rather than adding survivability constraints explicitly, the author defines several failure scenarios, each of which includes a restricted number of potential links only. For each failure scenario, a two-layer network is defined containing only the available links, with variables defined for each scenario. Taktak (2015) presents an in-depth study of the problem in the context of more recent telecom technology, in particular the polyhedral properties of the formulation.

Orlowski and Wessäly (2004) appears to the only contribution focusing on a general L-layer network design telecommunication application, where layers correspond to different technologies and several transmission protocols in the logical links/paths. Vertical links connect the layers and provide the means to route the commodities. The design connectivity relations in the integer network design model capture the associations between the logical paths and the supporting physical network, as well as the distribution of hardware capacity to the logical layers. A rich set of constraints is included to represent the hardware and technical characteristics and limitations of telecommunication designs.

4.2.2. MLND Formulation

We use the problem description of Dahl et al. (1999) to illustrate the MLND formulation (Section 2.2) for the two-layer network design telecommunications applications described above.

The notation and formulation are remarkably similar to those of the SSND-RM case (Section 4.1). The layer set $\mathcal{L} = \{1, 2\}$ includes the virtual layer ($l = 1$) and the physical layer ($l = 2$). One has the same concepts and notation for the connectivity requirements $\mathcal{C} = \{(2, 1)\}$, demand $\mathcal{K}$, arc costs $c^k_{a1}, k \in \mathcal{K}, a \in \mathcal{A}_1$ and $f_{al}, a \in \mathcal{A}_l, l \in \mathcal{L}$. The same decision variables are defined, $y_{al}, a \in \mathcal{A}_l, l \in \mathcal{L}$ to select arcs on the respective layers, while the flow distribution variables $x^k_{a1}, k \in \mathcal{K}, a \in \mathcal{A}_1$ are binary in this non-bifurcated flow case.

The model then (see Appendix A for the detailed formulation) minimizes the total cost objective function (12), subject to constraints (13) & (14), (19) (replacing (15), (16)), (20) (enforcing the single-path flow distribution), and (18).

$$y_{a2} \leq \sum_{b \in \mathcal{B}_1^{a2}} y_{b1} \leq v_{a2} y_{a2} \qquad \forall a \in \mathcal{A}_2, \tag{19}$$

$$x^k_{a1} \in \{0, 1\} \qquad \forall a \in \mathcal{A}_1, \quad \forall k \in \mathcal{K}. \tag{20}$$

Note that, when the links are undirected, the problem can still be modeled using the proposed formulation by replacing an undirected link with two directed arcs.

5. Two & Three-Layer Network Design with Flow Connectivity

Planning airline operations and resources involves several inter-connected decision processes. Briefly, the first step is *flight scheduling* to define the origins, destinations, and the departure and arrival times for each flight leg (i.e., no stopovers) to be flown during the schedule length. *Fleet assignment* then assigns an aircraft type to each flight leg to maximize the profit. *Aircraft routing* determines the sequence of flight legs to be covered by each individual plane of each aircraft type, such that each flight leg is covered exactly once while ensuring aircraft maintenance requirements. Finally, *crew scheduling* links crews and flights in two phases: *crew pairings* and *crew assignment*. A crew pairing is a sequence of flight legs separated by short and long (overnight) rest periods starting and ending at the same crew base. Crew assignment builds individual monthly schedules out of the generated pairings for each crew member, e.g., pilot, copilot, and flight attendants.

Traditionally, airlines addressed these problems sequentially, which reduces the complexity of the problem, but might result in a solution far from the global optimum of the integrated problem. Research therefore focuses on combining several of these problems and addressing them with comprehensive formulations. Most of these formulations take the form of two-layer network design models with flow connectivity requirements, and this section is mainly dedicated to this work.

The integrated aircraft routing and crew pairing problem is defined on a two-layer network, including an aircraft-routing layer and a crew-paring layer. In both networks, a node corresponds to a flight leg, while arcs represent the connections between two legs. The goal is to find the minimum total cost of aircraft and crew routing, one path for each aircraft and one path for each crew, such that 1) each flight leg is covered only once by a crew and only once by an aircraft, and 2) if a connection time for a link is too short, then the corresponding legs can be covered by the same crew only if both legs are covered by the same aircraft (otherwise, the connection time is insufficient for the crew). The second condition corresponds to the one-to-one type of flow-connectivity requirements.

Cohn and Barnhart (2003) proposed an extended crew pairing formulation, where the aircraft-routing variables represent a complete solution of a routing problem. Mercier et al. (2005); Mercier and Soumis (2007) enhanced the path-based formulation of Cordeau et al. (2001) by introducing restricted-connection arcs to represent short-time connections, together with penalties when the second condition above is not respected, and the possibility to select the flight-leg departure time to minimize the cost. Alternative approaches based on arcs were proposed by Salazar-González (2014) and Cacchiani and Salazar-González (2016). The fully arc-based formulation of the former requires a huge number of inequalities to avoid infeasible crew routes. Better results were obtained by the latter by returning to path representations, including an arc-path-based model with arc-based and path-based variables representing aircraft routes and crew pairings, respectively. Shao et al. (2015) took a step further by integrating fleet assignment to the joint aircraft routing and crew pairing problem, adding the aircraft-type selection decision to each flight leg (node) of the network.

Starting from the description of Cordeau et al. (2001), we formulate the corresponding two-layer MLND model with one-to-one type of flow-connectivity requirements. Let $\mathcal{L} = \{1, 2\}$ be the set of the crew ($l = 1$) and aircraft ($l = 2$) layers. The nodes in both layers, $\mathcal{N}_l, l \in \{1, 2\}$, are the flight legs, while arcs stand for the crew and aircraft connections in the crew and aircraft layers, respectively. The set $\mathcal{C}$ is defined as $\{(2, 1)\}$ representing the coupling constraints indicating that the aircraft layer supports the crew layer. A set of crews, $\mathcal{K}_1$, and planes, $\mathcal{K}_2$, must be routed on the crew and the aircraft layers, respectively. Flows are non-bifurcated. Partition each $\mathcal{N}_l, l \in \{1, 2\}$, into $\mathcal{N}_l^O = \{n \in \mathcal{N}_l \,|\, \exists\, k \in \mathcal{K}_l, n = O(k)\}$, $\mathcal{N}_l^D = \{n \in \mathcal{N}_l \,|\, \exists\, k \in \mathcal{K}_l, n = D(k)\}$ and $\mathcal{N}_l^I = \mathcal{N}_l \backslash (\mathcal{N}_l^O \cup \mathcal{N}_l^D)$. Binary decision variables x_{a1}^k and x_{a2}^k determine, if crew $k \in \mathcal{K}_1$ uses arc $a \in \mathcal{A}_1$ and if aircraft $k \in \mathcal{K}_2$ uses arc $a \in \mathcal{A}_2$, respectively. The MLND model:

$$\min \quad \Psi(x) \tag{21}$$

$$\sum_{a \in A_l^+(n)} x_{al}^k - \sum_{a \in A_l^-(n)} x_{al}^k = w_n^k \qquad \forall n \in \mathcal{N}_l, \quad \forall k \in \mathcal{K}_l, \quad l \in \{1, 2\}, \tag{22}$$

$$\sum_{k \in \mathcal{K}_1} x_{b1}^k \leq \sum_{k \in \mathcal{K}_2} x_{a2}^k \qquad \forall a \in \mathcal{A}_2, \quad \forall b \in \mathcal{B}_1^{a2}, \tag{23}$$

$$\sum_{a \in \mathcal{A}_l^+(n)} \sum_{k \in \mathcal{K}_l} x_{al}^k = 1 \qquad \forall l \in \mathcal{L}, \quad \forall n \in \mathcal{N}_l^O \cup \mathcal{N}_l^I, \tag{24}$$

$$\sum_{a \in \mathcal{A}_l^-(n)} \sum_{k \in \mathcal{K}_l} x_{al}^k = 1 \qquad \forall l \in \mathcal{L}, \quad \forall n \in \mathcal{N}_l^D \cup \mathcal{N}_l^I, \tag{25}$$

$$x_{al}^k \in \{0, 1\} \qquad \forall a \in \mathcal{A}_l, \quad \forall k \in \mathcal{K}. \tag{26}$$

The objective function (21) minimizes the total routing costs on both layers. Notice that, the objective function in airline applications is typically non-linear with respect to the arc flow variables,, while path-based formulations address this issue. We keep the arc-based model to facilitate comparison with the general framework (Section 2.2). Flow conservation equations (22) enforce the circulation of the crew and aircraft flows on therespective layers. Constraints (23) ensure that a crew does not change aircraft when the connection time is too short. These constraints correspond to the flow connectivity inequalities (10) of the general modeling framework. Constraints (24) and (25) are side constraints ensuring that a flight leg is covered by exactly one crew and one aircraft.

Zeighami and Soumis (2017) extend this methodology to address an integrated crew pairing and personalized (considering individual vacation requests, VRs) assignment problem for pilots and copilots. They define a three-layer network including a crew-pairing layer, a pilot-assignment layer, and a copilot-assignment layer. As previously, nodes in the crew-pairing network correspond to departure and arrival airports, while arcs represent the flights and the connections between then. Nodes in the pilot and copilot-assignment networks correspond to the start and end of pairings, while the arcs represent the pairings (paths in the crew-pairing layer), the connections between those, and the VRs of the pilots and copilots, respectively. A pilot or copilot assignment then corresponds to a path in the corresponding layer. The objective function aims for a trade-off between maximizing the number of satisfied VRs and minimizing the total cost of the pilot and copilot assignment, while guaranteeing that each flight is covered by exactly one pairing, and each pairing is covered by exactly one pilot and one copilot assignment. These conditions correspond to 1-to-many flow-connectivity requirements. The full MLND formulation is presented in Appendix B.

6. *L*-Layer Networks with Design-Flow Connectivity

The number of contributions proposing multi-layer networks with more than two layers is continuously raising. This research effort parallels, in particular, the evolution of transportation systems and the continuously increasing requirements and expectations of all types of stakeholders in terms of economic, service-quality, and societal impacts (on the environment, namely), which result in the need for comprehensive planning of activities and resource management. The resulting multi-layer scheduled *Service Network Design with Resource Management* (*SSND-RM*) models extend the research on two-layer network design reviewed in Section 4.1. The formulations represent commodities, resources, and services on particular layers, various design and flow connectivity constraints ensuring relevance and feasibility.

Figure 5 illustrates a number of possible problem settings and design-flow relations. To simplify the presentation, but without loss of generality, we will assume that all layers share the same time representation (discreet or continuous) and the same set of nodes making up the corresponding time-space network representation. Then, using the SSND-RM vocabulary for multi and intermodal freight carriers with consolidation, the figure includes a commodity layer capturing the entry and exit of multi-commodity origin-destination demands, as well as the handling of the commodity flows at the terminals of the system (e.g., loading, sorting, consolidation, waiting; see, e.g., Zhu et al., 2014).

The figure also displays two service layers. The design arcs of the time-space network on each design layer represent the services and their schedules one needs to select to satisfy the demand. More than one service layer is encountered in several settings, e.g., the railroad double consolidation policy (car to blocks and blocks to trains; see, e.g., Zhu et al., 2014) and the possibility of different service classes and tariffs (e.g., Bilegan et al.,

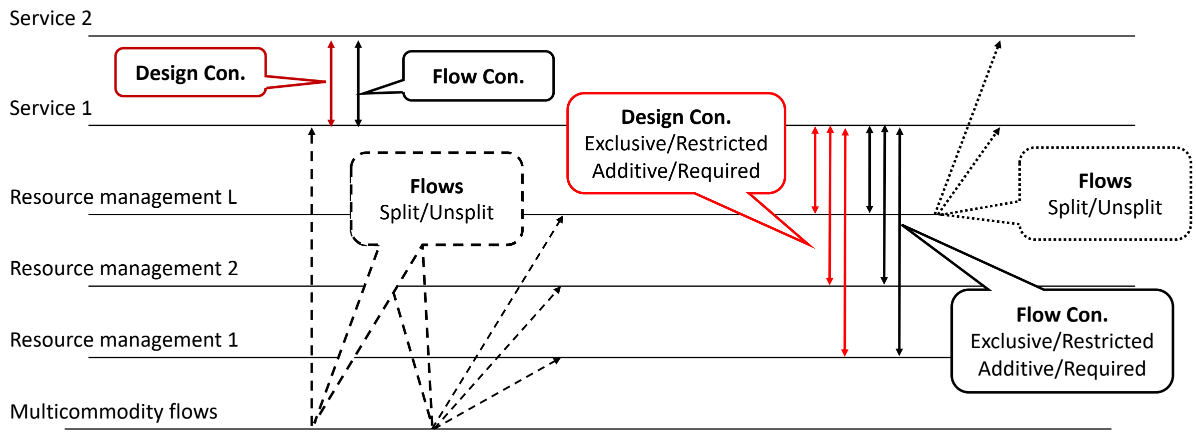

Figure 5: L-layer networks, connectivity types and degrees

2021), or services offered by different carriers (e.g., Crainic et al., 2018; Hewitt et al., 2019). The first setting is a typical case of design connectivity, where a service arc in the supported layer (Service 1 standing for blocks) corresponds to a path in the supporting layer (Service 2 standing for train services). The left-most vertical line connecting the service layers in Figure 5 illustrates this case. No such direct design connectivity exists in the two other cases, service layers being rather coupled indirectly through the flow-connectivity requirements linking the resource or flow layers to the service ones. The figure also displays the flow-connectivity requirements between the two service layers, as well as between these and the commodity layer (leftmost dashed arrow). The later is shown together with the flow-connectivity requirements between the commodity and the resource layers, the figure emphasizing that demand flows may be either split (bifurcated among several paths in the supporting layer) or unsplit (non-bifurcated, single path) when assigned to services or resources.

Many different resources may be involved is such problems, e.g., vessels with different capacities, speeds, and costs (Bilegan et al., 2021) or container rail cars characterized by number of platforms and length (Kienzle et al., 2021). Three resource layers in the figure illustrate the many possibilities and requirements in combining or selecting resources, and relating them to the supporting service layers and to the supported multicommodity demand layer. Note that, when several layers support the same commodity layer, the split/unsplit rule may be specific for each supported-supporting layer pair. Each resource layer is dedicated to managing a particular resource (or class of resources), that is, to optimizing its utilization and circulation given the characteristics of the particular application. This goal translates in most cases in defining selection decisions for each resource and, thus, in setting up a network design problem. The design decisions may involve arcs (Bilegan et al., 2021), paths or cycles (Crainic et al., 2018; Hewitt et al., 2019).

Design-connectivity relations couple the resource and service layers. Illustrated by the three double-arrow links between the resource layers and the Service 1 layer, the figure points to a number of main design-connectivity requirements one may encounter in L-layer networks, e.g., 1) *exclusive*, the resource class may be used on particular service(s) only; 2) *restricted*, the resource cannot be used on the service; 3) *additive*, several resource units of different classes may be used together on the same service, subject to one or several restrictions: capacity (maximum number of resources on a service), relevance (e.g., minimum total power of rail engine manifests), combination (groups of resources classes which may be combined or not), etc.; 4) *required*, the resource is needed for the service to be operated (one needs a pilot to fly a plane). The figure also illustrates, the three right-most double arrows, the flow-connectivity relations which may arise with the management of resources, e.g., when several resource units (railcars) may travel on the same arc, path, or cycle in the service layer (block or train; Kienzle et al., 2021). The resource flows may also be split/unsplit when assigned to a service layer.

6.1. Literature review

The literature on L-layer networks with design-flow connectivity is still scarce. We already referred to the work of Zhu et al. (2014) on the tactical planning problem of consolidation-based freight railroads. Briefly, the authors model the problem as a three-layer SSND problem, where train services are selected on the service layer, blocks are built and selected on the block layer, and the multicommodity car flows are handled on the car layer. The block layer supports the car layer, while the service layer supports the block layer. Design connectivity implies that a chain of services has to be opened in the service (block) layer to open a block (car) arc. Flow connectivity makes the flow on each service (block) equal to the sum of the flows on all its supported blocks (car links). Block flows are unsplit, while commodity flows may be split. The objective is to find a minimum cost block and service design, plus the cost of handling and transport cars, while considering the capacity of blocks, services, and terminals, and the flow and design-connectivity requirements between the layers.

Crainic et al. (2018) extend and generalize the SSND-RM methodology of Crainic et al. (2014) in two ways. First, by considering multiple classes of resources. Second, by including the strategic-tactical decisions of fleet acquisition, assignment, and repositioning. The L-layer time-space network is made up of the service (including outsourcing capabilities), resource classes, and multicommodity demand layers. Extra nodes, and the links connecting them to the first time instants, are used to model the acquisition, assignment, and repositioning decisions. Resource-specific cycles are build for each resource layer out of the potential services on the service layer. Hewitt et al. (2019) built on these ideas to propose an L-layer SSND-RM model explicitly integrating the uncertainty of demand when simultaneously addressing strategic (fleet sizing, acquisition, and allocation and service outsourcing) and tactical planning decisions.

Bilegan et al. (2021) address the issue of integrating revenue management into tactical planning SSND-RM models for intermodal consolidation-based freight transportation systems. The authors consider several customer, service, tariff, and operation classes. The problem includes two resource layers, i.e., two classes of vehicles with different capacities and speeds, plus service and multicommodity layers. Resources move according to cycles built out of service-leg arcs (and waiting-at-terminal arcs). Arc-flow decision variables are defined. One unit resource is to be exclusively assigned to each selected service for all its legs. Regular, contract-based, customer demand must be satisfied, while that of two other customer categories can be selected, if profitable, and serviced in whole or partially depending on the customer type. Commodity flows may be split. The goal is to simultaneously select the extra demand and determine the scheduled service network together with the resource circulation, in order to maximize the total net revenue.

Kienzle et al. (2021) focus on the block-planning problem for intermodal rail transportation, and propose a general modeling framework, which may be used to design the medium-term tactical plan and to adjust a given plan to new information concerning shorter time horizons (e.g., the next week). The time-space service layer includes a set of pre-defined intermodal services together with a set of potential extra services. A continuous time representation is used. The schedules of the services create the time instants of the nodes of the time-space networks on all layers. The commodity layer handles the origin-destination demands (volume, availability at origin, due-date at destination, etc.) for transportation of containers of several types. Several resource layers are considered, each handling the circulation of a specific intermodal car type. The loaded and empty cars are grouped to form blocks, which are then grouped to form trains (similar mechanisms to Zhu et al., 2014). Different from typical freight railroad applications (Chouman and Crainic, 2021), the demand (the containers) must be loaded on and unloaded from rail cars at terminals. Commodity flows may be split among resource types and blocks.

6.2. MLND formualtion

We use the work of Zhu et al. (2014) to illustrate the MLND formulations (Section 2.2) for L-layer network design problems with design-flow connectivity relations. Let $\mathcal{K}$ be the set of a single flow-type commodities. The three layers are the car ($l = 1$), the block ($l = 2$), and the service ($l = 3$), with $\mathcal{C} = \{(3, 2), (2, 1)\}$ connectivity requirements indicating, respectively, that the service layer supports the block layer, which supports the car layer. For each layer $l \in \mathcal{L}$, continuous flow variables x^k_{al} determine the flow of commodity $k \in \mathcal{K}$ on arc $a \in \mathcal{A}_l$. For the car layer, the binary design variables y_{a1} stand for the selection of a car arc or a projected block arc $a \in \mathcal{A}_1$. In the block ($l = 2$) and service ($l = 3$) layers, the binary design variables y_{al} equal 1 if arc $a \in \mathcal{A}_l$ is selected. Let T, $\mathcal{V}$, and $\mathcal{E}$ be the sets of time periods, yards, and track segments, respectively. Let $H(v, t)$ be the set of blocks built simultaneously at yard $v \in \mathcal{V}$, $S(e, t)$, the set of services moved simultaneously on track segment $e \in \mathcal{E}$, and h_v and s_e the maximum numbers of blocks and services that can be built at yard $v \in \mathcal{V}$ and moved on track segment $e \in \mathcal{E}$, respectively. The MLND formulation takes the form

$$\min \quad \sum_{l\in\mathcal{L}} \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}_l} c^k_{al} x^k_{al} + \sum_{l\in\mathcal{L}} \sum_{a\in\mathcal{A}_l} f_{al} y_{al} \tag{27}$$

$$\sum_{a\in A^+_1(n)} x^k_{a1} - \sum_{a\in A^-_1(n)} x^k_{a1} = w^k_n \qquad \forall n \in \mathcal{N}_1, \quad \forall k \in \mathcal{K}, \tag{28}$$

$$x^k_{al} = \sum_{b\in\mathcal{B}^{al}_{l'}} x^k_{bl'} \qquad \forall (l, l') \in C, \quad \forall a \in \mathcal{A}_l, \quad \forall k \in \mathcal{K}, \tag{29}$$

$$\sum_{k \in \mathcal{K}_l} d^k x^k_{al} \leq u_{al} y_{al} \qquad \forall l \in \mathcal{L}, \quad \forall a \in \mathcal{A}_l, \tag{30}$$

$$y_{a3} \leq \sum_{b \in \mathcal{B}_2^{a3}} y_{b2} \leq v_{a3} y_{a3} \qquad \forall a \in \mathcal{A}_3, \tag{31}$$

$$y_{bl'} \leq y_{al} \qquad \forall (l, l') \in \mathcal{C}, \quad \forall a \in \mathcal{A}_l, \quad \forall b \in \mathcal{B}^{al}_{l'}, \tag{32}$$

$$\sum_{a \in H(v,t)} y_{a2} \leq h_v \qquad \forall v \in \mathcal{V}, \quad \forall t \in \mathcal{T}, \tag{33}$$

$$\sum_{a \in S(e,t)} y_{a3} \leq s_e \qquad \forall e \in \mathcal{E}, \quad \forall t \in \mathcal{T}, \tag{34}$$

$$x^k_{al} \geq 0 \qquad \forall l \in \mathcal{L}, \quad \forall a \in \mathcal{A}_l, \quad \forall k \in \mathcal{K}, \tag{35}$$

$$y_{al} \in \{0, 1\} \qquad \forall l \in \mathcal{L}, \quad \forall a \in \mathcal{A}_l. \tag{36}$$

The objective function (27) minimizes the total design and transportation costs on the three layers. Constraints (28) guarantee that the demands are routed on the car layer, while constraints (29) compute the flow on each arc of the block and service layers based on the corresponding arcs of the car layer. Linking constraints (30) ensure that the flow on each arc is less than or equal to its capacity when selected. Constraints (32) ensure that to open an arc in the car (block) layer, all the corresponding block (service) arcs must be opened in the block (service) layer. Design-connectivity constraints (31) limit the number of blocks that can be moved on the corresponding arc of the service layer. Constraints (33) and (34) limit the number of blocks and services to be created at each yard and track segment, respectively.

7. Solution Approaches for Multilayer Network Design Problems

MLND belongs to the network design class and, thus, the solution methods developed for the latter may be applied to the former. Indeed, as the following survey shows, this is what most contributions in the literature propose, and most of the research taking advantage of the MLND problem structure is yet to come. We start with the exact methods in Section 7.1 and then move to heuristic approaches in Section 7.2.

7.1. Exact Solution Methods

Crainic and Gendron (2021) present a comprehensive overview of exact solution methods for network design, including relaxations and reformulations, valid inequalities, enumeration algorithms, Benders decomposition, connections to heuristics, and parallel algorithms. The methods proposed in the MLND belong to the first four categories (we address matheuristics in Section 7.2; no parallel exact algorithm for the MLND yet).

Dahl et al. (1999) proposed a Branch-and-Cut (*B&C*) algorithm for a two-layer telecommunications network design problem. Classical network-design valid inequalities (VIs) are added to the linear programming *(LP)* relaxation at each node of the B&C

tree, a variable fixing heuristic being called when no violated inequality is found. A similar approach is followed by Koster et al. (2008) for the two-layer telecom problem with survivability requirements against physical node and link failures. The authors observed that the survivability requirements increase the problem size dramatically and that in this case, the cutting plane algorithm only slightly improves the LP relaxation lower bounds. Further work along this line may be found in Mattia (2012), as well as in Mattia (2013); Taktak (2015), which focus on the of the two-layer network design formulation Raack and Koster (2009) followed a different approach in their study of a two-layer problem. The authors derive a bin packing problem to prove the NP-hardness of the problem and define two classes of facet-defining inequalities, which generalize the well-known cutset inequalities to two-layer network design.

It is noteworthy that, on the one hand, many applications in the MNDL literature involve routing (e.g., telecom messages) or scheduling (e.g., crews), path-based formulations being attractive in such cases. Multi-layer service network design problems, on the other hand, involve design-connection requirements involving an arc on a layer and a path or cycle on another. Not surprisingly, therefore, relaxation and column generation to solve the Dantzig-Wolfe reformulation appear prominently in the MLND literature. A first group of contributions are built around Branch-and-Price (*B&P)* and Branch-and-Price-and-Cut (*B&P&C*) algorithms. Cacchiani and Salazar-González (2016) introduced B&P&C for their arc-path and path-path two-layer models, introducing a bounding cut which accelerates the solution processes. Column generation is used for the path formulation on the crew layer. The authors show the superiority of the arc-path formulation. Andersen et al. (2011) also propose a B&P&C algorithm for a two-layer SSND-RM, where the selection of services is included into the resource-cycle selection, yielding a service-resource layer, and demand is moved on the commodity layer through commodity-specific paths built on the service arcs making up the cycles. The algorithm integrates two column-generation subproblems, for integer cycle design and continuous commodity path flows, as well as a number of branching strategies, a mechanism to dynamically add violated strong linear relaxation cuts, and an acceleration upper-bound identification technique. It performs very satisfactorily for instances of moderate dimensions, outperforming a well-known commercial software.

Solution methods based on Benders decomposition make up the second large group of contribution. Knippel and Lardeux (2007) proposed a classical Benders decomposition algorithm for a two-layer telecom MLND, where the master problem handles the design variables, two subproblems verifying the feasibility of the design for the logical layer and physical layers. The authors observed that solving the master problem was the most computing-intensive part and examined approaches for the generation of cuts. The contributions that followed in this line of research aimed to accelerate Benders, first by embed it into a B&C algorithm. Thus, the algorithm of Fortz and Poss (2009) solves the LP relaxation of the master problem at each node of the enumeration tree. It then adds the corresponding Benders cuts when the solution is integral, otherwise, it generates branches out of the node and adds the branching constraints to the master problem. VIs derived from the LP relaxation are added at the root node to, hopefully, reduce the size of the tree. Orlowski (2009); Orlowski et al. (2010) built of these contributions and proposed to combine Benders decomposition and column generation, to generate flow variables dynamically in the large-scale routing subproblems, within a B&P&C for a MLND problem with survivability requirements. The algorithm could find feasible solutions and lower bounds for instances that could not be solved by a commercial solver, but optimality gaps are still very high (57% and 28% on average for the instances with and without the survivability conditions, respectively) for large and dense instances.

Similar developments were proposed for the integrated passenger airline planning. Cordeau et al. (2001) embedded Benders decomposition into a B&P&C algorithm. The LP relaxation at each node is solved by Benders decomposition, and both the Benders master problem and the crew-scheduling subproblem are solved using column generation. Experimental results on instances derived from real data show that the algorithm yields significant savings in comparison to the traditional sequential approach. Mercier et al. (2005) followed the same methodology, but studied the importance of deciding what variables go into the master problem and which ones in the subproblem. They thus show that defining crew scheduling as the master problem and the aircraft routing as the subproblem yields outperfoems the reversed decomposition (used in Cordeau et al., 2001). The authors also show that Pareto-optimal cuts accelerate the convergence of Benders decomposition. Shao et al. (2015) returned to the crew-scheduling as subproblem, but added several acceleration techniques, and a stabilization technique for the column generation procedure. The resulting algoirhtm performed very well on real-world data obtained from a U.S.-based airline carrier. We conclude this part recalling the work of Zeighami and Soumis (2017) for the three-layer MLND model of the integrated crew pairing and assignment problem. The authors built on the previous contributions. The pairings are generated by the Benders master problem, while the schedules for pilots and copilots are generated by the Benders subproblems. Master and subproblems are solved by column generation.

7.2. Heuristic Solution Methods

Network design problems are complex and computationally difficult in all but the most trivial cases Crainic et al. (2021b). Multi-layer problems are no different. It is thus not surprising that heuristic methods are proposed, aiming to identify good-quality solutions within acceptable computation efforts for as large as possible instances. Crainic and Gendreau (2021) present a comprehensive overview of heuristic solution methods for network design problems, declined according to four main classes defined by the complexity (or the refinement) of the heuristic search: 1) *classical heuristics*, which rely on fairly simple rules to build and improve tentative solutions; 2) *metaheuristics*, which rely on sophisticated search strategies to derive very good (often near-optimal) solutions to the problem at hand; 3) *matheuristics*, which combine algorithmic components from metaheuristics with procedures derived from exact methods applied to the model formulation; 4) *parallel meta- and matheuristics*, which leverage the power of parallel computing to broaden and enhance the search and, thus, find better approximate solutions.

The methods described by Crainic and Gendreau (2021) may be extended, more or less directly, to multi-layer problem settings, as may be observed in most of the rather restricted number of contributions one may find in the literature. The work of Capone et al. (2007), for a two-layer telecommunications network design problem with node capacity and multicast traffic illustrates such an approach, where network-flow-based greedy and neighborhood-based Local Search (LS) heuristics are deployed on each layer, while a rather simple mechanism alternates between the two until a local optimum is reached. The idea of working separately on each layer, with simple coordination mechanism, is also to be found in Salazar-González (2014), which propose a two-phase heuristic solution method for the integrated crew scheduling and aircraft routing problem. The first phase is a greedy search on the crew layer to find sufficient pairings to cover all the flights. The pairings are then combined to yield aircraft routes.

Applying matheuristic concepts to MLND raises again the question of how to address the issue of inter-layer connectivity characteristics and layer-specific activities. The work on slope-scaling-based methodology (*SS*; Kim and Pardalos, 1999; Crainic et al., 2004; Kim et al., 2006) offers a perspective on possible approaches to this challenge. A straightforward approach of SS to MLND is to linearize the design variables on all layers simultaneously and address the resulting multi-commodity minimum-cost network flow problem either directly or by using a heuristic based on the idea of shortest augmenting-paths (on the residual-capacity network). This is approach proposed by Zhu et al. (2014) as the basic matheuristic for the integrated freight railroad service network design problem performed very well, outperforming a well-known MIP commercial solver on more than 90% of a set of small to medium-size instances based on the main-line network setting of a major North American railroad. The authors introduce a more sophisticated approach, taking advantage of the flow and design connectivity requirements of the MLND problem, to address the challenge of the large dimensions of the potential sets of design variables, which are paths in the time-space SSND networks. One layer (block selection) is thus "projected" on the defining one (service selection) and a Tabu Search-based matheuristic to dynamically generates blocks using this reformulation. Network-flow-based procedures are proposed to restore feasibility and to intensify the search around promising solutions, while long-term-memories guide the perturbation of the linearization parameters. This enhanced SS-based matheuristic outperformed a commercial software on smal to medium-size instances, and obtained very good solutions for larger settings.

Crainic et al. (2014, 2018) expanded this methodological idea of projecting one layer on the defining one and dynamically generating the design variables on the projected layer, and applied it to general deterministic and stochastic SSND-RM problems with several resource layers and cycle representations of resource activities. The dynamic generation of attractive new resource cycles is performed by shortest-path-inspired methods, based on the flows and reduced-costs yielded by the linearized approximation problem. Cycles are generated at several stages of the solution method, both to enrich the linearized approximation problem and when restoring feasibility y solving restricted network flow problems. The matheuristic performed very well, including when demand uncertainty is explicitly accounted for (Hewitt et al., 2019).

The problem-decomposition strategies proper of exact solution methods may be also used to build matheuristics. Lagrangian relaxation is such an approach, which can be used to separate the layer-specific problems and decompose the overall optimization. Belotti et al. (2008) illustrates this approach with a matheuristic for a two-layer MLND with design connectivity in telecommunications. Lagrangian relaxation is used to relax the virtual flow capacity constraints, disconnecting the layers and providing the means to decompose the problem into shortest path subproblems for each commodity, plus one capacity assignment subproblem to determine the design variables. Column generation is used to address the capacity assignment subproblem, while a subgradient method is applied to find the Lagrangian lower bound. The matheuristic then follows the Local Search idea, starting an initial solution and tring to improve it by rerouting the commodities on the virtual links and rerouting the virtual capacities on the physical links.

8. Conclusions and Perspectives

Multilayer network design represents an important class within the well-known network design combinatorial optimization field, as illustrated by applications to major applications domains, including transportation and telecommunications, as well as by the methodological challenges brought by the added complex problem structure.

This paper presents what we believe to be the first integrated study of the topic. It introduces a formal MLND problem definition, together with a general problem formulation, identifying main classes of connection requirements among the layers representing the various design and flow management decisions of the problem. It also structures the first version of a MLND taxonomy, emphasizing the multilayer features of the problem, in particular, the number of layers as well as the degree and type of connectivity among layers.

This taxonomy provides the means to synthesize the multilayer network design contributions found in the transportation and telecommunications literature relative to modeling and solution-method developments. The survey emphasizes the generality and applicability of the modeling framework we propose to the problem settings reviewed and possible extensions. It also offers a first appreciation of what the research achieved in the field, of the topics less studied up to now and, thus, of interesting research perspectives.

One first notices that, the transportation and telecommunication application fields yield very varied problem characteristics and modeling challenges. The integrated planning of transportation systems, airline passenger and consolidation-based freight transport mainly, are particularly active in this respect, yielding several design and resource layers with complex interconnection requirements. Most contributions address 2-layer MLND problems, however, the majority addressing cases with design-connectivity requirements. Few involve flow-connectivity requirements, none treating the case with design-flow connections. More research in this direction appears highly desirable. Another desirable research direction would apply and enrich this taxonomy and general model to other application areas, e.g., power system design, risk evaluation, and artificial neuronal networks.

The classification and survey point toward the need for a systematic research effort addressing MLND with more that two layers with a large pallet of design and flow connectivity requirements and constraints. Research should address modeling issues, e.g., how to combine efficiently (in terms of problem representation and impact on solution methods) various connectivity relations and constraints. It should also study the properties of the resulting mixed-integer formulations and derive meaningful insights for the development of efficient solution methods.

Indeed, the survey shows that the scope of the literature regarding solution methods for MLND is still very limited and many exact, heuristic, and matheuristic research avenues are wide open for exploration. The research avenues identified for general network design problems (Crainic and Gendron, 2021; Crainic and Gendreau, 2021) are, of course, highly relevant for multi-layer problem settings also. Exploiting the multi-layer structure of MLND problems offers, however, particular fascinating and challenges research avenues for the development of both exact and heuristic solution methods.

Exploring Lagrangian relaxation and decomposition methods for MLND with varying number of layers and complex connectivity requirements appears as a first promising but challenging research avenue. Taking advantage of the MLND problem structure to derive more and tighter valid inequalities appears as equally promising and challenging. Both lines of research combine in developing efficient Benders decomposition-based, B&C, and B&C&P algorithms. This latter research avenue is particularly challenging as the problem dimensions grow, both in the classical terms of number of arcs and, especially, commodities, and in the particular aspect of number of layers and connectivity requirements. We believe that solving medium-scale instances "exactly" will require a combination of relaxation, decomposition, and enumeration methods within a parallel optimization framework.

For larger problem instances, as well as to obtain good-quality solutions in relatively reduced computational efforts, the field needs a significant research effort to develop meta and matheuristics. Combining decomposition, metaheuristic search, and exact solution approaches applied to restricted subproblems appears particularly attracting. This is a vast area, however, and we point to a few major approaches only, to illustrate what could be successfully combined: Lagrangian relaxation and decomposition to separate the layers, dynamic time-space network generation, column generation, and integrative (parallel) cooperative search. We hope to report on some of these advancements in the near future.

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Appendix A. Telecommunication network design

Two layers are generally encountered in telecommunications networks, a *virtual* or *logical layer*) and a *physical layer*, or optical transport network. A network has to be designed in the logical layer to satisfy the multicommodity demand. Several links have to be opened, or facilities have to be installed, between different pairs of nodes of the physical network to support the logical network.

Figure A.6 illustrates a two-layer telecommunication network. Link a1 in the virtual layer corresponds to links b1 and b2 in the physical layer, and link a2 corresponds to links b3, b4, and b5. To open or install facilities on a link in the logical layer, all corresponding links in the physical layer have to be opened or need to have appropriate facilities. For example, to open link a2, all arcs b3, b4 and b5 have to be opened.

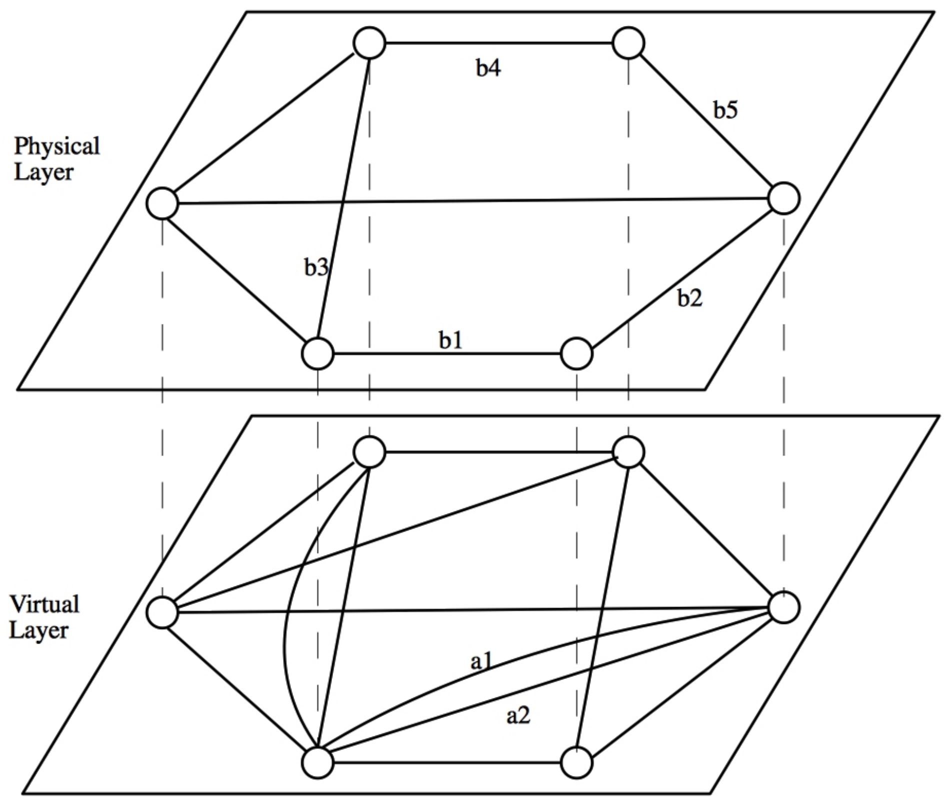

Figure A.6: A two-layer telecommunications network

We use the problem description of Dahl et al. (1999) to illustrate the MLND formulation (Section 2.2) of the two-layer network design with design connectivity telecommunications applications.

Let $\mathcal{L} = \{1, 2\}$ be the set of layers including the virtual layer ($l = 1$) and the physical layer ($l = 2$). Let $\mathcal{C} = \{(2, 1)\}$ be the set of connectivity requirements, where the ordered pair $(2, 1)$ means that the physical layer is supporting the virtual layer. Let again $\mathcal{K}$

be the set of single flow-type commodities to be routed on the virtual layer and d^k be the demand of each commodity $k \in \mathcal{K}$. Let c^k_{a1} be the flow cost of routing one unit of commodity $k \in \mathcal{K}$ on arc $a \in \mathcal{A}_1$, and f_{al} be the fixed cost of opening arc $a \in \mathcal{A}_l, l \in \mathcal{L}$. The binary flow variable x^k_{a1} determines if the demand of commodity $k \in \mathcal{K}$ flows on the virtual arc $a \in \mathcal{A}_1$. The binary design variables y_{a1} and y_{a2} determine, respectively, if the virtual arc $a \in \mathcal{A}_1$ and the physical arc $a \in \mathcal{A}_2$ is open. The problem can then be formulated as the following MLND model:

$$\min \quad \sum_{k\in\mathcal{K}} \sum_{a\in\mathcal{A}_1} c^k_{a1} x^k_{a1} + \sum_{l\in\mathcal{L}} \sum_{a\in\mathcal{A}_l} f_{al} y_{al} \tag{A.1}$$

subject to

$$\sum_{a\in\mathcal{A}_1^+(n)} x^k_{a1} - \sum_{a\in\mathcal{A}_1^-(n)} x^k_{a1} = w^k_n \qquad \forall n \in \mathcal{N}_1, \quad \forall k \in \mathcal{K}, \tag{A.2}$$

$$\sum_{k\in\mathcal{K}} d^k x^k_{a1} \leq u_{a1} y_{a1} \qquad \forall l \in \mathcal{L}, \quad \forall a \in \mathcal{A}_1, \tag{A.3}$$

$$y_{a2} \leq \sum_{b\in\mathcal{B}_1^{a2}} y_{b1} \leq v_{a2} y_{a2} \qquad \forall a \in \mathcal{A}_2, \tag{A.4}$$

$$x^k_{a1} \in \{0,1\} \qquad \forall a \in \mathcal{A}_1, \quad \forall k \in \mathcal{K}, \tag{A.5}$$

$$y_{al} \in \{0,1\} \qquad \forall a \in \mathcal{A}_l. \tag{A.6}$$

The objective function (A.1) minimizes the total cost including the total routing cost on the virtual layer and the summation of the design costs of the virtual and physical layers. Constraints (A.2) are the flow conservation equations that ensure demand is satisfied in the virtual layer. Flow capacity constraints (A.3) are imposed for the virtual layer. Design capacity constraints (A.4) are coupling constraints ensuring that, to open an arc in the virtual layer, the supporting arcs in the physical layer should be open and that the maximum number of selected virtual arcs is limited to the design capacity of the corresponding physical arc. These constraints correspond to the design capacity constraints (7) of the general modeling framework Constraints (A.5) and (A.6) define the feasible domains of the decision variables. In telecommunications applications, the flow variables are binary, to ensure that the flow of each commodity follows a single path from the origin to the destination. Note that, when the links are undirected, the problem can still be modeled using the proposed formulation by replacing an undirected link with two directed arcs.

Appendix B. Three Layers & 1toM Flow Connectivity for Crew Pairing & Assignment

Zeighami and Soumis (2017) address an integrated crew pairing and personalized assignment problem for a given set of pilots and copilots, considering sets of vacation requests (VRs) for each pilot and copilot. The problem is defined on a three-layer network including 1) a crew-pairing layer, 2) a pilot-assignment layer, and 3) a copilot-assignment layer. Nodes in the crew pairing network correspond to departure and arrival flight stations. The arcs represent the flights and the connections between the flights. Nodes in the pilot and copilot-assignment networks correspond to the start and end of pairings, the arcs representing the pairings, the connections between pairings, and the VRs of the pilots and copilots, respectively. The objective function aims to find a trade-off between maximizing the number of satisfied VRs and minimizing the total cost of the pilot and copilot pairing assignments (one path for each pairing and one path for each pilot and copilot assignment), while two main conditions are satisfied: 1) each flight is covered by exactly one pairing, and 2) each pairing is covered by exactly one pilot and one copilot assignment.

To model the problem using the general modeling framework of Section 2.2, we let $\mathcal{L} = \{1, 2, 3\}$ be the set of the crew pairing ($l = 1$), pilot assignment ($l = 2$), and copilot assignment ($l = 3$) layers. $\mathcal{G}_l = (\mathcal{N}_l, \mathcal{A}_l)$ defines the network of each layer $l \in \mathcal{L}$. We partition the arcs in the crew-pairing layer into the sets of flight arcs $\mathcal{A}_1^f$ and connection arcs $\mathcal{A}_1^c$. We assume that all potential pairings exist in the pilot and copilot-assignment layers. Let $\mathcal{K}_2$ and $\mathcal{K}_3$ be the sets of pilots and copilots, respectively, which need to be routed in their associated assignment layers. $O(k) \in \mathcal{N}_l$ and $D(k) \in \mathcal{N}_l$ are the origin and the destination, respectively, of each crew $k \in \mathcal{K}_l$ in layers $l \in \{2, 3\}$.

Let $\mathcal{C} = \{(1, 2), (1, 3)\}$ be the set of connectivity requirements, where $(1, 2)$ and $(1, 3)$ mean that, respectively, the pilot and copilot-assignment layers are supported by the crew-pairing layer. To use a pairing arc in an assignment layer, all the corresponding arcs need to be selected in the crew-pairing layer. Let $\mathcal{B}_2^{a1}$ and $\mathcal{B}_3^{a1}$ be the sets of arcs (pairings) in the pilot and copilot layers, respectively, which are supported by arc $a \in \mathcal{A}_1$ in the pairing layer. Binary flow variable y_{a1} determines whether arc $a \in \mathcal{A}_1$ is selected or not. Binary flow variables x_{a2}^k and x_{a3}^k determine whether or not, respectively, pilot $k \in \mathcal{K}_2$ and copilot $k \in \mathcal{K}_3$ selects arc $a \in \mathcal{A}_2$ and $a \in \mathcal{A}_3$. The model may then be written as:

$$\min \quad \Psi(x, y) \tag{B.1}$$

subject to

$$\sum_{a \in A_2^+(n)} x_{a2}^k - \sum_{a \in A_2^-(n)} x_{a2}^k = w_n^k \qquad \forall n \in \mathcal{N}_2, \quad \forall k \in \mathcal{K}_2, \tag{B.2}$$

$$\sum_{a \in A_3^+(n)} x_{a3}^k - \sum_{a \in A_3^-(n)} x_{a3}^k = w_n^k \qquad \forall n \in \mathcal{N}_3, \quad \forall k \in \mathcal{K}_3, \tag{B.3}$$

$$\sum_{k \in \mathcal{K}_2} x_{b2}^k \leq y_{a1} \qquad \forall a \in \mathcal{A}_1, \quad \forall b \in \mathcal{B}_2^{a1}, \tag{B.4}$$

$$\sum_{k \in \mathcal{K}_3} x_{b3}^k \leq y_{a1} \qquad \forall a \in \mathcal{A}_1, \quad \forall b \in \mathcal{B}_3^{a1}, \tag{B.5}$$

$$\sum_{b \in \mathcal{B}_2^{a1}} \sum_{k \in \mathcal{K}_2} x_{b2}^k = 1 \qquad \forall a \in A_1^f, \tag{B.6}$$

$$\sum_{b \in \mathcal{B}_3^{a1}} \sum_{k \in \mathcal{K}_3} x_{b3}^k = 1 \qquad \forall a \in A_1^f, \tag{B.7}$$

$$x_{al}^k \in \{0,1\} \qquad \forall l \in \{2,3\}, \quad \forall a \in \mathcal{A}_l, \quad \forall k \in \mathcal{K}_l, \tag{B.8}$$

$$y_{a1} \in \{0,1\} \qquad \forall a \in \mathcal{A}_1, \tag{B.9}$$

$$x \in X. \tag{B.10}$$

The objective function (B.1) minimizes the total routing and design costs on the three layers. Pilot flow conservation equations (B.2) guarantee the routing of each pilot $k \in \mathcal{K}_2$ in the second layer, while constraints (B.3)enforce the same conditions in the copilot layer, with $w_n^k = 1$ if $n = O(k)$, $w_n^k = -1$ if $n = D(k)$, and 0 otherwise. Constraints (B.4) and (B.5) are the coupling constraints ensuring the flow connectivity between layers. Constraints (B.6) and (B.7) are the covering constraints which ensure each flight arc is covered by exactly one pairing. Constraints (B.8) and (B.9) define the domain of the decision variables. Side constraints (B.10) capture problem-specific aspects, including those corresponding to the person