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Combinatorial Bids Generation in Truckload Transportation Procurement Auctions with Uncertainty on Clearing Prices, Bids Success and Contracts Materialization

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Abstract. We address a new variant of the Bid Construction Problem (BCP) with stochastic clearing prices in combinatorial auctions for the procurement of truckload transportation service that tackles uncertainty on bids success and auctioned contracts materialization. We propose the first exact method to generate multiple nonoverlapping OR bids while ensuring a non-negative profit for the carrier regardless of the auction outcomes and contracts materialization. Our exact method iteratively solves a relaxed then a restricted version of the BCP with stochastic clearing prices while checking all routes generated in each optimal solution and dynamically adds cuts to forbid what we define as risky routes. Risky routes are the ones that could yield a negative profit if some bids are not won or some contracts do not materialize later. Our experimental study highlights the good performance of our exact approach and points out the potential gains in profits yielded by our new variant compared to a standard BCP.

Keywords: Combinatorial auctions, transportation procurement, Bids generation, Stochastic prices, contracts materialization, Bids success

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1 Introduction

Combinatorial Auctions (CA) are mechanisms where one can bid on packages of items rather than on each item separately. Combinatorial bids become particularly interesting when the value of a set of items for the bidder is larger than the sum of the values of the items (De Vries and Vohra, 2003). This is the case for transportation markets where the value of a shipment for a carrier depends on whether other shipments are won. Past research has already well established the important cost savings achieved by both shippers and carriers when CA are used as a trading mechanism for transportation services procurement (Caplice and Sheffi, 2006; Karaenke et al., 2019).

Our paper addresses the so-called Bid Construction Problem (BCP) – also known as the bid generation problem – for reverse one-sided CA for truckload (TL) transportation services procurement. We consider a CA where the shipper acts as the auctioneer and carriers compete by submitting combinatorial bids on the shipper's transportation requests (referred to as contracts in the following). In each bid, the carrier specifies the set of transportation contracts to serve and the associated ask price. If the bid is won, all the contracts submitted in the package are allocated to the carrier, otherwise none of the contracts is allocated.

A BCP must be solved by each carrier participating into the auction. When generating combinatorial bids, the carrier must take into account its existing commitments, its operational constraints but also other competing carriers bids. The latter are generally not known by the carrier when solving the BCP. The carrier rather relies on its experience and knowledge of its competitors to evaluate the clearing price associated with auctioned contracts. Generally, contracts clearing prices cannot be known with certainty. To the best of our knowledge only Triki et al. (2014) and more recently Hammami et al. (2021) addressed a BCP with stochastic clearing prices.

Moreover, the majority of published papers in transportation CA propose solution approaches for the BCP so that a single combinatorial bid is generated. The BCP should then be run many times to generate multiple bids with no guarantee that the package of contracts covered by these bids do not overlap. However, when determining the winning carriers, the shipper solves the so-called Winner Determination Problem (WDP) in which each auctioned contract is generally assigned to a single carrier's bid, making non-overlapping bids mandatory for a carrier to win multiple bids. Besides, depending on the auction rules and the bidding language used, a bidder may be permitted to submit either *XOR* bids, *OR* bids or a combination of *OR* and *XOR* bids (Nisan, 2010). *XOR* bids imply that at most one of the submitted bids can be allocated to the carrier. On the opposite, *OR* bids enable a carrier to express its preferences for different subsets of auctioned contracts with the possibility to win all or a part of them.

The few papers assuming *OR* bidding consider deterministic clearing prices (Ben Othmane et al., 2019; Hammami et al., 2019). To the best of our knowledge, the paper by Hammami et al. (2021) is the sole to propose an exact method for a BCP with stochastic prices where multiple non-overlapping *OR* bids can be generated. They determine package bids and their associated prices so as to maximize a global expected profit assuming that all the generated *OR* bids would be won and all the contracts put on the auction by the shipper would materialize. Our paper extends this work by proposing an exact method for the BCP with stochastic clearing prices that ensures a non-negative profit for the carrier regardless of the *OR* bids effectively won, and independently of the auctioned contracts that effectively materialize.

Since 2006, Caplice and Sheffi (2006) have already pointed out the uncertainty surrounding auctioned contracts materialization. The authors reported that from a shipper's perspective, there is uncertainty on the realization of auctioned contracts since it is difficult to forecast the demand and the freight volume given that they are highly disaggregated. They stated that the shippers have "*the right but not the obligation*" to use the carriers as determined through the auction process. Thus, the carrier may be awarded a transportation contract at the strategic auction stage that does not materialize at the operational stage. This may yield monetary losses for the carrier and reduce the benefits attributable to combinatorial bidding. Contracts materialization can also be affected by sudden unexpected events: during the Coronavirus pandemic, transportation businesses in the United States faced a decline in the active business linked to the permanent shutdown of many shopping business following the COVID-19 restrictions (Kalogiannidis, 2020; Fairlie, 2020). This market turmoil, which has a negative impact on carriers, is the result of a simultaneous disturbance in both supply and demand Ivanov (2020).

The approach proposed in our paper enables a carrier to submit multiple combinatorial *OR* bids for which the packages of covered contracts do not overlap such that the carrier has enough capacity to serve all of them. More importantly, given that the carrier is ensured to have a non-negative profit even if not all of its submitted *OR* bids are won, it can adopt an aggressive pricing strategy by submitting large ask prices while taking into account uncertainty on contracts clearing prices. Finally, with the proposed approach, uncertainty on contracts materialization is also handled by ensuring a non-negative expected profit for the carrier even if the contracts won at the auction are not allocated during operations. To the best of our knowledge, our paper is the first to simultaneously address all these issues while providing an exact method to handle this complex problem.

Beyond the good computational performance of the proposed exact method, our experimental study evaluates the merits/drawbacks of incorporating the risks elimination feature on the carrier expected and real profits. This is done by comparing two contexts: one where the risks associated with bids success and contracts materialization are eliminated, referred to as a risk-averse bidding, and another where such risks are permitted, referred to as a risky bidding. We evaluate both contexts with two pricing strategies: a conservative and an aggressive one. Unlike the conservative pricing strategy, the aggressive one fixes bids ask prices to relatively large values with regard to bids clearing prices. Our results show that even though at first glance a risk-averse bidding context requires larger CPU times to generate no-risky bids and yields lower maximum expected profits when compared to a risky bidding context, it generally results in larger real profits when different scenarios of bids success are simulated. It is also noteworthy that the decrease in the maximum expected profit when changing from the risky bidding context to the risk-averse one is observed when the same pricing strategy is adopted. These results should thus be interpreted with some restraint since each bidding context should be combined with an appropriate pricing strategy to illustrate more realistically a bidding behavior of a carrier. In practice, a moderately risky behavior consists in either: (i) adopting a risky bidding (as traditionally addressed in the literature) coupled with a conservative pricing strategy to increase the chance of winning the submitted bids but with the risk to incur monetary losses if the bids are not won, or (ii) using a risk-averse bidding (our new approach) with an aggressive pricing strategy yielding substantially larger profits but with less certainty that the price asked in the submitted bids is sufficiently low to win the submitted bids but at the same time knowing that no monetary losses will occur if bids are not allocated. When comparing the maximum expected profits yielded by these two alternatives, the second one (based on our new method) yields larger maximum profits for almost all the instances with an average relative increase of 42.57% and a maximum increase of 96.60%.

The remainder of this paper is organized as follows. Section 2 presents a literature review on the BCP for TL transportation services procurement CA and some of the recent related works. In Section 3, we define and mathematically model the BCP with stochastic clearing prices. Section 4 describes our approach to handle risks on contracts materialization and bids success. Section 5 details our exact solution method. In Section 6, we report and analyze the results of our experimental study. Section 7 concludes the paper and offers insights for future research.

2 Literature review

In recent years, several works studied the BCP for TL transportation services procurement, but few of them addressed a stochastic context. In the following, we review the major works addressing the BCP in a deterministic context and then in a stochastic one.

Song and Regan (2003) presented an optimization-based approximation model to help the carrier identify the most profitable auctioned contract packages. The BCP is formulated as a set partitioning problem where the objective is to select the set of routes minimizing the carrier's empty traveling costs. Later, Song and Regan (2005) addressed a BCP where two scenarios are considered: with and without pre-existing contracts. The problem is also formulated as a set partitioning problem then relaxed as a Set Covering Problem (SCP) where the objective is to minimize empty traveling costs. The SCP is solved by a commercial solver and near-optimal solutions are obtained for instances with up to 10 auctioned contracts.

Wang and Xia (2005) addressed a BCP where the objective function is to minimize the empty traveling costs. Two heuristics are proposed. The first one is based on a fleet assignment model which is a generic formulation for the routing and scheduling problem. The second heuristic is based on the nearest insertion method. The two heuristics are tested and compared on instances with up to 30 auctioned contracts.

Lee et al. (2007) proposed a quadratic formulation for the BCP that integrates routes generation and selection where the objective function is to maximize the carrier's profit defined as the difference between the revenue from serving contracts minus the traveling costs. In their model, the authors assumed that each carrier generates only one package bid. The proposed solution approach is a heuristic based on column generation and Lagrangian relaxation. Experiments were run on instances with up to 335 auctioned contracts. Reported results show a relatively large variation in the optimality gap (between 0.00% and 32.84%) for large instances accompanied with relatively large computational times (reaching up to 120 hours).

Chang (2009) proposed a *bidding advisor* which helps carriers evaluate the most profitable bid packages. The developed tool consists on converting the BCP into a synergistic minimum cost network flow problem by estimating the average synergy values between the auctioned contracts. According to the author, the main contribution of the proposed approach is that it can easily determine the desirable bid packages without evaluating all $2^n - 1$ possible bid packages where n denotes the number of auctioned contracts. A column generation method with a synergistic shortest path algorithm is proposed to solve the problem and generate bid packages. A bid is simply formed by the auctioned contracts served by the same vehicle.

Chen et al. (2009) proposed a method allowing carriers to implicitly consider the complete set of all possible bids without evaluating their exponential number. This approach requires that each carrier submits a Bid Generation Function (BGF) which will be embedded directly into the shipper's WDP. The BGF underlines the minimal cost to serve a set of transportation contracts and is obtained by solving a minimum cost flow problem. The main advantage of the proposed approach is that it allows the carrier to submit its BGF parameters instead of computing and submitting an exponential number of bids. However, this approach forces the carrier to share a substantial amount of information concerning its network structure, its transportation costs and how auctioned contracts valuation is made.

Rekik et al. (2017) proposed a route-based model for the BCP with two types of business constraints related to CA. The first one limits the total number of contracts covered by the generated bids. The second one limits the number of contracts submitted in each bid. To solve the problem, they proposed a branch-price-and-cut algorithm. The exact solution approach was tested on a set of 20 instances with up to 131 auctioned contracts. Computational results showed that optimal solutions are obtained within 730 seconds.

Ben Othmane et al. (2019) addressed the BCP with pre-existing routes where the carrier's objective is to optimize its operations by integrating profitable auctioned contracts. The authors proposed a three-stage heuristic to solve the problem and tested it on instances with up to 600 contracts. The proposed approach identifies profitable contracts then inserts them into the predefined routes and/or build new ones for unused vehicles. Ben Othmane et al. (2019) report feasible solutions within relatively short computational times however these solutions were not compared to optimal solutions for most instances given the problem size.

Hammami et al. (2019) were the first to consider a BCP with a heterogeneous fleet where the objective is to maximize the carrier's profit. The authors developed two approaches to solve the problem. The first one is a hybrid adaptive large neighborhood search heuristic based on a destroy-repair principle coupled with several local search procedures and hybridized with a set packing problem. The second one is an exact branch-and-cut which starts from the best heuristic solution to solve an arc-based formulation of the BCP. Computational results on different sets of small and large-scale instances with up to 500 contracts show that the proposed solution methods perform well in terms of computational times and provide optimal or near-optimal solutions.

Only two papers address the BCP with stochastic clearing prices. In Triki et al. (2014), the problem is formulated as a probabilistic mixed integer optimization model where chance constraints are used to model the probability of winning bids. The model generates a single combinatorial bid and requires enumerating all possible package bids so that chance constraints could be formulated making it intractable to solve to optimality. To solve the problem, the authors rather developed two heuristics and tested them on instances with up to 400 auctioned contracts. The quality of the solutions obtained is not reported.

Recently, Hammami et al. (2021) addressed a BCP with stochastic clearing prices that allows generating multiple non-overlapping *OR* bids. The problem is formulated with a chance constraint model inspired by the work of Triki et al. (2014) without requiring an exhaustive enumeration of all package bids. The paper of Hammami et al. (2021) is the first to propose an exact method for solving a BCP with stochastic prices. The method is based on an iterative process where restricted and relaxed problems are iteratively solved to yield valid lower and upper bounds. Results for instances with up to 50 contracts show that the exact method reported optimality for 74% of them within two hours on average. A simulation of 704 auctions also derived insights on how the parameters of the exact method should be tuned so that theoretical expected profits are minimally realized.

Our paper considers a BCP with stochastic clearing prices where the aim is to construct multiple non-overlapping *OR* bids to maximize a global net profit while considering pre-existing commitments and the carrier fleet capacity. Our problem extends that of Hammami et al. (2021) and differs from it at two levels. First, we aim to generate combinatorial bids so that a non-negative profit is always ensured for the carrier in case: (i) some of the bids submitted to the auction are not effectively won or (ii) a contract covered by a winning bid does not eventually materialize. Second, given that non-negative profits are certain, we consider the case where the carrier adopts what we call an *aggressive* pricing strategy when fixing bids prices, i.e., a strategy where ask prices are relatively large compared to competitors. In Hammami et al. (2021), the authors model uncertainty on auctioned contracts prices by considering a chance constraint where the probability of the bid ask price to be larger than the bid clearing price must be lower than or equal to 50%. In our paper, we extend their methodology to all pricing strategies (more than, less than or equal to 50%).

3 The BCP with stochastic clearing prices

The BCP with stochastic clearing prices, denoted as Stochastic BCP (SBCP), is defined as follows. Let $K_e = \{1, \ldots, |K_e|\}$ and $K_n = \{|K_e| + 1, \ldots, |K_n| + |K_e|\}$ denote the sets of the carrier's pre-existing and auctioned contracts, respectively. Let $K = K_e \cup K_n$ be the set of all contracts. To each contract $k \in K$ is associated a unique pair of origin and destination nodes denoted (o_k, d_k). Given the TL context, each volume picked up at a contract's origin node $(o_k, k \in K)$ must be directly delivered to the contract's destination node $(d_k, k \in K)$. To each pre-existing contract $k \in K_e$ is associated a known revenue p_k. To each contract $k \in K_n$ is associated a random clearing price variable $\tilde{p}_k$ following a normal distribution $N(\overline{p_k}, \sigma_k^2)$.

Let $O = \{o_k, k \in K\}$ and $D = \{d_k, k \in K\}$ denote respectively the sets of contracts' origin and destination nodes. Nodes 0 and N represent respectively the carrier's start and end depot. Hence, the problem is formally defined on a directed graph $G = (V, A)$ where $V = O \cup D \cup \{0, N\} = V^* \cup \{0, N\}$ is the set of nodes and A is the set of arcs defined as: $A = \{(o_k, d_k), k \in K\} \cup \{(0, i) : i \in O\} \cup \{(i, j) : i = d_k \in D, j = o_{k'} \in O : k, k' \in K, k \neq k'\} \cup \{(j, N), j \in D\}$. To each arc are associated a travel cost c_{ij} and a travel time t_{ij}. The carrier's fleet is assumed homogeneous and is denoted by $L = \{1, \ldots, |L|\}$. To each vehicle is associated a fixed usage fee f and a maximum tour duration T_{max}.

To model clearing prices uncertainty, a parameter $\alpha \in [0, 1]$ affects the probability of a bid ask price to be larger than its clearing price. This parameter value is fixed by the carrier and represents its *aggressiveness* in bidding. The larger is the value of α, the larger are the bids ask prices are when compared to competitors and the more *aggressive* is the carrier's pricing strategy.

We define $\mathfrak{B} = \{1, \ldots, |\mathfrak{B}|\}$ as the set of indices of the different combinatorial non-overlapping *OR* bids that could be generated by the carrier. Each bid is defined by a pair (K_b, p_b) where $K_b \subseteq K_n$ denotes the set of auctioned contracts covered by a bid of index $b \in \mathfrak{B}$ and p_b is the price asked by the carrier for serving all the contracts in K_b. Here, it is important to mention that a first-price CA is considered implying that if the carrier wins a bid of index $b \in \mathfrak{B}$ then it must be paid a price p_b to serve the set of auctioned contracts K_b. Indeed, K_b and $p_b, b \in \mathfrak{B}$ are not known in advance and are determined by solving the problem. A random variable $\tilde{C}_b$ is thus defined for each bid $b \in \mathfrak{B}$ to represent its clearing price.

3.1 Mathematical formulation

Hammami et al. (2021) propose a chance-constraint to model the SBCP above with five sets of decision variables:

x_{ij}^l binary variables defined for each arc $(i,j) \in A$ and vehicle $l \in L$; $x_{ij}^l = 1$ if arc (i,j) is traversed by vehicle l and zero otherwise,

z_{kb} binary variables defined for each auctioned contract $k \in K_n$ and bid of index $b \in \mathfrak{B}$; $z_{kb} = 1$ if k is part of bid of index b and zero otherwise,

w_b binary variables for each bid of index $b \in \mathfrak{B}$; $w_b = 1$ if bid of index b is selected and zero otherwise,

B_i positive integer variables defined for each node $i \in V$ indicating the order of visiting node i,

p_b positive continuous variables for each bid of index $b \in \mathfrak{B}$ indicating the carrier's ask price for the bid of index b.

A chance-constrained mathematical model, denoted M_p, is then formulated as follows:

$$M_p : \max \sum_{k \in K_e} p_k + \sum_{b \in \mathfrak{B}} p_b w_b - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x_{ij}^l - \sum_{l \in L} \sum_{j \in O} f x_{0j}^l \tag{1}$$

$$\text{s.t. } P(p_b w_b \leq \tilde{C}_b) \geq 1 - \alpha \qquad \forall b \in \mathfrak{B} \tag{2}$$

$$\sum_{b \in \mathfrak{B}} z_{kb} \leq 1 \qquad \forall k \in K_n \tag{3}$$

$$w_b \leq \sum_{k \in K_n} z_{kb} \qquad \forall b \in \mathfrak{B} \tag{4}$$

$$z_{kb} \leq w_b \qquad \forall k \in K_n, b \in \mathfrak{B} \tag{5}$$

$$\sum_{l \in L} x_{o_k d_k}^l = \sum_{b \in \mathfrak{B}} z_{kb} \qquad \forall k \in K_n \tag{6}$$

$$w_b \leq w_{b-1} \qquad \forall b \in \mathfrak{B} \setminus \{1\} \tag{7}$$

$$p_b \leq p_{b-1} \qquad \forall b \in \mathfrak{B} \setminus \{1\} \tag{8}$$

$$\sum_{l \in L} x_{o_k d_k}^l = 1 \qquad \forall k \in K_e \tag{9}$$

$$x_{o_k d_k}^l \leq \sum_{j \in O} x_{0j}^l \leq 1 \qquad \forall l \in L, k \in K \tag{10}$$

$$x_{o_k d_k}^l \leq \sum_{i \in D} x_{i,N}^l \leq 1 \qquad \forall l \in L, k \in K \tag{11}$$

$$\sum_{j:(j,i) \in A} x_{ji}^l = \sum_{j:(i,j) \in A} x_{ij}^l \qquad \forall l \in L, i \in V^* \tag{12}$$

$$\sum_{(i,j)\in A} t_{ij}x_{ij}^l \leq T_{max} \qquad \forall l \in L \tag{13}$$

$$\sum_{l\in L}\sum_{(i,j)\in A} t_{ij}x_{ij}^l \leq |L|T_{max} \tag{14}$$

$$B_i + \sum_{l\in L} x_{ij}^l - |V|\left(1 - \sum_{l\in L} x_{ij}^l\right) \leq B_j \qquad \forall (i,j) \in A \tag{15}$$

$$B_0 = 0 \tag{16}$$

$$x_{o_k d_k}^l \leq \sum_{k'\in K, k'\leq k} x_{o_{k'} d_{k'}}^{(l-1)} \qquad \forall k \in K, l \in L \setminus \{1\} \tag{17}$$

$$\sum_{j\in O} x_{0j}^l \leq \sum_{j\in O} x_{0j}^{(l-1)} \qquad \forall l \in L \setminus \{1\} \tag{18}$$

$$x_{o_1 d_1}^1 = 1 \tag{19}$$

$$x_{ij}^l \in \{0,1\} \qquad \forall (i,j) \in A, l \in L \tag{20}$$

$$0 \leq B_i \leq |V| \qquad \forall i \in V \tag{21}$$

$$z_{kb} \in \{0,1\} \qquad \forall b \in \mathfrak{B}, k \in K_n \tag{22}$$

$$w_b \in \{0,1\} \qquad \forall b \in \mathfrak{B} \tag{23}$$

$$p_b \geq 0 \qquad \forall b \in \mathfrak{B}. \tag{24}$$

The objective function (1) maximizes the carrier's net profit defined as the difference between the revenues collected from servicing the pre-existing contracts, the bidding prices, the traveling costs and the fixed costs associated with the use of vehicles. Probabilistic chance constraints (2) express a winning probability of $(1-\alpha)$ for each generated bid of index b. Constraints (3) imply that each auctioned contract is allocated to at most one bid. Constraints (4) and (5) link the variables w_b and z_{kb} so that a bid is generated if and only if it covers at least one auctioned contract. Constraints (6) link routing variables x_{ij}^l to z_{kb} variables. Constraints (7) and (8) are bids' symmetry breaking constraints. Constraints (9) ensure that all pre-existing contracts are served exactly once. Constraints (10) and (11) imply that each route starts and ends at the depot. Flow conservation is ensured by constraints (12). Constraints (13) impose maximum route duration. Constraint (14) is imposed on the global duration of routes to strengthen the formulation (Bianchessi et al., 2018; Hammami et al., 2020). Constraints (15) forbid sub-tours and impose an order for visiting nodes. Constraint (16) implies that each route begins at the depot. Constraints (17)–(19) are routing's symmetry breaking constraints. Constraints (17) impose an order for serving contracts. Constraints (18) imply that vehicle l is used if and only if vehicle $l-1$ is already used to break symmetry. Constraint (19) arbitrarily and without loss of generality forces the first pre-existing contract to be assigned to the lowest indexed vehicle. Finally, constraints (20)–(24) define the domain of the decision variables.

3.2 Model relaxation

Model M_p is not linear given the quadratic objective function (1) and the probabilistic chance constraints (2). As observed by Hammami et al. (2021), linearizing the objective function can be done by simply replacing the product $p_b w_b$ by a continuous variable $\zeta_b \geq 0, \forall b \in \mathfrak{B}$ and adding appropriate linking constraints. For chance constraints (2), like in Triki et al. (2014) and Hammami et al. (2021), it is assumed that auctioned contracts' prices are independent random variables and that the bid clearing price $\tilde{C}_b, b \in \mathfrak{B}$ can be approximated as:

$$\tilde{C}_b = S_b \sum_{k \in K_b} \tilde{p_k} = S_b \sum_{k \in K_n} z_{kb} \tilde{p_k} \qquad \forall b \in \mathfrak{B},$$

where $S_b \in]0, 1]$ denotes the estimated synergy value between the auctioned contracts forming the bid package of index $b \in \mathfrak{B}$. The synergy factor S_b is computed so that the lower its value, the larger is the degree of complementarity between the contracts covered by bid b. We refer the reader to Hammami et al. (2021) for additional information on synergy factors, how they can be estimated, and their impact on the auction outputs.
Given that auctioned contracts' prices follow a normal distribution and using the inverse cumulative distribution function for a standard normal distribution, denoted Φ^{-1}, the

probabilistic chance constraints (2) can be formulated as follows:

$$\zeta_b - S_b \sum_{k \in K_n} \overline{p_k} z_{kb} \leq S_b \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n} z_{kb}^2 \sigma_k^2} \qquad \forall b \in \mathfrak{B}. \tag{25}$$

Constraints (25) are still not linear but were relaxed in Hammami et al. (2021) who propose to bound this non-linear expression, yielding thus a relaxation of model M_p. Then, an iterative solution method is proposed in which a relaxed and a restricted problem are solved at each iteration to determine valid upper and lower bounds. The process iterates until the upper and lower bounds are equal. In Hammami et al. (2021), the relaxed constraints are valid in a context where $\alpha \leq \frac{1}{2}$ and are given by:

$$\zeta_b - \sum_{k \in K_n} \overline{p_k} z_{kb} \ \leq \ \Phi^{-1}(\alpha) \sigma_{min} w_b \qquad \forall b \in \mathfrak{B}, \tag{26}$$

where $\sigma_{min} = \min\{\sigma_k, k \in K_n\}$.

Choosing a value of $\alpha \leq \frac{1}{2}$ was motivated by the fact that the carrier may incur a negative profit if some of the bids it submitted are not won.

We propose to extend the same approach to the case where $\alpha \geq \frac{1}{2}$. In what follows, we describe how constraints (25) can be relaxed when $\alpha \geq \frac{1}{2}$.

If $\alpha \geq \frac{1}{2}$, then $\Phi^{-1}(\alpha) \geq 0$. Given that $0 < S_b \leq 1, \forall b \in \mathfrak{B}$, we have:

$$S_b \sum_{k \in K_n} \overline{p_k} z_{kb} \leq \sum_{k \in K_n} \overline{p_k} z_{kb} \qquad \forall b \in \mathfrak{B}$$

and

$$S_b \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n} z_{kb}^2 \sigma_k^2} \leq \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n} z_{kb}^2 \sigma_k^2} \qquad \forall b \in \mathfrak{B}.$$

Given constraints (22) and $\sigma_k > 0, \forall k \in K_n$, we have:

$$\sum_{k \in K_n} (z_{kb}\sigma_k)^2 \leq \left(\sum_{k \in K_n} z_{kb}\sigma_k\right)^2 \qquad \forall b \in \mathfrak{B}.$$

Hence,

$$\Phi^{-1}(\alpha)\sqrt{\sum_{k \in K_n} z_{kb}^2\sigma_k^2} \leq \Phi^{-1}(\alpha)\sum_{k \in K_n} z_{kb}\sigma_k \qquad \forall b \in \mathfrak{B}.$$

Then, constraints (25) can be relaxed using the following linear constraints:

$$\zeta_b \leq \sum_{k \in K_n} \overline{p_k} z_{kb} + \Phi^{-1}(\alpha)\sum_{k \in K_n} z_{kb}\sigma_k \qquad \forall b \in \mathfrak{B}. \tag{27}$$

4 Uncertainty on contracts materialization and bids success

Model M_p enables generating multiple non-overlapping *OR* bids to maximize a global expected net profit. However, there is no guarantee that a non-negative profit is obtained if one or multiple submitted bids are not won or if the contracts submitted in a winning bid do not materialize. This section presents the main concepts and models used by our proposed solution approach to eliminate both risks. A formal description of the proposed algorithm is given in Section 5.

4.1 Overview of the method to guarantee positive profit bids

When a combinatorial bid is won, all the contracts covered by the bids are generally allocated to the carrier, if these contracts materialize. These contracts will be served through one or multiple routes as modeled by constraints (9)–(21). Our idea is to add constraints to model M_p that forbid generating what we call *risky* routes. A *risky* route is a route that is no longer profitable if any subset of its contracts is not awarded. These constraints are referred to as Risky Routes (RR) constraints.

Considering RR constraints eliminates at the same time the risk of a negative profit when a contract does not materialize or when a bid is not won. As will explained in Section 5, RR constraints are dynamically added to model M_p during the solution process: at each iteration, an extended M_p model (with additional RR constraints) is solved to optimality and the current optimal candidate solution is evaluated. If the current solution includes risky routes, the corresponding RR constraints are added. The process iterates until identifying an optimal solution with no risky routes. We now explain how risky routes are identified (Section 4.2), and how RR constraints are modeled and generated (Section 4.3).

4.2 Risky routes via stochastic constraint satisfaction

Let $S = (w, p, z, \zeta, x)$ denote a feasible solution of model M_p. Let R be the set of routes associated with solution S. Let K_r denote the set of contracts covered by route $r \in R$, and K_{e_r} (K_{n_r}) be the set of corresponding existing (auctioned) contracts. That is $K_r = K_{e_r} \cup K_{n_r}$, where $K_{e_r} = K_e \cap K_r$ and $K_{n_r} = K_n \cap K_r$. Route $r \in R$ is considered as risky if there is no sequence with a non-negative expected profit that can be generated to visit only a subset of auctioned contracts in K_{n_r}.

To check if a given route $r \in R$ is risky, we first enumerate all the different subsets of contracts in K_{n_r}. There are $2^{|K_{n_r}|}$ such subsets in total. Let $\mathcal{P}(r) = \{\mathcal{P}^1(r), \ldots, \mathcal{P}^{2^{|K_{n_r}|}}(r)$ $\}$ denote the set of all these combinations. For example, if a route r serves the set of contracts $K_r = \{k_1, k_2, k_3, k_4\}$ where $K_{e_r} = \{k_1\}$ and $K_{n_r} = \{k_2, k_3, k_4\}$, then, $\mathcal{P}(r) = \{\emptyset, \{k_2\}, \{k_3\}, \{k_4\}, \{k_2, k_3\}, \{k_2, k_4\}, \{k_3, k_4\}, \{k_2, k_3, k_4\}\}$. Considering a subset $\mathcal{P}^u(r), u = 1, \ldots, 2^{|K_{n_r}|}$, models the case where contracts in $K_{n_r} \setminus \mathcal{P}^u(r)$ are not awarded. Consequently, if there exists $u \in \{1 \ldots 2^{|K_{n_r}|}\}$ such that the optimal expected profit of serving the contracts in $\mathcal{P}^u(r)$ is negative, then route r is risky and must be forbidden by adding the corresponding RR constraints.

We consider a stochastic Constraint SAtisfaction Problem (CSAP) for each subset of $\mathcal{P}(r)$ associated with a route $r \in R$. The CSAP associated with subset $\mathcal{P}^u(r)$, $u \in \{1, \ldots, 2^{|K_{n_r}|}\}$ checks if there exists a tour in which all the contracts in $\mathcal{P}^u(r) \cup K_{e_r}$ can be served while ensuring a non-negative expected profit. This tour must be identified on a graph $G^u(r) = (V^u(r), A^u(r))$, where the set of nodes $V^u(r)$ includes the two nodes representing the depot, and all the origin and destination nodes in V corresponding to the existing contracts covered by r and the auctioned contracts in $\mathcal{P}^u(r)$. Formally, $V^u(r) = \{0, N\} \cup \{i : i = o_k \in O \text{ and } k \in K_e \cup \mathcal{P}^u(r)\} \cup \{j : j = d_k \in D \text{ and } k \in K_e \cup \mathcal{P}^u(r)\}$. The set of arcs $A^u(r)$ is the subset of arcs in A linking all the nodes in $V^u(r)$.

Uncertainty on contracts prices can be handled by generating plausible future scenarios of contracts prices and considering the equivalent deterministic model. This is possible since it is assumed that auctioned contracts prices are random variables and $\tilde{p}_k, k \in K_n$ follows a normal distribution $N(\overline{p_k}, \sigma_k^2)$. A scenario in this case is defined as a compound event which is the result of the juxtaposition of random processes related to the different contracts. Let Ω denote a set of scenarios. For a scenario $\omega \in \Omega$, the instance of the auctioned contracts clearing prices is denoted p_k^ω. We demonstrate how this can be handled exactly without enumerating Ω.

The stochastic CSAP associated with subset $u \in \{1 \ldots 2^{|K_{n_r}|}\}$ and scenarios in Ω is modeled using two sets of decision variables:

- χ_{ij}^ω =1 if arc $(i,j) \in A^u(r)$ is traversed in scenario $\omega \in \Omega$; and $\chi_{ij}^\omega = 0$; otherwise,
- β_i^ω ≥ 0 and integer representing the order of visiting node $i \in V^u(r)$ for scenario $\omega \in \Omega$,

and formulated with model $M_s^u(r, \Omega)$ as follows:

$$\sum_{k \in K_{e_r}} p_k - f + \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \left(\sum_{k \in \mathcal{P}^u(r)} p_k^\omega \chi_{o_k d_k}^\omega - \sum_{(i,j) \in A^u(r)} c_{ij} \chi_{ij}^\omega \right) \geq 0 \tag{28}$$

$$\chi_{o_k d_k}^\omega = 1 \qquad \forall k \in K_{e_r} \cup \mathcal{P}^u(r), \omega \in \Omega \tag{29}$$

$$\sum_{k \in K_{e_r} \cup \mathcal{P}^u(r)} \chi_{0 o_k}^\omega = 1 \qquad \forall \omega \in \Omega \tag{30}$$

$$\sum_{k\in K_{e_r}\cup\mathcal{P}^u(r)} \chi^\omega_{d_k,N} = 1 \qquad \forall \omega\in\Omega \tag{31}$$

$$\sum_{j:(j,i)\in A^u(r)} \chi^\omega_{ji} = \sum_{j:(i,j)\in A^u(r)} \chi^\omega_{ij} \qquad \forall i\in V^u(r),\omega\in\Omega \tag{32}$$

$$\sum_{(i,j)\in A^u(r)} t_{ij}\chi^\omega_{ij} \le T_{max} \qquad \forall \omega\in\Omega \tag{33}$$

$$\beta^\omega_i + \chi^\omega_{ij} - 2(1+|K_{e_r}\cup\mathcal{P}^u(r)|)(1-\chi^\omega_{ij}) \le \beta^\omega_j \qquad \forall (i,j)\in A^u(r),\omega\in\Omega \tag{34}$$

$$0 \le \beta^\omega_i \le 2(1+|K_{e_r}\cup\mathcal{P}^u(r)|) \qquad \forall i\in V^u(r),\omega\in\Omega \tag{35}$$

$$\chi^\omega_{ij}\in\{0,1\} \qquad \forall (i,j)\in A^u(r),\omega\in\Omega. \tag{36}$$

Constraint (28) imposes that the expected profit of the tour visiting all the contracts in $\mathcal{P}^u(r)\cup K_{e_r}$ is non-negative. Constraints (29) ensure that each contract of $K_{e_r}\cup\mathcal{P}^u(r)$ is served once. Constraints (30) and (31) ensure a route to start and end at the depot. Flow conservation is ensured via constraints (32). Constraints (33) impose a maximum route duration. Sub-tours are eliminated with constraints (34). Finally, constraints (35) and (36) define the domain of decision variables.

Observe that auctioned contracts clearing prices p^ω_k are only involved in constraints (28). Given constraints (29), constraints (28) can be rewritten as :

$$\sum_{k\in K_{e_r}} p_k - f + \sum_{k\in\mathcal{P}^u(r)} \frac{1}{|\Omega|}\sum_{\omega\in\Omega} p^\omega_k - \frac{1}{|\Omega|}\sum_{\omega\in\Omega}\sum_{(i,j)\in A^u(r)} c_{ij}\chi^\omega_{ij} \ge 0.$$

Knowing that $\tilde{p}_k, k\in K_n$ follows a normal distribution $N(\overline{p_k},\sigma^2_k)$, we have:

$$\sum_{k\in K_{e_r}} p_k - f + \sum_{k\in\mathcal{P}^u(r)} \overline{p_k} - \frac{1}{|\Omega|}\sum_{\omega\in\Omega}\sum_{(i,j)\in A^u(r)} c_{ij}\chi^\omega_{ij} \ge 0.$$

The resulting $M^u_s(r,\Omega)$ model is then separable by scenario and all subproblems associated with scenarios $\omega\in\Omega$ have identical variables and constraints and thus identical solutions. Hence to determine if the stochastic CSAP is feasible, one needs to solve a single deterministic model as follows:

$$\sum_{k\in K_{e_r}} p_k - f + \sum_{k\in\mathcal{P}^u(r)} \overline{p_k} - \sum_{(i,j)\in A^u(r)} c_{ij}\chi_{ij} \geq 0 \tag{37}$$

$$\chi_{o_k d_k} = 1 \qquad \forall k \in K_{e_r} \cup \mathcal{P}^u(r) \tag{38}$$

$$\sum_{k\in K_{e_r}\cup\mathcal{P}^u(r)} \chi_{0 o_k} = 1 \tag{39}$$

$$\sum_{k\in K_{e_r}\cup\mathcal{P}^u(r)} \chi_{d_k,N} = 1 \tag{40}$$

$$\sum_{j:(j,i)\in A^u(r)} \chi_{ji} = \sum_{j:(i,j)\in A^u(r)} \chi_{ij} \qquad \forall i \in V^u(r) \tag{41}$$

$$\sum_{(i,j)\in A^u(r)} t_{ij}\chi_{ij} \leq T_{max} \tag{42}$$

$$\beta_i + \chi_{ij} - 2(1 + |K_{e_r} \cup \mathcal{P}^u(r)|)(1 - \chi_{ij}) \leq \beta_j \qquad \forall (i,j) \in A^u(r) \tag{43}$$

$$0 \leq \beta_i \leq 2(1 + |K_{e_r} \cup \mathcal{P}^u(r)|) \qquad \forall i \in V^u(r) \tag{44}$$

$$\chi_{ij} \in \{0,1\} \qquad \forall (i,j) \in A^u(r). \tag{45}$$

4.3 Risky route constraints

If the CSAP associated with subset $\mathcal{P}^u(r)$, $u \in \{1, \ldots, 2^{|K_{n_r}|}\}$ of a route $r \in R$ is infeasible, it implies that the new auctioned contracts of $\mathcal{P}^u(r)$ must not be visited by the same vehicle route that serves exactly the same existing contracts as in K_{e_r}. This is ensured by adding appropriate RR constraints to model M_p. The RR constraints associated with subset $\mathcal{P}^u(r)$, $u \in \{1, \ldots, 2^{|K_{n_r}|}\}$ of a route $r \in R$ are formulated as:

$$\sum_{k\in K_{e_r}\cup\mathcal{P}^u(r)} x^l_{o_k d_k} \leq |K_{e_r} \cup \mathcal{P}^u(r)| - 1 + \sum_{k\in K_e\setminus K_{e_r}} x^l_{o_k d_k}, \quad l \in L. \tag{46}$$

Observe that if $\sum_{k\in K_e\setminus K_{e_r}} x^l_{o_k d_k} = 0$, constraints (46) reduce to:

$$\sum_{k\in K_{e_r}\cup\mathcal{P}^u(r)} x^l_{o_k d_k} \leq |K_{e_r} \cup \mathcal{P}^u(r)| - 1, \quad l \in L,$$

which avoids generating a route that serves exactly the same contracts as in $K_{e_r} \cup \mathcal{P}^u(r)$. However, if $\sum_{k \in K_e \setminus K_{e_r}} x^l_{o_k d_k} \geq 1$, inequalities (46) are not binding since considering additional existing contracts requires reevaluating the route profitability.

Theorem 1. *For a given route r, if $|\mathcal{P}(r)| \geq 2$ then for all $u, u' \in \{1, \ldots, 2^{|K_{n_r}|}\}$ such that $u \neq u'$ and $\mathcal{P}^u(r) \subset \mathcal{P}^{u'}(r)$, the RR constraint associated with $\mathcal{P}^{u'}(r)$ is dominated by the RR constraint associated with $\mathcal{P}^u(r)$.*

Proof. Our proof consists in showing that if the RR constraints associated with $\mathcal{P}^u(r)$ are satisfied then the RR constraints associated with $\mathcal{P}^{u'}(r)$ are also satisfied.
The RR constraints associated with $\mathcal{P}^u(r)$ are given by:

$$\sum_{k \in K_{e_r} \cup \mathcal{P}^u(r)} x^l_{o_k d_k} \leq |K_{e_r} \cup \mathcal{P}^u(r)| - 1 + \sum_{k \in K_e \setminus K_{e_r}} x^l_{o_k d_k}, \quad l \in L.$$

Given that $\mathcal{P}^u(r) \subset \mathcal{P}^{u'}(r)$, we have:

$$\sum_{K_{e_r} \cup \mathcal{P}^{u'}(r)} x^l_{o_k d_k} \leq |K_{e_r} \cup \mathcal{P}^u(r)| - 1 + |\mathcal{P}^{u'}(r) \setminus \mathcal{P}^u(r)| + \sum_{k \in K_e \setminus K_{e_r}} x^l_{o_k d_k}, \quad l \in L$$

and

$$|K_{e_r} \cup \mathcal{P}^u(r)| - 1 + |\mathcal{P}^{u'}(r) \setminus \mathcal{P}^u(r)| = |K_{e_r} \cup \mathcal{P}^{u'}(r)| - 1.$$

It follows that:

$$\sum_{k \in K_{e_r} \cup \mathcal{P}^{u'}(r)} x^l_{o_k d_k} \leq |K_{e_r} \cup \mathcal{P}^{u'}(r)| - 1 + \sum_{k \in K_e \setminus K_{e_r}} x^l_{o_k d_k}, \quad l \in L,$$

which correspond to the RR constraints associated with $\mathcal{P}^{u'}(r)$. $\square$

5 Exact solution approach

Our solution approach extends that of Hammami et al. (2021) to eliminate risks related to contracts materialization and bids success. We also adapt their approach to the case where the parameter α is larger than $\frac{1}{2}$ which would be an interesting alternative for a carrier given a guarantee of non-negative profits independently of the auction outcomes. In the following, we roughly recall the main steps of the exact algorithm proposed by Hammami et al. (2021) and the way we adapt it to consider an $\alpha > \frac{1}{2}$ (Algorithm 1). Then, we describe our new algorithm to handle risks (Algorithm 2).

At each iteration of Algorithm 1, valid lower and upper bounds (denoted respectively LB and UB) are updated by solving appropriate restricted and relaxed problems. The relaxed and restricted problems formulations are given respectively in A and B. The process iterates until $LB = UB$ or a time limit is met. At each iteration, a relaxed problem ($\overline{M_p}(\overline{\mathfrak{B}})$) is solved for which the relaxed constraints, (27) for $\alpha \geq \frac{1}{2}$ and (26) for $\alpha < \frac{1}{2}$, are considered instead of chance constraints (25). For each solution of $\overline{M_p}(\overline{\mathfrak{B}})$, denoted $(w^0, p^0, z^0, \zeta^0, x^0)$, Algorithm 1 computes for each generated bid of index $b \in \mathfrak{B}$ the associated synergy value S_b as described in Hammami et al. (2021). Recall that $\mathfrak{B}$ represents the set of indices of different bids that could be generated, two bids being different if they do not cover the same set of contracts. Afterwards, Algorithm 1 verifies if the chance constraint (25) associated with the bid of index $b \in \mathfrak{B}$ is violated. If so, it generates a bid of index $\bar{b}$ covering the same auctioned contracts as the bid of index b and updates the set of indices of partial bids $\overline{\mathfrak{B}}$. To avoid generating the same bids (i.e., bids that cover the same package of contracts), no-good cuts are added to the relaxed problems. A new restricted problem, denoted by $\underline{M_p}(\overline{\mathfrak{B}})$, is obtained by restricting the set of bids of M_p to $\overline{\mathfrak{B}}$ (for which the covered contracts are known) and considering the corresponding chance constraints (25).

To handle risks on contracts materialization and bids success, we adapt Algorithm 1 by modifying the relaxed and restricted problems $\overline{M_p}(\overline{\mathfrak{B}})$ and $\underline{M_p}(\overline{\mathfrak{B}})$ (lines 3 and 15

Algorithm 1 General structure of the exact solution method for the SBCP

$\overline{\mathfrak{B}} \leftarrow \emptyset$; $LB \leftarrow -\infty$; $UB \leftarrow +\infty$
while a time limit is not met and $LB < UB$ **do**
  $UB \leftarrow$ Solve $\overline{M_p}(\overline{\mathfrak{B}})$
  for each feasible solution $(w^0, p^0, z^0, \zeta^0, x^0)$ of $\overline{M_p}(\overline{\mathfrak{B}})$ **do**
    for each generated bid of index $b \in \mathfrak{B}$ deduced from $(w^0, p^0, z^0, \zeta^0, x^0)$ **do**
      Compute the corresponding synergy value S_b
      if $\zeta_b^0 > S_b \left(\sum_{k \in K_n} \overline{p_k} z_{kb}^0 + \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n} z_{kb}^0 \sigma_k^2} \right)$ **then**
        Generate a new bid $\bar{b}$ such that $K_{\bar{b}} = K_b$ without fixing its price
        $\overline{\mathfrak{B}} \leftarrow \overline{\mathfrak{B}} \cup \{\bar{b}\}$
        Add to $\overline{M_p}(\overline{\mathfrak{B}})$ no-good cuts (52) to forbid regenerating bid $\bar{b}$
        Add to $\underline{M_p}(\overline{\mathfrak{B}})$ the chance constraint (64) corresponding to bid $\bar{b}$
      end if
    end for
  end for
  $LB \leftarrow$ Solve $\underline{M_p}(\overline{\mathfrak{B}})$
end while

of Algorithm 1) so that risky routes are not accepted. The proposed modification is described in Algorithm 2. In sum, model $\underline{M_p}(\overline{\mathfrak{B}})$ is first solved by branch-and-cut. Then, for each route r generated in its optimal solution $S = (w, p, z, \zeta, x)$, Algorithm 2 checks if route r is risky or not as described in Section 4. If route r is risky, all the corresponding non-dominated RR constraints (as described in Section 4.3) are added to $\underline{M_p}(\overline{\mathfrak{B}})$ and $\overline{M_p}(\overline{\mathfrak{B}})$. RR cuts are added until all the routes generated in S are checked. If at least one RR cut is added to model $\underline{M_p}(\overline{\mathfrak{B}})$, the resulting extended model is solved again.

Observe that ordering the set of auctioned contracts combinations in $\mathcal{P}(r)$ (line 21) and updating it (line 26) enables generating non-dominated cuts as explained in Section 4.3. Proposition 1 hereafter establishes the convergence of Algorithm 2.

Proposition 1. *Algorithm 2 always identifies optimal OR bids ensuring a non-negative profit for the carrier, independently of contracts materialization or auction outcomes, provided that a solution exists for the carrier existing network in which all the routes serving exclusively the existing contracts have a non-negative profit.*

Proof. Adding RR constraints to the restricted model $\underline{M_p}(\overline{\mathfrak{B}})$ and to the relaxed model

Algorithm 2 General structure of the exact solution method for the SBCP with risks elimination

$\overline{\mathfrak{B}} \leftarrow \emptyset$; $LB \leftarrow -\infty$; $UB \leftarrow +\infty$
while a time limit is not met and $LB < UB$ **do**
  $UB \leftarrow$ Solve $\overline{M_p}(\overline{\mathfrak{B}})$
  for each feasible solution $(w^0, p^0, z^0, \zeta^0, x^0)$ of $\overline{M_p}(\overline{\mathfrak{B}})$ **do**
    for each generated bid of index $b \in \mathfrak{B}$ deduced from $(w^0, p^0, z^0, \zeta^0, x^0)$ **do**
      Compute the corresponding synergy value S_b
      if $\zeta_b^0 > S_b \left(\sum_{k \in K_n} \overline{p_k} z_{kb}^0 + \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n} z_{kb}^0 \sigma_k^2} \right)$ **then**
        Generate a new bid $\overline{b}$ such that $K_{\overline{b}} = K_b$ without fixing its price
        $\overline{\mathfrak{B}} \leftarrow \overline{\mathfrak{B}} \cup \{\overline{b}\}$
        Add to $\overline{M_p}(\overline{\mathfrak{B}})$ no-good cuts (52) to forbid regenerating bid $\overline{b}$
        Add to $\underline{M_p}(\overline{\mathfrak{B}})$ the chance constraint (64) corresponding to bid $\overline{b}$
      end if
    end for
  end for
  $LB \leftarrow$ Solve $\underline{M_p}(\overline{\mathfrak{B}})$
  Let $S = (w, p, z, \zeta, x)$ be its optimal solution
  Define the set R of routes associated with S (Section 4.2)
  for each route $r \in R$ **do**
    Determine the set $\mathcal{P}(r)$ associated with r (Section 4.2)
    while $\mathcal{P}(r) \neq \emptyset$ **do**
      Order the elements of $\mathcal{P}(r)$ in an ascending order with respect to their size
      Let $\mathcal{P}^u(r)$ denote the first element of $\mathcal{P}(r)$
      Solve the CSAP associated with subset $\mathcal{P}^u(r)$ (Section 4.2)
      if CSAP is infeasible **then**
        Add the RR cut (46) associated with $\mathcal{P}^u(r)$ to models $\underline{M_p}(\overline{\mathfrak{B}})$ and $\overline{M_p}(\overline{\mathfrak{B}})$
        Remove from $\mathcal{P}(r)$ all the subsets including all elements of $\mathcal{P}^u(r)$
      end if
    end while
  end for
  if at least one RR cut is added **then**
    Solve the extended model $\underline{M_p}(\overline{\mathfrak{B}})$ (with RR cuts)
  end if
end while

$\overline{M_p}(\overline{\mathfrak{B}})$ at each iteration forbids generating routes with a negative profit. Either these routes cover exclusively existing contracts (when $\mathcal{P}^u(r) = \emptyset$), or both existing and new auctioned contracts. Assuming that an initial solution, say S^e, exists for the carrier existing network in which all the routes serving exclusively the existing contracts have a non-negative profit, implies that the extended models $\underline{M_p}(\overline{\mathfrak{B}})$ and $\overline{M_p}(\overline{\mathfrak{B}})$ incorporating all non-dominated RR constraints are always feasible. Indeed, at worst, the set of generated bids $\overline{\mathfrak{B}}$ would be empty and a solution would be S^e. □

6 Experimental study

Our experimental study has two main goals. First, we evaluate the performance of Algorithms 1 and 2 in terms of computing time and solution quality. Second, we study the merits/drawbacks of incorporating the risk elimination procedure on the carrier expected and real profits. This is done by comparing two contexts: one where the risks associated with bids success and contracts materialization are eliminated, referred to as a risk-averse bidding, and another where such risks are permitted, referred to as a risky bidding. Observe that in a risk-averse context, bids are generated with Algorithm 2. They are generated with Algorithm 1 in a risky context. For each context, we consider two strategies for fixing bids prices: a *conservative* pricing strategy where the parameter α is relatively small, and an aggressive one where it is large. Combining the two risk-behavior bidding contexts and the two pricing strategies enables considering different carriers profiles.

Our experimental study considers 23 problem tests obtained by varying the auctioned contracts (K_n), the carrier's existing contracts (K_e), its fleet size ($|L|$) and the fixed vehicles' fees (f). Contracts origins and destinations are defined from a list of existing cities from Canada. Traveling times (t_{ij}) and costs (c_{ij}) associated with arcs $(i, j) \in A$ are generated using Google Maps features. For each problem test, the maximum route duration T_{max} is set to 1,500 (time units). The revenue associated with each existing contract $k \in K_e$ and the mean value $\overline{p_k}$ of the normal distribution function associated with auctioned contracts $k \in K_n$ clearing prices are uniformly generated within the interval $[1.5 \times c_{o_k d_k}, 2 \times c_{o_k d_k}]$. For each auctioned contract $k \in K_n$, the standard deviation σ_k is set to $15\%\overline{p_k}$. Synergy factors S_b are computed as in Hammami et al. (2021) by considering pairwise synergies within $[0.01, 1.00]$.

Each problem test is solved for two values of the parameter α: 5% and 95%. All the instances are solved to optimality using our exact method implemented in Java. The mathematical models of the relaxed, the restricted and the constraint satisfaction problems are solved using the branch-and-cut procedure of CPLEX 20.10. All experiments were conducted on computers mounted in parallel and equipped with Intel(R) Xeon(TM) Gold 6148 processors clocked at 2.40 GHz and 186 Gigabyte of RAM.

6.1 Computational performance of Algorithms 1 and 2

Table 1 describes the instances characteristics (the number of vehicles ($|L|$), the vehicle's fixed fee (f), the number of existing contracts ($|K_e|$) and the number of auctioned contracts ($|K_n|$)) and reports for each instance, the total CPU time in seconds required by each algorithm and each pricing strategy ($\alpha = 5\%$ and 95%). Table 1 also displays the number of RR cuts generated when Algorithm 2 is considered.

Table 1: Computing times of Algorithms 1 and 2

Pricing					Conservative ($\alpha = 5\%$)			Aggressive ($\alpha = 95\%$)		
Instance characteristics					Risky (Algorithm 1)	Risk-averse (Algorithm 2)		Risky (Algorithm 1)	Risk-averse (Algorithm 2)	
#	$\|L\|$	f	$\|K_e\|$	$\|K_n\|$	CPU	CPU	Cuts	CPU	CPU	Cuts
1	2	500	10	20	6671.34	9918.73	296	3697.05	5969.20	294
2	2	750	10	20	3946.16	4402.51	0	4156.01	4203.74	0
3	3	500	10	20	10046.63	10346.68	0	3169.58	3200.04	0
4	3	750	10	20	10368.60	10528.84	0	6998.35	7035.73	0
5	4	500	10	20	8924.64	9556.08	0	2766.14	2769.10	0
6	4	750	10	20	10059.79	10849.58	0	6859.02	6920.14	0
7	3	500	15	15	4.89	5.77	0	4.80	10.17	0
8	3	750	15	15	8.11	11.38	0	5.57	8.94	0
9	4	500	15	15	120.34	1027.27	424	123.24	21904.05	2744
10	4	750	15	15	42.18	5317.72	2288	164.24	113067.49	3680
11	5	500	15	15	2779.57	2967.33	0	2464.82	24218.51	1885
12	5	750	15	15	16750.25	20415.34	1025	3490.43	5413.37	2260
13	6	500	15	15	987.45	1053.46	0	808.29	2506.12	2070
14	6	750	15	15	170.66	11644.34	906	1074.34	13991.63	3042
15	4	500	15	20	2706.78	71245.00	1236	2333.56	109854.66	1864
16	4	750	15	20	3545.35	42964.93	1272	1349.18	29540.51	1444
17	5	750	20	15	332.80	1867.43	635	952.20	2926.84	675
18	6	500	20	15	128.12	259.85	0	879.93	23952.92	1254
19	4	500	15	25	75.28	266.60	32	40.90	320.07	36
20	4	750	15	25	130.38	201.01	0	52.99	447.44	0
21	5	500	20	20	103.89	367.22	25	142.10	436.48	25
22	5	750	20	20	139.73	258.43	10	110.31	571.41	30
23	6	750	25	20	16229.84	56842.39	282	5975.53	48393.96	396
Average					**4098.82**	**11839.81**	**367**	**2070.37**	**18594.02**	**943**

The results of Table 1 clearly show that ensuring a non-negative profit for the carrier independently of the auction outcomes and the contracts materialization (Algorithm 2) comes at the expense of computing times yielding an increase that averages 2.15 hours for $\alpha = 5\%$ and 4.59 hours for $\alpha = 95\%$ when compared to Algorithm 1. This increase is particularly significant when the number of RR cuts is relatively large. As one can notice, considering an aggressive pricing strategy ($\alpha = 95\%$) results in a larger number of RR cuts implying that risky routes are more likely to be generated when the carrier fixes bidding prices more aggressively. When no cuts are added, this implies that the solutions obtained from Algorithm 1 are already "safe" and their safety is established with Algorithm 2. This was observed for 11 instances if $\alpha = 5\%$ and 8 instances if $\alpha = 95\%$. For these instances, the increase in CPU times yielded by Algorithm 2 compared to Algorithm 1 is relatively small: 254 seconds, respectively, 73 seconds, in average for $\alpha = 5\%$, respectively, $\alpha = 95\%$.

Table 2 displays the solutions obtained by Algorithms 1 and 2, or equivalently the risky and the risk-averse bidding contexts, for the different pricing strategies (values of α). It reports for each instance, each bidding context (algorithm), and each pricing strategy (value of α), the maximum expected profit (under the columns Profit) that could be obtained by the carrier if all the bids generated by the corresponding algorithm are won. Table 2 also reports for each bidding context, the number of generated bids ($|\mathfrak{B}^*|$) and the total number of auctioned contracts to bid on ($|K(\mathfrak{B}^*)|$). The last column (Gap_p) displays the relative difference between the two maximum profit's values $\left(Gap_p(\%) = \frac{Profit(risky) - Profit(risk-averse)}{Profit(risky)}\right)$. It is noteworthy that the results of the instances for which no RR cuts were generated are identical since both contexts give exactly the same solutions. We display them to analyze the impact of the pricing strategy (value of α) on the maximum expected profit.

The results of Table 2 first show that the number of generated bids, their structures and the maximum profit they could generate vary, sometimes substantially, with the bidding context. Given that the risk-averse context is more restrictive than the risky context, it was predictable that the maximum profit generated by Algorithm 1 would be at least as large as that yielded by Algorithm 2. Although the gap in profits could be relatively large

Table 2: SBCP solutions for the different contexts and pricing strategies

Pricing	**Conservative** (α = 5%)							**Aggressive** (α= 95%)						
Context	**Risky (Algorithm 1)**			**Risk-averse (Algorithm 2)**				**Risky (Algorithm 1)**			**Risk-averse (Algorithm 2)**			
Instance	*Profit*	$\lvert\mathfrak{B}^*\rvert$	$\lvert K(\mathfrak{B}^*)\rvert$	*Profit*	$\lvert\mathfrak{B}^*\rvert$	$\lvert K(\mathfrak{B}^*)\rvert$	Gap_p	*Profit*	$\lvert\mathfrak{B}^*\rvert$	$\lvert K(\mathfrak{B}^*)\rvert$	*Profit*	$\lvert\mathfrak{B}^*\rvert$	$\lvert K(\mathfrak{B}^*)\rvert$	Gap_p
1	580	4	7	571	3	9	1.55	1130	6	8	1067	5	9	5.58
2	875	4	10	875	4	10	0.00	1447	5	8	1447	5	8	0.00
3	1078	3	8	1078	3	8	0.00	1600	5	8	1600	5	8	0.00
4	847	5	8	847	5	8	0.00	1493	7	8	1493	7	8	0.00
5	701	4	10	701	4	10	0.00	1302	5	7	1302	5	7	0.00
6	901	4	9	901	4	9	0.00	1382	6	9	1382	6	9	0.00
7	753	1	2	753	1	2	0.00	981	2	2	981	2	2	0.00
8	499	1	1	499	1	1	0.00	644	1	2	644	1	2	0.00
9	1059	3	7	961	1	1	9.25	2515	7	7	2082	6	6	17.22
10	369	1	1	136	1	1	63.14	817	5	7	514	3	5	37.09
11	1087	1	1	1087	1	1	0.00	1995	5	7	1809	4	6	9.32
12	723	1	1	723	1	1	0.00	1062	4	6	1004	1	1	5.46
13	1483	1	1	1483	1	1	0.00	2072	4	6	2072	4	6	0.00
14	783	1	1	783	1	1	0.00	1242	5	6	1064	1	1	14.33
15	1068	2	6	925	2	7	13.39	1706	3	7	1377	4	6	19.28
16	1129	3	5	968	3	6	14.26	1956	4	7	1560	5	6	20.25
17	1795	3	7	1795	3	7	0.00	2580	4	7	2580	4	7	0.00
18	1845	1	5	1845	1	5	0.00	2295	2	7	2233	3	7	2.70
19	749	2	3	488	1	3	34.85	1130	2	3	794	2	3	29.73
20	876	2	3	876	2	3	0.00	1151	3	3	1151	3	3	0.00
21	694	1	3	438	1	1	36.89	942	2	3	440	1	1	53.29
22	470	2	3	407	1	2	13.40	770	2	3	502	1	2	34.81
23	1440	3	7	1409	3	6	2.15	2217	3	7	2164	3	6	2.39
Average	**948**	**2.30**	**4.74**	**893**	**2.09**	**4.48**	**8.21**	**1497**	**4.00**	**6.00**	**1359**	**3.52**	**5.17**	**10.93**

for certain problem tests (reaching 63% for problem test 10 with $\alpha = 5\%$, for example), it is relatively small (less than 3%) for eight instances in total when considering both the conservative and aggressive pricing strategies. For four of these instances, both algorithms yielded the same profits although RR cuts were added by Algorithm 2. This means that Algorithm 2 rearranged the contracts selected by Algorithm 1 among routes so that a non-negative profit is ensured even if one or multiple contracts do not materialize.

The results of Table 2 also show that adopting an aggressive pricing strategy rather than a conservative one yields larger maximum profits with an average increase of 61.21% when considering the bids generated in a risky context and 60.74% when considering those generated in a risk-averse context. However, observe that it might be too risky for a carrier to consider an aggressive pricing strategy ($\alpha = 95\%$) without having the guarantee of a non-negative profit in case one or more submitted bids are not won. In practice, a moderately risky behavior consists in either: (i) adopting a risky bidding (i.e., use Algorithm 1) coupled with a conservative pricing strategy (i.e., a low value of α) to increase the chance of winning the submitted bids but with the risk to incur monetary losses if the bids are not won, or (ii) using a risk-averse bidding (Algorithm 2) with an aggressive pricing strategy (i.e., a large value of α) yielding substantially larger profits but with less guarantee that the price asked in the submitted bids would be sufficiently low to win the submitted bids but knowing that there is no risk of monetary losses if bids are not allocated. When comparing the maximum expected profits yielded by these two alternatives, the second one (Algorithm 2 with $\alpha = 95\%$) yields larger profits for almost all the instances (22 over 23) with an average relative increase of 42.57% and a maximum increase of 96.60%.

When analyzing the results of Table 2, one should keep in mind that the profits reported correspond to those that could be won by the carrier if all the generated bids are won after the auction clears. In practice, it is not likely that the carrier wins all of its submitted bids. In the next section we analyze the impact of a risk-averse bidding on the carrier's effective profit for the instances where it yields lower maximum expected profits than a risky bidding for the same pricing strategy (instances for which $Gap_p > 0\%$ in Table 2).

6.2 Effective profits for the risk-averse and the risky bidding contexts

For each instance, we simulate all the possible scenarios of bids success and determine the profit effectively realized by the carrier for each scenario and each bidding context. For example, if we consider problem test 1, four combinatorial bids are generated when a risky bidding is used ($|\mathfrak{B}^*| = 4$ in Table 2). In this case, 15 different scenarios for bids success are possible: (i) none of the bis are won (in this case, the effective profit corresponds to that of the carrier existing network), (ii) only one bid is won (four possible scenarios), (ii) two bids are won (six possible scenarios), (iii) three bids are won (three possible scenarios), and (iv) all the bids are won (in this case, the effective profit corresponds to the maximum expected profit reported in Table 2 for Algorithm 1).

Table 3 summarizes the results obtained for each bidding context and each pricing strategy. Scenarios of bids success are grouped over all the problem tests with regard to the number of bids won (column Bids won). For each line, the first three columns associated with each bidding context report the average (P_{Avg}), minimum (P_{Min}) and maximum (P_{Max}) profits effectively realized by the carrier over all the scenarios and all the problem tests. Observe that the results corresponding to the scenarios where none of the generated bids are won are not reported since both bidding contexts yield a profit equal to the carrier initial profit in its existing network. Table 3 also reports, under the columns Dec, the number of times there is a decrease in the carrier's profit with regard to its initial profit (the profit realized when serving only the existing contracts). For the risky bidding context, we additionally display the number of times the carrier's profit is negative (column Neg). Recall that this may only happen for the risky bidding context and never for the risk-averse one. Finally, the last four columns of Table 3 report the relative difference (in percentage) between the average (S_{Avg}), minimum (S_{Min}) and maximum (S_{Max}) profits when comparing the risk-averse context to the risky bidding one $\left(\frac{Value(risk-averse)-Value(risky)}{Value(risk-averse)}\right)$ as well as the absolute difference (S_{Dec}) between the number of times there is a decrease in the profit with regard to the existing network ($Dec(risk-averse) - Dec(risky)$). Detailed results for each problem test are given in C (Table 4 for $\alpha = 5\%$ and Table 5 for $\alpha = 95\%$).

Table 3: Effective profits for the different scenarios of bids success obtained with Algorithms 1 and 2

Context	**Risky (Algorithm 1)**					**Risk-averse (Algorithm 2)**				**Saving/loss**			
						Conservative pricing ($\alpha = 5\%$)							
Bids won	P_{Avg}	P_{Min}	P_{Max}	Dec	Neg	P_{Avg}	P_{Min}	P_{Max}	Dec	S_{Avg}	S_{Min}	S_{Max}	S_{Dec}
1	440.05	-2.00	1198.00	5	1	497.00	112.00	1137.00	1	11.46	101.79	-5.36	-4
2	635.72	1.00	1424.00	3	0	709.90	199.00	1212.00	0	10.45	99.50	-17.49	-3
3	738.00	143.00	1440.00	0	0	982.67	571.00	1409.00	0	24.90	74.96	-2.20	0
4	580.00	580.00	580.00	0	0	–	–	–	–	–	–	–	–
						Aggressive pricing ($\alpha = 95\%$)							
Bids won	P_{Avg}	P_{Min}	P_{Max}	Dec	Neg	P_{Avg}	P_{Min}	P_{Max}	Dec	S_{Avg}	S_{Min}	S_{Max}	S_{Dec}
1	492.70	-628.00	2188.00	16	10	644.55	1.00	1840.00	8	23.56	62900.00	-18.91	-8
2	521.12	-560.00	2295.00	39	15	765.32	67.00	2187.00	12	31.91	935.82	-4.94	-27
3	739.45	-385.00	2217.00	17	9	987.54	269.00	2233.00	0	25.12	243.12	0.72	-17
4	1045.42	-6.00	1956.00	1	1	1244.22	505.00	1809.00	0	15.98	101.19	-8.13	-1
5	1461.26	568.00	2100.00	0	0	1590.50	1067.00	1908.00	0	8.13	46.77	-10.06	0
6	1946.37	1130.00	2274.00	0	0	2082.00	2082.00	2082.00	0	6.51	45.73	-9.22	0
7	2515.00	2515.00	2515.00	0	0	–	–	–	–	–	–	–	

The results of Table 3 show that the trend is reversed in comparison to Table 2 when real profits associated with the two bidding contexts are compared for the different scenarios of bids success: a risk-averse bidding results in an average profit that is always larger than that obtained with a risky one. One should notice, however, that such comparison is only possible for the scenarios where the value of the number of bids won is possible for both contexts (this value must be lower than or equal to the the number of bids generated in the two contexts). Moreover, a risky bidding yielded a negative profit once when a conservative pricing strategy is considered and 35 times when an aggressive pricing is rather adopted. Finally, a risk-averse bidding results in a decrease in profits with regard to the initial existing network for 21 instances in total compared to 81 instances for the risky bidding context.

In summary, our experimental study shows that even though at first glance a risk-averse bidding context requires larger CPU times to generate no-risky bids and yields lower maximum expected profits when compared to a risky bidding context, it generally results in larger real profits when different scenarios of bids success are simulated. While a risky context may yield negative profits for 36 scenarios and monetary losses with regard to the initial profit for 81 scenarios, a risk-averse bidding ensures a non-negative profit for all the scenarios and yields monetary losses for only 21 scenarios. Besides, it is noteworthy that the decrease in the maximum expected profit is observed for the same pricing strategy. These results should thus be interpreted with some restraint since each bidding context should be combined with an appropriate pricing strategy to illustrate more realistically a bidding behavior of a carrier: in practice, it is unlikely that a carrier adopts a risky bidding with an aggressive pricing thus taking a very big risk of losing its bids and incurring monetary losses. It is also less likely that a carrier chooses a risk-averse bidding with a conservative pricing strategy if it has the guarantee of a non-negative profit even if its bids are not won.

7 Conclusion

This paper addresses a new variant of the Bid Construction Problem (BCP) with stochastic clearing prices for transportation services procurement in combinatorial auctions. It extends previous works on stochastic BCP by proposing an exact method that ensures a non-negative profit for the carrier regardless of the combinatorial *OR* bids effectively won and the auctioned contracts that effectively materialize. To the best of our knowledge, our paper is the first to address such a problem. Our experimental results show that for both conservative and aggressive pricing strategies that could be adopted by the carrier, our new bid construction procedure yields on average larger effective profits when different scenarios of bids success are simulated than a procedure where risks on bids loss are not handled.

The approach we propose is conservative in the sense that the risks of a negative profit yielded by contracts non-materialization or loss are eliminated. An interesting research avenue would be to conceive a less conservative approach where contracts materialization are modeled as random variables with known probability distribution functions.

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A Relaxed problem

The relaxed model $\overline{M_p}(\overline{\mathfrak{B}})$ is given by:

$$\overline{M_p}(\overline{\mathfrak{B}}) : \max \sum_{k\in K_e} p_k + \sum_{b\in\mathfrak{B}\cup\overline{\mathfrak{B}}} \zeta_b - \sum_{l\in L}\sum_{(i,j)\in A} c_{ij}x^l_{ij} - \sum_{l\in L}\sum_{j\in O} f x^l_{0j} \tag{47}$$

$$\text{s.t. } (4)-(5), (7)-(22) \text{ and to}$$

$$\zeta_b \le S_b \left(\sum_{k\in K_n} \overline{p_k}\delta_{kb} + \Phi^{-1}(\alpha)\sqrt{\sum_{k\in K_n}\delta_{kb}\sigma_k^2} \right) \qquad \forall b \in \overline{\mathfrak{B}} \tag{48}$$

$$(26) \text{ if } \alpha \le \frac{1}{2}$$

$$(27) \text{ if } \alpha > \frac{1}{2}$$

$$\sum_{b\in\mathfrak{B}\cup\overline{\mathfrak{B}}} w_b \le \gamma \tag{49}$$

$$\sum_{b\in\mathfrak{B}} z_{kb} + \sum_{b\in\overline{\mathfrak{B}}} \delta_{kb}w_b \le 1 \qquad \forall k \in K_n \tag{50}$$

$$\sum_{l\in L} x^l_{o_k d_k} = \sum_{b\in\mathfrak{B}} z_{kb} + \sum_{b\in\overline{\mathfrak{B}}} \delta_{kb}w_b \qquad \forall k \in K_n \tag{51}$$

$$\sum_{\substack{k\in K_n:\\ \delta_{kb'}=0}} z_{kb} + \sum_{\substack{k\in K_n:\\ \delta_{kb'}=1}} (1-z_{kb}) \ge 1 \qquad \forall b \in \mathfrak{B}, b' \in \overline{\mathfrak{B}} \tag{52}$$

$$\zeta_b \le M w_b \qquad \forall b \in \mathfrak{B}\cup\overline{\mathfrak{B}} \tag{53}$$

$$\zeta_b \le p_b \qquad \forall b \in \mathfrak{B}\cup\overline{\mathfrak{B}} \tag{54}$$

$$p_b + M(w_b - 1) \le \zeta_b \qquad \forall b \in \mathfrak{B}\cup\overline{\mathfrak{B}} \tag{55}$$

$$\sum_{b\in\mathfrak{B}} w_b + \psi \ge 1 \tag{56}$$

$$\left|\overline{\mathfrak{B}}^*\right| (\psi - 1) \le \sum_{b\in\overline{\mathfrak{B}}^*} w_b - \left|\overline{\mathfrak{B}}^*\right| \tag{57}$$

$$\sum_{b\in\overline{\mathfrak{B}}^*} w_b - \left|\overline{\mathfrak{B}}^*\right| \le \left(\left|\overline{\mathfrak{B}}^*\right| - \gamma\right)(\psi - 1) \tag{58}$$

$$\sum_{b\in\overline{\mathfrak{B}}\setminus\overline{\mathfrak{B}}^*} w_b \le \gamma\,(1-\psi) \tag{59}$$

$$\psi \in \{0,1\} \tag{60}$$

$$p_b \geq 0 \qquad \forall b \in \mathfrak{B} \cup \overline{\mathfrak{B}} \tag{61}$$

$$\zeta_b \geq 0 \qquad \forall b \in \mathfrak{B} \cup \overline{\mathfrak{B}}. \tag{62}$$

Constraints (56)–(60) are accelerating constraints. Here, set $\overline{\mathfrak{B}}^*$ contains the indices of the selected bids in the previous iteration of the algorithm (previous call to this model). $\overline{\mathfrak{B}}^*$ is initially empty. Observe that acceleration constraints (56)–(60) impose that if no bid of index $b \in \mathfrak{B}$ is selected (i.e., all $w_b = 0$) then the bids of indices $b \in \overline{\mathfrak{B}}^* = \{\bar{b} \in \overline{\mathfrak{B}} : w^*_{\bar{b}} = 1\}$ of the previous iteration (best solution so far) gets selected because $\psi = 1$ in (56).

B Restricted problem

We define a restricted problem as a CSPP for which a set of indices of partial bids, denoted by $\overline{\mathfrak{B}}$, is known by the auctioned contracts that form it but not the associated price. Hence, $\forall b, b' \in \overline{\mathfrak{B}}$: $b \neq b'$, $K_b \neq K_{b'}$ where K_b denotes the set of auctioned contracts covered by a bid of index b.

Let δ_{kb} a set of binary parameters defined for each auctioned contract $k \in K_n$ and each partial bid of index $b \in \overline{\mathfrak{B}}$: $\delta_{kb} = 1$ if contract $k \in K_n$ is generated within a bid of index $b \in \overline{\mathfrak{B}}$, and $\delta_{kb} = 0$ otherwise. As in Hammami et al. (2021), we assume that partial bids are sorted in an ascending order and the lowest partial bid index is denoted $\bar{b}_{min}$. The restricted model, denoted by $\underline{M_p}$, can be formulated as follows:

$$\underline{M_p} : \max \sum_{k \in K_e} p_k + \sum_{b \in \overline{\mathfrak{B}}} \zeta_b - \sum_{l \in L} \sum_{(i,j) \in A} c_{ij} x^l_{ij} - \sum_{l \in L} \sum_{j \in O} f x^l_{0j} \tag{63}$$

$$\text{s.t. } (9)-(21), \text{ and to}$$

$$\zeta_b \leq S_b \left(\sum_{k \in K_n} \overline{p_k} \delta_{kb} + \Phi^{-1}(\alpha) \sqrt{\sum_{k \in K_n} \delta_{kb} \sigma^2_k} \right) \qquad \forall b \in \overline{\mathfrak{B}} \tag{64}$$

$$\sum_{b \in \overline{\mathfrak{B}}} \delta_{kb} w_b \leq 1 \qquad \forall k \in K_n \tag{65}$$

$$w_b \leq w_{b-1} \qquad \forall b \in \overline{\mathfrak{B}} \setminus \{\bar{b}_{min}\} \tag{66}$$

$$p_b \leq p_{b-1} \qquad \forall b \in \overline{\mathfrak{B}} \setminus \{\bar{b}_{min}\} \tag{67}$$

$$\sum_{l \in L} x^l_{o_k d_k} = \sum_{b \in \overline{\mathfrak{B}}} \delta_{kb} w_b \qquad \forall k \in K_n \tag{68}$$

$$\zeta_b \leq M w_b \qquad \forall b \in \overline{\mathfrak{B}} \tag{69}$$

$$\zeta_b \leq p_b \qquad \forall b \in \overline{\mathfrak{B}} \tag{70}$$

$$p_b + M(w_b - 1) \leq \zeta_b \qquad \forall b \in \overline{\mathfrak{B}} \tag{71}$$

$$w_b \in \{0, 1\} \qquad \forall b \in \overline{\mathfrak{B}} \tag{72}$$

$$p_b \geq 0 \qquad \forall b \in \overline{\mathfrak{B}} \tag{73}$$

$$\zeta_b \geq 0 \qquad \forall b \in \overline{\mathfrak{B}}. \tag{74}$$

C Detailed simulation results

Tables 4 and 5 summarize the results obtained when the parameter $\alpha = 5\%$ and $\alpha = 95\%$, respectively. For each problem test, scenarios of bids success are grouped with regard to the number of bids won (column Bids won) and the columns corresponding to each line and each context report the average (P_{Avg}), minimum (P_{Min}) and maximum profit (P_{max}) effectively realized by the carrier for these scenarios. Observe that the results corresponding to scenarios where none of this bids are won are not reported since both contexts yield a profit equal to the carrier initial profit in its existing network. Tables 4 and 5 also report, under the columns Dec, the number of times, for each group of scenarios (line), there is a decrease in the carrier's profit with regard to its initial profit (the profit realized when serving only the existing contracts). For Algorithm 1, we additionally display the number of times the carrier's profit is negative (column Neg). Recall that this may happen for Algorithm 1 but never for Algorithm 2. Finally, the last four columns of Tables 4 and 5 report the relative difference (in percentage) between the average (S_{Avg}), minimum (S_{Min}) and maximum (S_{Max}) profits when comparing Algorithm 2 to Algorithm 1 $\left(\frac{Value(Risk-averse)-Value(Risky)}{Value(Risk-averse)}\right)$ and the absolute difference (S_{Dec}) between the number of times there is a decrease in the profit with regard to the existing network $(Dec(Risk-averse)-Dec(Risky))$.

Table 4: Impact on profits for the different scenarios of bids success for $\alpha = 5\%$

Ins	**Context**	**Risky (Algorithm 1)**					**Risk-averse (Algorithm 2)**				**Saving/loss (%)**			
	Bids won	P_{Avg}	P_{Min}	P_{Max}	Dec	Neg	P_{Avg}	P_{Min}	P_{Max}	Dec	S_{Avg}	S_{Min}	S_{Max}	S_{Dec}
1	1	161.00	-2.00	456.00	1	1	176.00	112.00	290.00	0	9.32	5700.00	-36.40	-1
	2	261.33	1.00	456.00	1	0	345.33	199.00	481.00	0	32.14	19800.00	5.48	-1
	3	384.50	143.00	577.00	0	0	571.00	571.00	571.00	0	48.50	299.30	-1.04	0
	4	580.00	580.00	580.00	0	0	–	–	–	–	–	–	–	0
9	1	321.33	68.00	665.00	2	0	961.00	961.00	961.00	0	199.07	1313.24	44.51	-2
	2	571.33	527.00	637.00	2	0	–	–	–	–	–	–	–	-2
	3	1059.00	1059.00	1059.00	0	0	–	–	–	–	–	–	–	0
10	1	369.00	369.00	369.00	0	0	136.00	136.00	136.00	0	-63.14	-63.14	-63.14	0
15	1	444.00	131.00	757.00	1	0	605.00	371.00	839.00	0	36.26	183.21	10.83	-1
	2	1068.00	1068.00	1068.00	0	0	925.00	925.00	925.00	0	-13.39	-13.39	-13.39	0
16	1	548.67	398.00	641.00	0	0	484.33	339.00	607.00	0	-11.73	-14.82	-5.30	0
	2	838.00	714.00	987.00	0	0	730.33	673.00	823.00	0	-12.85	-5.74	-16.62	0
	3	1129.00	1129.00	1129.00	0	0	968.00	968.00	968.00	0	-14.26	-14.26	-14.26	0
19	1	386.50	340.00	433.00	0	0	488.00	488.00	488.00	0	26.26	43.53	12.70	0
	2	749.00	749.00	749.00	0	0	–	–	–	–	–	–	–	0
21	1	694.00	694.00	694.00	0	0	438.00	438.00	438.00	0	-36.89	-36.89	-36.89	0
22	1	336.00	240.00	432.00	0	0	407.00	407.00	407.00	0	21.13	69.58	-5.79	0
	2	470.00	470.00	470.00	0	0	–	–	–	–	–	–	–	0
23	1	863.67	542.00	1198.00	1	0	777.00	419.00	1137.00	1	-10.03	-22.69	-5.09	0
	2	1120.00	820.00	1424.00	0	0	982.33	616.00	1212.00	0	-12.29	-24.88	-14.89	0
	3	1440.00	1440.00	1440.00	0	0	1409.00	1409.00	1409.00	0	-2.15	-2.15	-2.15	0

Table 5: Impact on profits for the different scenarios of bids success for $\alpha = 95\%$

Ins	Context	Risky (Algorithm 1)					Risk-averse (Algorithm 2)				Saving/loss (%)			
	Bids won	P_{Avg}	P_{Min}	P_{Max}	Dec	Neg	P_{Avg}	P_{Min}	P_{Max}	Dec	S_{Avg}	S_{Min}	S_{Max}	S_{Dec}
1	1	233.50	43.00	627.00	0	0	265.00	111.00	627.00	0	13.49	158.14	0.00	0
	2	402.33	144.00	758.00	0	0	444.70	103.00	758.00	0	10.53	-28.47	0.00	0
	3	542.20	166.00	968.00	0	0	607.80	269.00	909.00	0	12.10	62.05	-6.10	0
	4	714.53	332.00	1026.00	0	0	802.20	505.00	916.00	0	12.27	52.11	-10.72	0
	5	880.67	568.00	1126.00	0	0	1067.00	1067.00	1067.00	0	21.16	87.85	-5.24	0
	6	1130.00	1130.00	1130.00	0	0	–	–	–	–	–	–	–	0
9	1	578.29	50.00	1251.00	4	0	633.50	50.00	1251.00	3	9.55	0.00	0.00	-1
	2	689.95	238.00	1309.00	9	0	711.33	288.00	1309.00	7	3.10	21.01	0.00	-2
	3	985.71	474.00	1429.00	1	0	994.05	637.00	1429.00	0	0.85	34.39	0.00	-1
	4	1313.29	898.00	1633.00	0	0	1326.87	995.00	1630.00	0	1.03	10.80	-0.18	0
	5	1661.90	1321.00	2100.00	0	0	1682.83	1462.00	1908.00	0	1.26	10.67	-9.14	0
	6	2063.00	1932.00	2274.00	0	0	2082.00	2082.00	2082.00	0	0.92	7.76	-8.44	0
	7	2515.00	2515.00	2515.00	0	0	–	–	–	–	–	–	–	0
10	1	-299.00	-628.00	248.00	4	4	177.67	1.00	450.00	2	159.42	100.16	81.45	-2
	2	-201.70	-560.00	230.00	9	8	80.33	67.00	105.00	3	139.83	111.96	-54.35	-6
	3	50.80	-385.00	525.00	6	6	514.00	514.00	514.00	0	911.81	233.51	-2.10	-6
	4	357.20	-6.00	572.00	1	1	–	–	–	–	–	–	–	-1
	5	817.00	817.00	817.00	0	0	–	–	–	–	–	–	–	0
11	1	801.80	41.00	1381.00	2	0	967.00	136.00	1381.00	1	20.60	231.71	0.00	-1
	2	726.40	40.00	1020.00	3	0	827.67	637.00	1020.00	2	13.94	1492.50	0.00	-1
	3	1097.20	549.00	1406.00	1	0	1191.75	1008.00	1313.00	0	8.62	83.61	-6.61	-1
	4	1448.00	1249.00	1580.00	0	0	1809.00	1809.00	1809.00	0	24.93	44.84	14.49	0
	5	1994.00	1994.00	1994.00	0	0	–	–	–	–	–	–	–	0
12	1	142.00	-499.00	1004.00	3	3	1004.00	1004.00	1004.00	0	607.04	301.20	0.00	-3
	2	-2.90	-384.00	371.00	9	4	–	–	–	–	–	–	–	-9
	3	304.10	-208.00	722.00	6	2	–	–	–	–	–	–	–	-6
	4	578.00	425.00	772.00	0	0	–	–	–	–	–	–	–	0
	5	1062.00	1062.00	1062.00	0	0	–	–	–	–	–	–	–	0
14	1	226.00	-319.00	1064.00	3	3	1064.00	1064.00	1064.00	0	370.80	433.54	0.00	-3
	2	105.10	-204.00	431.00	9	3	–	–	–	–	–	–	–	-9
	3	436.10	-28.00	782.00	3	1	–	–	–	–	–	–	–	-3
	4	734.00	605.00	832.00	0	0	–	–	–	–	–	–	–	0
	5	1242.00	1242.00	1242.00	0	0	–	–	–	–	–	–	–	0
15	1	694.33	531.00	782.00	0	0	497.00	189.00	954.00	1	-28.42	-64.41	21.99	1
	2	1086.33	810.00	1349.00	0	0	785.50	345.00	1110.00	0	-27.69	-57.41	-17.72	0
	3	1706.00	1706.00	1706.00	0	0	1052.50	608.00	1222.00	0	-38.31	-64.36	-28.37	0
	4	–		–		–	1377.00	1377.00	1377.00	0	–	–	–	0
16	1	683.50	345.00	994.00	0	0	513.40	276.00	834.00	0	-24.89	-20.00	-16.10	0
	2	1106.83	645.00	1567.00	0	0	772.60	426.00	1104.00	0	-30.20	-33.95	-29.55	0
	3	1531.00	1188.00	1872.00	0	0	1035.40	650.00	1327.00	0	-32.37	-45.29	-29.11	0
	4	1956.00	1956.00	1956.00	0	0	1298.80	1060.00	1477.00	0	-33.60	-45.81	-24.49	0
	5	–	–	–		–	1560.00	1560.00	1560.00	0	–	–	–	0
18	1	1532.00	876.00	2188.00	0	0	1284.67	695.00	1840.00	1	-16.14	-20.66	-15.90	1
	2	2295.00	2295.00	2295.00	0	0	1714.67	1177.00	2187.00	0	-25.29	-48.71	-4.71	0
	3	–	–	–		–	2233.00	2233.00	2233.00	0	–	–	–	0
19	1	577.00	547.00	607.00	0	0	504.00	452.00	556.00	0	-12.65	-17.37	-8.40	0
	2	1130.00	1130.00	1130.00	0	0	794.00	794.00	794.00	0	-29.73	-29.73	-29.73	0
21	1	698.00	683.00	713.00	0	0	440.00	440.00	440.00	0	-36.96	-35.58	-38.29	0
	2	942.00	942.00	942.00	0	0	–	–	–	–	–	–	–	0
22	1	486.50	471.00	502.00	0	0	502.00	502.00	502.00	0	3.19	6.58	0.00	0
	2	771.00	771.00	771.00	0	0	–	–	–	–	–	–	–	0
23	1	1122.67	580.00	1497.00	0	0	1028.67	678.00	1502.00	0	-8.37	16.90	0.33	0
	2	1638.00	1298.00	2163.00	0	0	1485.67	1006.00	1836.00	0	-9.30	-22.50	-15.12	0
	3	2217.00	2217.00	2217.00	0	0	2164.00	2164.00	2164.00	0	-2.39	-2.39	-2.39	0