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June 2022

Document de travail également publié par la Faculté des sciences de l'administration de l'Université Laval, sous le numéro FSA-2022-006.

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An SDDP-based Solution Approach for Carriers' Selection and Shipments Assignment under Dynamic Stochastic Demand

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Abstract. We consider a distribution network where shipment services between warehouses and distribution centers must be ensured by external carriers taking into account demand uncertainty. The problem is tackled at the operational decision level and is governed by contractual restrictions and decisions already taken at the strategic level. Strategic decisions provide a set of core carriers, and their shipping conditions, selected using a combinatorial auction mechanism. Operational decisions involve the set of the selected core carriers and a set of spot carriers available to procure transportation services. The main objective of the addressed problem is to determine optimal transportation decisions that minimizes inventory, backorder, and transportation costs over a finite planning horizon under a dynamic stochastic demand at distribution centers. The problem is modeled using a stochastic linear multistage formulation and solved by adapting the Stochastic Dual Dynamic Programming (SDDP) algorithm. Our paper is the first to adapt the SDDP algorithm to solve a multi-period and multi-product carriers' selection and shipments assignment problem for distribution networks under demand uncertainty. Our computational results highlight the good performance of SDDP as near-optimal solutions are obtained within a reasonable computational time. We also study the benefits of using our SDDP-based method rather than an average scenario-based approach.

Keywords: Operational decision making, transportation procurement, multistage model, carriers' selection problem, shipment assignment problem, stochastic dual dynamic programming.

Acknowledgements. This project was funded by the Natural Sciences and Engineering Council of Canada (NSERC) under grant 2016-04482. This support is greatly acknowledged.

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1. Introduction

In the past 30 years, companies have been outsourcing many logistic functions to cope with the complexity of managing the whole supply chain. Freight transport outsourcing was particularly encouraged by the deregulation of road haulage in the 1980s. Transportation services procurement markets, in general, include two main actors: (1) the shippers that outsource their transportation operations to a third-party logistics, and (2) carriers offering transportation services of different types and at various rates. Our paper deals with the procurement of Truckload (TL) operations services in a distribution network. We consider a context where a company uses the TL services of external carriers to deliver a set of products from a set of warehouses to a set of distribution centers. TL operations imply that shipments are moved in full trailers from origins to destinations without any intermediate stops.

Transportation services procurement follows a three-level process: strategic, tactical, and operational. Shippers generally select for-hire carriers at the strategic/tactical levels and commit with them on a long-time period. This would ensure that some carriers are available to satisfy their shipment requests at the operational level with interesting transportation rates. At the strategic level, the shipper starts by constituting a small data basis of potential carriers with which it would engage on a long-term period. These carriers are pre-selected based on a number of criteria such as the carrier's financial stability, its shipment capacity and the geographical regions it covers. Once the strategic pre-selection phase is achieved, the shipper and the "shortlisted" carriers start an information exchange phase. The objective is to select carriers with which the company will engage on for the next planning horizon (one to three years).

In the last decades, auctions, and more particularity combinatorial ones, have gained popularity as an efficient market mechanism for strategic carriers' selection. In these auctions, the shipper acts as the auctioneer and presents its shipping requests to the set of pre-selected carriers. Participating carriers compete by submitting their offers in the form of bids. Combinatorial auctions have the particularity to permit bids on packages of shipping requests so that either all the requests in the package are allocated to the carrier if its bid is won, or nothing at all. When compared to simple bidding, combinatorial bidding enables a carrier to better express its preferences and to take advantage from the economy of scope characterizing transportation operations.

When organizing the auction, the shipper uses historical data and estimations of future outcomes for the upcoming planning horizon to provide the participating carriers with some information on the shipments to be auctioned (e.g., pick-up/delivery locations, products types, approximate shipment volumes). At this strategic level, such information is generally aggregated and not known with certainty (Remli et al., 2019). The shipper also gives some details on the auction format and the bids structure (Abrache et al., 2007). Based on this, the carriers solve a Bid Construction Problem (BCP) to determine which bids to submit. Once all the carriers' bids are received, the shipper determines the winners of the auction by solving the so-called Winner Determination Problem (WDP) and approximately knows which carrier would probably serve which shipping request and at which price for the next planning horizon. The winning carriers of the auction determined at the strategic level are referred to as "strategic carriers" in the rest of the paper.

At the operational level, the planning horizon is discretized in shorter time periods (weeks or months) compared to the strategic level. The shipper has then more disaggregated and precise information on its distribution network when selecting carriers and allocating shipments. The final contracts with strategic carriers remain valid but the shipper may allocate some shipments to spot carriers to meet the actual demand observed for each month or each week of the planning horizon. Moreover, at the strategic level, the shipper has selected strategic carriers with the objective to only minimize transportation costs. At the operational level, other costs such as the inventory and backorder costs as well as spot carriers transportation costs must be taken into account.

This paper addresses a Carriers' Selection and Shipments' Assignment Problem (CSSAP) that must be solved by a company to distribute its products from its warehouses to its DCs. The problem is tackled at the operational level and incorporates a number of restrictions deriving from the strategic selection level. More precisely, CSSAP consists in finding the best way to select and allocate shipments to external carriers so that the total expected cost of transportation, inventory and backordering is minimized over a finite planning horizon. The demand at each distribution center is assumed stochastic and is modeled as a random variable with known probability distribution for each time period. The decisions of shipments allocation are made at the beginning of each period of the planning horizon with available short-term information (e.g., inventory levels and costs, spot carriers' transportation costs). They are governed by contractual restrictions and decisions from the strategic level (e.g., transportation rates of the carriers selected at the strategic level, shipping contracts won, etc.). To the best of our knowledge our paper is the first to address such a problem in a stochastic dynamic context.

CSSAP is formulated as a Multi-Stage Stochastic Program (MSSP) that can be solved by a Dynamic Programming (DP) algorithm. However, for such multi-period, multi-product, multi-warehouse and multi-DC problem, DP suffers from the curse of dimensionality. To handle this issue, Approximate Dynamic Programming (ADP) algorithms and more precisely the well-known Stochastic Dual Dynamic Programming (SDDP) algorithm is considered and adapted. This algorithm models the stochastic process as a scenario tree that approximates the distribution of demands using a Monte Carlo Simulation, splits the problem into small and tractable sub-problems solved in each stage separately, and helps approximating the future cost function iteratively. Our experimental results prove the efficiency of the SDDP in finding robust solutions in reasonable computing times. To the best of our knowledge, our paper is the first to adapt and evaluate the performance of the SDDP approach to a transportation services procurement problem.

The reminder of the paper is organized as follows. Section 2 reviews relevant and recent papers in the literature dealing with transportation services procurement. Section

3 defines the problem addressed and presents a mathematical formulation to model it. The SDDP algorithm is presented in Section 4. Experimental results are reported and discussed in Section 5. Finally, Section 6 concludes the paper and describes possible extensions for future work.

2. Literature review

In the last decades, several works have been proposed in the literature to help shippers select carriers either at the strategic or the operational decision levels. Electronic markets and more specifically reverse combinatorial auctions have been proved to be efficient mechanisms for the strategic procurement of TL services (Caplice and Sheffi, 2006). A number of interesting papers proposed either stochastic programming or robust optimization approaches to deal with demand uncertainty for strategic carriers' selection trough combinatorial auctions (Remli and Rekik, 2013; Zhang et al., 2014, 2015). A recent paper by Remli et al. (2019) addressed the strategic carriers' selection problem with two uncertain parameters: the shipper's demand and the carriers' capacity. Short-term transportation procurement problems present in spot markets were also addressed by various researchers (Figliozzi et al., 2003; Garrido, 2003; Mes et al., 2009; Berger and Bierwirth, 2010; Lindsey and Mahmassani, 2017; Budak et al., 2017). However a limited number of works have tackled the stochastic version of the problem. In the rest of this section, we report and discuss relevant papers addressing carriers' selection and/or shipments assignment at the operational level within stochastic environments.

(Agralı et al., 2008) proposed a two-stage stochastic model in a logistics spot market with two types of carriers: local carriers based at the same region as the logistic center and in-transit carriers that may be called by the logistics center while traveling to their bases. In the first stage, an auction is analyzed depending on the given number of carriers of each type that bid for an order and their cost distributions. In the second stage, a continuous Markov Chain model is developed to determine the steady-state probability distribution of the number of carriers in the logistics center. Finally, they demonstrate the effects of various parameters such as order and carrier arrival and abandonment rates on the performance of the system.

(Tsai et al., 2011) applied concepts from the theory of Real Options to deal with uncertainty in transportation capacity and costs using derivative contracts called truckload options. They propose a mean-reverting stochastic process to model the spot prices of truckloads and develop explicit pricing formulas for truckloads calls. Further, they provided an approach to estimate the potential value of truckload options to both shippers and carriers for selected lanes.

(Xu and Huang, 2013) proposed a periodic double auction model to address transportation service procurement in a dynamic single-lane transportation spot market taking into account the stochastic arrivals of demand and supply. In dynamic transportation context, sequential auctions allow the purchase of transportation service to be real-time or periodic based on dynamic pricing strategies. They develop asymptotically efficient double auction mechanisms for transportation service procurement under two scenarios: (i) symmetric demand and supply and (ii) supply-demand imbalance. The proposed method determines the optimal operational strategy for the logistic e-marketplace to gain higher myopic profit from a relatively short auction length.

(Feki et al., 2016) proposed an adaptive carriers' selection strategy that minimizes transportation, inventory and shortage costs in dynamic stochastic environment while considering a random carriers' availability and market demand. Within a long-short term framework, their study emphasis on the allocation of freight shipments in a short-term continuous time.

Later, (Collado et al., 2020) study a dynamic transportation procurement planning problem under limited information of future demand for transportation services in the presence of a shorter commitment horizon than the planning horizon for procurement contracts. The authors show the value of the availability of partial information in contracting policies and also identify settings when this value is negligeable. The evolution of the system can be formulated as discounted infinite time horizon Markov decision process. We refer the reader to (Lafkihi et al., 2019) and the references therein for more details regarding the existing freight transportation organization and procurement mechanisms and their challenges and opportunities in the context of E-commerce.

Recently, (Wang et al., 2021) suggest an investigation into shipping structure and risk management issues of the ocean freight industry with demand and freight rate uncertainties. Three cases are studied: considering only the long-term contract with the carrier, only the spot freight market and a combination of the two. Stackelberg games model is formulated by taking into account both the carrier's long-term decision (on freight rate) as well as the shipper's long-term decision (on shipment capacity procurement amount) and spot market supplementary procurement decision. The study shows that long-term contract with carrier combining short-term replenishment from spot freight market can increase for the carrier-shipper's overall performance.

A limited discussion on the integration of the strategic and operational planning in freight transportation problem is provided in the literature. Our study belongs to this category, more particularly the carriers' selection and shipments assignment problem under uncertainy. Kantari et al. (2021) address the problem of transportation services sourcing using the mix of contract-based and on-demand sourcing. The proposed model includes the complex shipment problem from plant to customers where the demand is fluctuating following a seasonal pattern and the shipment time is uncertain. Due to the dynamic nature of the problem, discrete event simulation was proposed to model the problem. The effect of the demand fluctuation was measured by three performance indicators: product fill rate, shipment reliability and truck utilization.

(Lopez, 2021) proposed an integrated planning framework considering tactical and operational decisions with minimum commitment contracts in the filed of e-commerce logistics distribution. A multi-period distribution problem is formulated with mixedinteger linear programs in order to minimize the total costs along a finite time horizon with time varying demand. The management problem includes selecting carriers and deciding on the distribution planning for every period along the time horizon. Several solution methods such as the combinatorial Benders algorithm and a heuristic-based relaxation of time-linking constraints are compared.

Our literature review shows that the problem of operationalizing the strategic carrier's selection decisions yielded by a combinatorial auction mechanism has not been addressed before in a stochastic dynamic context. We mean by operationalizing, the allocation of shipments on shorter periods while considering: (i) the decisions made at the strategic level, (ii) the operational constraints and costs other than those related to transportation operations (e.g., inventory and backorder, desegregated demand, depot capacity, replenishment, etc.). Our paper fills these gaps by considering a multi-stage stochastic problem in a dynamic context that incorporates appropriate information and constraints arising form the strategic decision level.

Our paper makes a number of contributions to the field of CSSAPs. First, it is the first to address a CSSAP with a stochastic demand and incorporating restrictions resulting from a combinatorial-auction based strategic selection stage. Second, our paper is the first to use, adapt and prove the good performance of the SDDP approach to solve the CSSAP with stochastic demand. The SDDP approach has been intensively applied for the stochastic dynamic hydrothermal planning and the related power system problems with random events and stochastic processes (Hjelmeland et al., 2019; Morillo et al., 2020; Mbeutcha et al., 2021) and recently used in several applications including pastoral dairy farms (Dowson et al., 2019), portfolio optimization (Valladao et al., 2019) and multi-echelon lot-sizing problem under demand uncertainty (Thevenin et al., 2021). Finally, our paper proves, through an extensive experimental study, the relevance of tackling demand uncertainty for the CSSAP addressed with an SDDP approach and the potential profit it would generate for the shipper when compared to an average scenario-based approach.

3. PROBLEM DEFINITION

3.1. Context and assumptions

The CSSAP addressed in this paper considers a set of warehouses I from which a set of products P must be directly shipped (TL context) to a set of DCs J. The planning horizon T is assumed to be finite and discretized into equal time periods $t \in T$. The demands for all products at DCs are assumed to occur at the beginning of each period. They are assumed to be known with certainty for the first time period. They are stochastic for the remaining periods with known distribution functions. The demands at different time periods, for different products and for different DCs, are assumed to be mutually independent random variables.

Each unit of unsatisfied demand for a current period is backlogged to the next period with a per-unit backordering cost. Transshipment between DCs is not allowed. Inventory is allowed at both the warehouses and the DCs. The excess inventories after shipping decisions are carried to the next period with a per-unit holding cost. Production planning is not integrated in the problem addressed. The quantities of products arriving at warehouses (from the production units) are assumed to be an input parameter assuming a periodic replenishment strategy.

The company has no private fleet. It must outsource all its transportation operations to external carriers with which it already engages on at the strategic level (strategic carriers) or those selected from the spot market at the operational level (spot carriers). The set of strategic carriers is known at the beginning of the planning horizon. It derives from a combinatorial auction process performed by the company at a strategic level. The output of the auction specifies the set of winning carriers and the bids they win. A winning bid includes the set of lanes (origin-destination pairs) won by the strategic carrier, the shipping price for each volume unit shipped on that lane, and the minimum and maximum total volumes that must or can be allocated to the strategic carrier on each lane won for each period of the planning horizon T. The set of all carriers (including both strategic and spot ones) is denoted C.

It is assumed that the final contracts between the shipper and the strategic carriers specify the following terms for the operational level: (1) the shipper must assign a minimum volume for each period of the planning horizon as specified by the carrier for each lane of a winning bid. Otherwise, the shipper pays the carrier a penalty cost for each lacking unit, (2) the carrier commits to providing a capacity per period for each lane of a winning bid at the transporting rates proposed at the strategic level. If the shipper wants to allocate larger volumes, it has to pay the exceeding amount at possibly higher rates.

We assume that there are no losses during transportation (products are not damaged) and that the time required to ensure a shipment between a warehouse and a DC is relatively small (collect and delivery are done within the same discretization period). Finally, the products are assumed homogeneous in size and infinitely divisible (to yield a Linear Programming (LP) model).

3.2. Mathematical model for the deterministic context

When no uncertainty is considered, CSSAP can be formulated with an LP model, denoted M^{det}. Model M^{det} uses the sets, parameters and decision variables described in Table 1.

Table 1: Sets, parameters and decision variables of model (M^{det})

Set	Description
I	Set of warehouses
J	Set of DCs
P	Set of products
CS	Set of strategic carriers
CO	Set of spot carriers
T	Set of discretized periods of equal lengths
B^c	Set of bids won by a strategic carrier $c \in CS$
$L^{c,b}$	Set of lanes l covered by a bid b won by a strategic carrier $c \in CS$
L^o	Set of all lanes (i,j) that can be allocated to the spot carriers
Parameter	**Description**
$Q_{p,i,t}$	Quantity of product p arriving at warehouse i at the beginning of period t
$D_{p,j,t}$	Demand of product p at DC j for period t
R_i	Storage capacity of warehouse i
R_j	Storage capacity of DC j
$C_{c,t}$	Capacity of carrier c in period t
$CW_{p,i}$	Unit inventory cost of product p at warehouse i
$CD_{p,j}$	Unit inventory cost of product p at DC j
$CB_{p,j}$	Unit back order cost product p at DC j
$LB^{c,b}_{i,j}$	Lower bound on the volume per period that must be allocated to strategic carrier $c \in CS$ on lane (i,j) of a winning bid $b \in B^c$
$UB^{c,b}_{i,j}$	Upper bound on the volume per period that can be allocated to strategic carrier $c \in CS$ on lane (i,j) of a winning bid $b \in B^c$
$CT^{c,b}_{p,i,j}$	Transportation cost per unit of product p from warehouse i to DC j proposed by strategic carrier $c \in CS$ in its winning bid b
$CT^{c,o}_{p,i,j}$	Transportation cost per unit of product p from warehouse i to DC j proposed by spot carrier $c \in CO$ at the operational level
$\theta^{c,b}_{i,j}$	Penalty cost associated with strategic carriers if the lower bound imposed on the volume per period that must be allocated to strategic carrier c on lane (i,j) of a winning bid $b \in B^c$ is not respected
$\chi^{c,b}_{i,j}$	Penalty cost associated with core carriers if the upper bound imposed on the volume per period that must be allocated to strategic carrier c on lane (i,j) of a winning bid $b \in B^c$ is not respected
Decision variable	**Description**
$X^b_{p,c,i,j,t}$	Quantity of product p shipped by strategic carrier $c \in CS$ in period t on lane (i,j) where (i,j) is covered by a bid $b \in B^c$ won by carrier c
$X^o_{p,c,i,j,t}$	Quantity of product p shipped on lane (i,j) by a spot carrier $c \in CO$ in period t
$S_{p,i,t}$	Inventory level of product p at warehouse i at the beginning of period t
$S^+_{p,j,t}$	Inventory level of product p at DC j at the beginning of period t
$S^-_{p,j,t}$	Backorder level of product p at DC j at the beginning of period t
$Y^b_{c,i,j,t}$	Lacking amount on a lane $(i,j) \in L^{c,b}$, $b \in B^c$, in period t, with respect to the lower bound $\mathrm{LB}^{c,b}_{i,j}$ promised to strategic carrier $c \in CS$ in its winning bid b
$Z^b_{c,i,j,t}$	Exceeding amount on a lane $(i,j) \in L^{c,b}$, $b \in B^c$, in period t, with respect to the upper bound $UB^{c,b}_{i,j}$ offered by strategic carrier $c \in CS$ in its winning bid b

Model M^{det} is given by:

$$
\begin{aligned}
\min \quad & \sum_{t\in T}\sum_{p\in P}\sum_{i\in I} CW_{p,i}S_{p,i,t} + \sum_{t\in T}\sum_{p\in P}\sum_{j\in J}(CD_{p,j}S^+_{p,j,t} + CB_{p,j}S^-_{p,j,t}) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} CT^{c,b}_{p,i,j}X^b_{p,c,i,j,t} + + \sum_{t\in T}\sum_{p\in P}\sum_{c\in CO}\sum_{(i,j)\in L^o} CT^{c,o}_{p,i,j}X^o_{p,c,i,j,t} \\
& + \sum_{t\in T}\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} (\theta^{c,b}_{i,j}Y^b_{c,i,j,t} + \chi^{c,b}_{i,j}Z^b_{c,i,j,t}) \qquad (1)
\end{aligned}
$$

$$\text{s.t.} \sum_{p\in P}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} X^b_{p,c,i,j,t} \leq C_{ct} \qquad \forall t\in T, c\in CS \tag{2}$$

$$\sum_{p\in P}\sum_{(i,j)\in L^o} X^o_{p,c,i,j,t} \leq C_{ct} \qquad \forall t\in T, c\in CO \tag{3}$$

$$\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} X^b_{p,c,i,j,t} + \sum_{c\in CO}\sum_{(i,j)\in L^o} X^o_{p,c,i,j,t} - S_{p,i,t} + S_{p,i,t+1} - Q_{p,i,t} = 0$$
$$\forall t\in T, p\in P,\ i\in I \tag{4}$$

$$\sum_{p\in P} S_{p,i,t} + \sum_{p\in P} Q_{p,i,t} \leq R_i \qquad \forall i\in I, t\in T \tag{5}$$

$$\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} X^b_{p,c,i,j,t} + \sum_{c\in CO}\sum_{(i,j)\in L^o} X^o_{p,c,i,j,t}$$
$$+ S^+_{p,j,t} - S^-_{p,j,t} - S^+_{p,j,t+1} + S^-_{p,j,t+1} = D_{p,j,t} \qquad \forall t\in T, p\in P, j\in J \tag{6}$$

$$\sum_{p\in P}\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} X^b_{p,c,i,j,t} + \sum_{p\in P}\sum_{c\in CO}\sum_{(i,j)\in L^o} X^o_{p,c,i,j,t} + \sum_{p\in P} S^+_{p,j,t} \leq R_j$$
$$\forall j\in J, t\in T \tag{7}$$

$$\sum_{p\in P} X^b_{p,c,i,j,t} + Y^b_{c,i,j,t} \geq LB^{c,b}_{i,j} \qquad \forall t\in T, c\in CS, b\in B^c, (i,j)\in L^{c,b} \tag{8}$$

$$\sum_{p\in P} X^b_{p,c,i,j,t} - Z^b_{c,i,j,t} \leq UB^{c,b}_{i,j} \qquad \forall t\in T, c\in CS, b\in B^c, (i,j)\in L^{c,b} \tag{9}$$

$$X^b_{p,c,i,j,t}, Y^b_{c,i,j,t}, Z^b_{c,i,j,t} \geq 0 \qquad \forall p\in P, t\in T, c\in CS, b\in B^c, (i,j)\in L^{c,b} \tag{10}$$

$$X^o_{p,c,i,j,t} \geq 0 \qquad \forall p\in P, t\in T, c\in CO, (i,j)\in L^o \tag{11}$$

$$S_{p,i,t} \geq 0 \qquad \forall p\in P, i\in I, t\in T \tag{12}$$

$$S^+_{p,j,t}, S^-_{p,j,t} \geq 0 \qquad \forall p\in P, j\in J, t\in T \tag{13}$$

The objective function (1) aims at minimizing the total expected logistic costs on the whole planning horizon. The logistic costs are composed of the warehouses and DCs inventory costs, the DCs backordering costs, the transportation costs for strategic carriers, the transportation costs for spot carriers, and the penalty costs incurred for not satisfying the minimum and maximum volume commitments established with strategic carriers. Constraints (2), respectively (3), ensure that the total amount transported by strategic carriers, respectively, spot carriers, at each period is lower than or equal to the carrier capacity at each period. Constraints (4) ensure that the inventory level of product p at warehouse i at the beginning of period (t+1) is equal to its inventory level at period t plus the quantity of product p arriving at period t minus the total quantity of product p leaving (transported from) warehouse i during period t. Constraints (5) ensure that the maximum amount that can be stored at warehouse i during period t respects the warehouse capacity. Constraints (6) ensure that the inventory (or backorder) level of product p at DC j at the beginning of period (t+1) is equal to the sum of its inventory (backorder) level at the beginning of period t and the corresponding quantity entering DC j during period t (the quantity transported to it) minus the product demand at period t. Constraints (7) ensure that the maximum amount that can be stored at DC j during period t respects the DC capacity. Constraints (8) define the shortage volume of shipments assigned to a strategic carrier $c \in CS$ at the operational level with respect to the lower bound promised to it, for each bid $b \in B^c$ won by this carrier at the strategic level, and each pair (i, j) covered by b. Constraints (9) are similar to (8) but consider the surplus with respect to upper bounds. Finally, constraints (10)-(13) are non-negativity constraints on the decision variables.

The CSSAP addressed in this paper is a dynamic multi-stage stochastic distribution problem, for which decisions, at the beginning of each period (stage) t, are taken based on the information revealed up to that time and before knowing the value of the actual demand at DCs in the upcoming periods. Next section describes the formulation proposed to model it.

3.3. Multi-stage stochastic model

To deal with the dynamic nature of the problem addressed, we use the SDP paradigm. SDP is a variant of dynamic programming that handles uncertainty on problem parameters. It applies the Bellman recursion (Bellman, 1957) to the stochastic case to determine an optimal policy over a certain period of time. A policy is a rule or function that indicates the decision to take for each possible system state/period t. A state variable is generally a particular variable, from the previous stage (t-1), that fully describes the system state at the beginning of the stage t (Defourny et al., 2012) so that the decision at stage t can be taken without requiring any further information.

In dynamic programming, an optimal policy is constructed in a recursive way. Given the initial system state at the beginning of period/ stage t, an optimal decision is taken yielding a change in the system state. The resulting state will be the initial state for the following period/stage and an optimal decision is made, and so on. The relation between the state value at a stage t+1 with the state value at stage t given the optimal decisions taken at stage t define the transition equation.

For CSSAP, knowing the levels of stocks in the warehouses and in the DCs is sufficient to decide on the quantities to ship. In other words, the vector of state variables at period t is given by $S_t = (S_{p,i,t}, S^+_{p,j,t}, S^-_{p,j,t})$ (recall that these variables represent the inventories and backorders at the beginning of period t). The decisions that must be taken at period t correspond to the quantity to be transported by each carrier within period t from the different warehouses to the different distribution centres and the corresponding excess and lack quantities with regard to the strategic level. Formally, the vector of decision variables at stage t is given by: $X_t = (X^o_{p,c,i,j,t}, X^b_{p,c,i,j,t}, Y^b_{c,i,j,t}, Z^b_{c,i,j,t})$.

Based on this, we formulate the dynamic stochastic CSSAP with a multistage SDP model, denoted by M^{SDP}, as follows :

$$M^{SDP} : \min \ \mathbb{E}_{\widetilde{D}_t}[\sum_{t\in T}\sum_{p\in P}\sum_{i\in I} CW_{p,i}S_{p,i,t} + \sum_{p\in P}\sum_{j\in J}(CD_{p,j}S^+_{p,j,t} + CB_{p,j}S^-_{p,j,t})$$
$$+\sum_{p\in P}\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} CT^{c,b}_{p,i,j}X^b_{p,c,i,j,t} + \sum_{p\in P}\sum_{c\in CO}\sum_{(i,j)\in L^o} CT^{c,o}_{p,i,j}X^o_{p,c,i,j,t}$$
$$+\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}}(\theta^{c,b}_{i,j}Y^b_{c,i,j,t} + \chi^{c,b}_{i,j}Z^b_{c,i,j,t})] \tag{14}$$

$$\text{s.t.} \quad (2), (3), (5), (7) - (13)$$

$$S_{p,i,t+1} = S_{p,i,t} + Q_{p,i,t} - \sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} X^b_{p,c,i,j,t} - \sum_{c\in CO}\sum_{(i,j)\in L^o} X^o_{p,c,i,j,t}$$
$$\forall t \in T, p \in P, i \in I \tag{15}$$

$$S^-_{p,j,t+1} - S^+_{p,j,t+1} = S^-_{p,j,t} - S^+_{p,j,t} + D_{p,j,t} - \sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} X^b_{p,c,i,j,t}$$

$$- \sum_{c \in CO} \sum_{(i,j) \in L^o} X^o_{p,c,i,j,t} \forall t \in T, p \in P, j \in J \tag{16}$$

The objective (14) aims to minimize the cost of the first stage plus the expected cost of the optimal decisions of all subsequent stages resulting from the decision of the first stage, the recourse decisions and the random variables $\widetilde{D}_2 ... \widetilde{D}_{|T|}$. Constraints (15) and (16) are the transition equations.

By applying the Bellman's principal of optimality, objective function (14) is reformulated as:

$$\begin{aligned} F_1(X_1, S_1, \widetilde{D}_1) = \min & \sum_{p \in P} \sum_{i \in I} CS_{p,i} S_{p,i,1} + \sum_{p \in P} \sum_{j \in J} (CD_{p,j} S^+_{p,j,1} + CB_{p,j} S^-_{p,j,1}) + \\ & \sum_{p \in P} \sum_{c \in CS} \sum_{b \in B^c} \sum_{(i,j) \in L^{c,b}} CT^{c,b}_{p,i,j} X^b_{p,c,i,j,1} + \sum_{p \in P} \sum_{c \in CO} \sum_{(i,j) \in L^o} CT^{c,o}_{p,i,j} X^o_{p,c,i,j,1} + \\ & \sum_{c \in CS} \sum_{b \in B^c} \sum_{(i,j) \in L^{c,b}} (\theta^{c,b}_{i,j} Y^b_{c,i,j,1} + \chi^{c,b}_{i,j} Z^b_{c,i,j,1}) + \mathbb{E}_{\widetilde{D}_2} [F_2(X_2, S_2, \widetilde{D}_2)] \end{aligned} \tag{17}$$

$F_1(X_1, S_1, \widetilde{D}_1)$ corresponds to the optimal costs for the entire horizon, which are incurred by the immediate first stage decisions. $F_2(X_2, S_2, \widetilde{D}_2)$ represents the second stage costs associated with decision X_2 and realization $\widetilde{D}_2$ starting from period 2 until the end of the planning horizon. The same decomposition process is repeated for every stage of the horizon. The problem at stage $t \in T$, can thus be formulated using model M^{SDP}_t as follows:

$$\begin{aligned} M^{SDP}_t : F_t(X_t, S_t, \widetilde{D}_t) = \min & \sum_{p \in P} \sum_{i \in I} CS_{p,i} S_{p,i,t} + \sum_{p \in P} \sum_{j \in J} (CD_{p,j} S^+_{p,j,t} + CB_{p,j} S^-_{p,j,t}) + \\ & \sum_{p \in P} \sum_{c \in CS} \sum_{b \in B^c} \sum_{(i,j) \in L^{c,b}} CT^{c,b}_{p,i,j} X^b_{p,c,i,j,t} + \sum_{p \in P} \sum_{c \in CO} \sum_{(i,j) \in L^o} CT^{c,o}_{p,i,j} X^o_{p,c,i,j,t} + \\ & \sum_{c \in CS} \sum_{b \in B^c} \sum_{(i,j) \in L^{c,b}} (\theta^{c,b}_{i,j} Y^b_{c,i,j,t} + \chi^{c,b}_{i,j} Z^b_{c,i,j,t}) + \mathbb{E}_{\widetilde{D}_{t+1}} [F_{t+1}(X_{t+1}, S_{t+1}, \widetilde{D}_{t+1})] \end{aligned} \tag{18}$$

$$\text{s.t.} \quad (2), (3), (5), (7) - (13), (15), (16)$$

The objective function (18) is the sum of the expected immediate costs at stage t and the future cost from $t + 1$ to the end of the horizon. $F_{t+1}(.)$ is referred to as the cost-to-go function from $t+1$ to the end of the horizon and $\mathbb{E}_{\widetilde{D}_{|T|+1}}[F_{|T|+1}] = 0$. The future cost function $F_{t+1}(.)$ depends on the inventory level in warehouses $S_{p,i,t+1}$, the inventory and the backorder levels in DCs ($S^{+}_{p,j,t+1}, S^{-}_{p,j,t+1}$) at the beginning of stage $t+1$ and the random demand $\widetilde{D}_{t+1} \dots \widetilde{D}_{|T|+1}$. This function also implicitly depends on the realizations of the random parameters in the previous stages.

As mentioned before, the solution of the proposed multi-stage problem is based on the approximation of the recourse functions using a finite set of outcomes. However, the DP technique suffers from the curse of dimensionality: the problem size increases exponentially with the state space and becomes thus very difficult to solve ((Bellman, 1957)). To overcome this difficulty, (Pereira and Pinto, 1991) propose the SDDP approach that uses an outer approximation of the cost to go function based on Benders cuts without discretizing the state space. Section 4 describes in details how the SDDP algorithm is adapted to our problem.

4. STOCHASTIC DUAL DYNAMIC PROGRAMMING METHOD

The SDDP is a sampling-based algorithm that can be used to solve stochastic linear problems with a large number of stages. It approximates the Bellman function with a set of piecewise linear functions by iteratively sampling a finite number of scenarios using two passes: a forward and a backward passes(Pereira and Pinto, 1991). At each iteration, the algorithm generates a sequence of feasible decisions, called trial solutions, starting at the first stage and moving forward up to the last stage. At the completion of the forward pass, a statistical upper bound of the optimal objective value of the problem is estimated. Then a backward pass is performed to refine the approximation of the cost to go function using the trial solutions. This is done by adding new cuts (Benders cuts) to each of the sub-problems visited in the forward pass, starting from the last stage and moving backward to the first stage. At the end of the backward pass, a valid lower bound of the optimal objective value of the problem is computed. The algorithm terminates when the lower and upper bounds satisfy a predefined convergence criterion. More details on the SDDP approach and the convergence criteria are available in (Pereira and Pinto, 1991) and (Shapiro, 2011).

In our case, the SDDP algorithm defines at each stage t an approximate value for function $F_t(X_t, S_t, \widetilde{D}_t)$ by replacing $\mathbb{E}\left[F_{t+1}(X_{t+1}, S_{t+1}, \widetilde{D}_{t+1})\right]$ in model M_t^{SDP} by variable ϕ_{t+1} and constraining ϕ_{t+1} by the following set of Benders cuts:

$$\begin{aligned}\phi_{t+1} \geq & \sum_{p\in P}\sum_{i\in I}\pi^l_{p,i,t+1}(S_{p,i,t+1} - S^l_{p,i,t+1}) + \sum_{p\in P}\sum_{j\in J}(S^-_{p,j,t+1} - S^l_{p,j,t+1})\lambda^l_{p,j,t+1} \\ & - \sum_{p\in P}\sum_{j\in J}(S^+_{p,j,t+1} - S^l_{p,j,t+1})\lambda^l_{p,j,t+1} + F_{t+1}(X^l_{t+1}, S^l_{t+1}, \widetilde{D}_{t+1}) \qquad \forall l \in \mathcal{L}_t\end{aligned} \tag{19}$$

In inequalities (19), $\mathcal{L}_t$ denotes the set of trial solutions and $\pi_{p,i,t}$ and $\lambda_{p,j,t}$ are the Simplex multipliers associated with constraints (15) and (16), respectively, of model M_t^{SDP} (more details on how these multipliers are determined are given next). Based on this, the problem at stage $t \in T$, can be approximated using model M_t^{ASDP} as follows:

$$\begin{aligned}M_t^{ASDP}: F_t(X_t, S_t, \widetilde{D}_t) = \min & \sum_{p\in P}\sum_{i\in I} CS_{p,i}S_{p,i,t} + \sum_{p\in P}\sum_{j\in J}(CD_{p,j}S^+_{p,j,t} + CB_{p,j}S^-_{p,j,t}) + \\ & \sum_{p\in P}\sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}} CT^{c,b}_{p,i,j}X^b_{p,c,i,j,t} + \sum_{p\in P}\sum_{c\in CO}\sum_{(i,j)\in L^o} CT^{c,o}_{p,i,j}X^o_{p,c,i,j,t} \\ & + \sum_{c\in CS}\sum_{b\in B^c}\sum_{(i,j)\in L^{c,b}}(\theta^{c,b}_{i,j}Y^b_{c,i,j,t} + \chi^{c,b}_{i,j}Z^b_{c,i,j,t}) + \phi_{t+1}\end{aligned} \tag{20}$$

$$\text{s.t.} \quad (2), (3), (5), (7) - (13), (15), (16), (19)$$

At each iteration of the SDDP algorithm, and for each stage $t \in T$, N trial solutions are considered. These trial solutions are obtained (through a forward pass) by solving models M_t^{ASDP}, $t \in T$ taking into account the Benders cuts added in the previous iterations (through the backward pass). To handle uncertainty on demand, the number of realizations of demand at each stage t is assumed finite and the corresponding set of plausible scenarios is denoted by Ω_t. It is also assumed that the random data process is stagewise independent, i.e., random vector $\widetilde{D}_{t+1}$ is independent of $\widetilde{D}_1, \ldots, \widetilde{D}_t$. Formally, for each iteration k of the SDDP algorithm, a trial solutions $n = 1 \ldots N$ is obtained by randomly generating a vector of demand values $\widetilde{D}_{t,k,n}$ in Ω_t for each stage $t \in T$. Then, the trial solution is progressively constructed by solving a deterministic version of models M_t^{ASDP} - starting with the first stage ($t = 1$) and moving forward to the last stage ($|T|$)- where the value of the demand in constraints (16) is taken equal to $\widetilde{D}_{t,k,n}$. At iteration k, models M_t^{ASDP} include the Benders cuts (19) generated in iterations $1, \ldots, (k-1)$.

To generate the Benders cuts used to approximate the cost-to-go functions at stages $t \in T$, a modified version of models M_t^{ASDP} are solved backward, for each trial solution n, starting from $t = |T|$ and moving backward to $t = 2$. The modified version of model M_t^{ASDP} at stage t associated with a trial solution n and solved in a backward pass (denoted by $\overline{M}_{t,n}^{ASDP}$) uses the same variables and constraints as M_t^{ASDP} except that the right-hand sides of transition constraints (15) and (16) consider the values of the trial solution n obtained at stage $t-1$. Formally, constraints (15), respectively, (16), are replaced by constraints (21), respectively, (22) as follows:

$$S_{p,i,t+1} = \overline{S}_{p,i,t}^{n} + Q_{p,i,t} - \sum_{c \in CS} \sum_{b \in B^c} \sum_{(i,j) \in L^{c,b}} X_{p,c,i,j,t}^{b} - \sum_{c \in CO} \sum_{(i,j) \in L^o} X_{p,c,i,j,t}^{o}$$

$$\forall t \in T, p \in P, i \in I \tag{21}$$

$$S_{p,j,t+1}^{-} - S_{p,j,t+1}^{+} = \overline{S}_{p,j,t}^{-n} - \overline{S}_{p,j,t}^{+n} + D_{p,j,t} - \sum_{c \in CS} \sum_{b \in B^c} \sum_{(i,j) \in L^{c,b}} X_{p,c,i,j,t}^{b}$$

$$- \sum_{c \in CO} \sum_{(i,j) \in L^o} X_{p,c,i,j,t}^{o} \forall t \in T, p \in P, j \in J \tag{22}$$

where $(\overline{S}_{p,i,t}^{n}, \overline{S}_{p,j,t}^{-n}, \overline{S}_{p,j,t}^{+n})$ are the values of inventory/backorder variables obtained for trial solution n at stage $t-1$.

Solving model $\overline{M}_{t,n}^{ASDP}$ to optimality enables determining the simplex multipliers associated with constraints (21) and (22). To handle uncertainty, it is rather an approximation of these multipliers that is used in inequalities (19). Formally, a pre-specified number Ξ of demand scenarios are generated from Ω_t for each stage t. Then, for each trial solution n, a deterministic version of model $\overline{M}_{t,n}^{ASDP}$ is solved Ξ times, one time for each demand scenario. The simplex multipliers values associated with scenario ξ are denoted by $\pi_{p,i,t,\xi}^{n}$ and $\lambda_{p,j,t,\xi}^{n}$. The expected simplex multipliers values used in constraints (19) to approximate the cost-to-go function ϕ_{t+1} are then computed as:

$$\pi_{p,i,t}^{n} = \sum_{\xi=1}^{\Xi} \rho_{\xi} \pi_{p,i,t,\xi}^{n} \tag{23}$$

$$\lambda_{p,j,t}^{n} = \sum_{\xi=1}^{\Xi} \rho_{\xi} \lambda_{p,j,t,\xi}^{n}, \tag{24}$$

where ρ_{ξ} denotes the probability of occurrence of scenario ξ.

At each iteration of the SDDP algorithm, a valid lower bound is obtained by solving the first stage model M_1^{ASDP} which is a relaxation of M_1^{SDP} given that there is no guarantee that all the benders cuts required to fully define the cost-to-go functions are considered (Pereira and Pinto, 1991). Upper bounds are estimated based on the N problems solved in each forward pass to derive the N trial solutions. As already mentioned, each trial solution is obtained by randomly choosing demand scenarios in $\in \Omega_t, t \in T$ and performing a forward pass. Based on this, one can compute an average upper bound value, $UB = \frac{1}{N}\sum_{n=1}^{N} UB_n$, where UB_n is the value of the total cost (cost of stage $t = 1$) obtained with trial solution n in the forward pass. The corresponding (1-α)% confidence interval is given by $[UB - z_{\alpha} * \frac{\sigma}{\sqrt{N}};\ UB + z_{\alpha} * \frac{\sigma}{\sqrt{N}}]$ where $z_{\alpha} = \Phi^{-1}(1-\alpha)$ and $\Phi(.)$ is the cumulative distribution function of the standard normal distribution. These lower and upper bounds are updated at each iteration of the SDDP algorithm. As in Shapiro (2011), the algorithm is stopped when e the lower bound lies in the confidence interval $[UB - z_{\alpha} * \frac{\sigma}{\sqrt{N}};\ UB + z_{\alpha} * \frac{\sigma}{\sqrt{N}}]$ and the gap between the upper bound of the confidence interval and the lower bound is smaller than a pre-specified threshold ϵ. An overview of the the SDDP algorithm is illustrated in Figure 1.

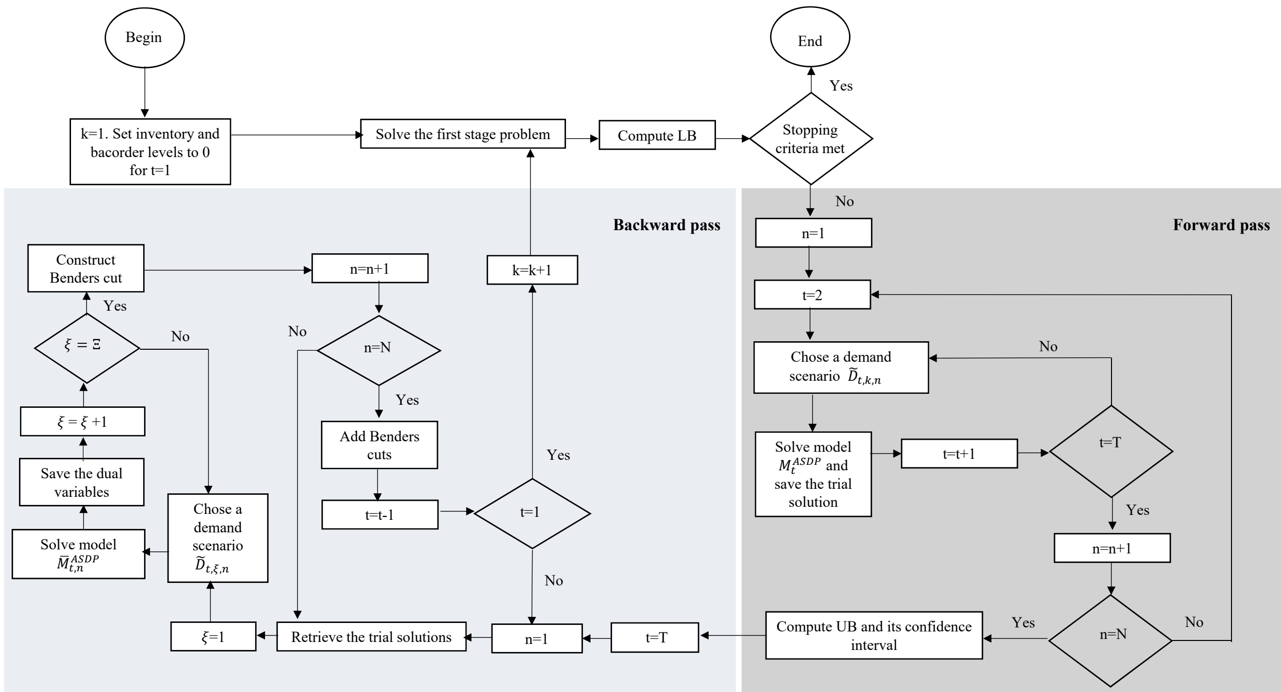

Figure 1: Flowchart of the SDDP algorithm

5. COMPUTATIONAL RESULTS

Our experimental study is twofold. First, we want to assess the computational performance of the proposed algorithm (in terms of computing time and costs approximation) to solve the dynamic stochastic CSSAP within different contexts. Our second objective is to evaluate the quality of the solutions obtained by our approach compared to an average scenario-based approach and assess thus its relevance and its efficiency in addressing CSSAPs in dynamic stochastic environments.

The SDDP algorithm is implemented in Microsoft Visual C++ using ILOG CPLEX

12.6 optimization library. All experiments were conducted on a dual Intel Xeon X5650 processor 2.66 GHZ and 72 GB DDR3 ECC Reg Memory RAM.

5.1. Problem tests

Eight distribution contexts are generated by varying the number of Dcs, the number of warehouses, the number of products and the number of strategic carriers. Table 2 gives a formal description of these contexts. For all the contexts, we consider a 12-period planning horizon (one year with a discretization period of one month). The demand of each product p at each DC j for each period t ($D_{p,j,t}$) is randomly generated following a normal distribution $N(700, 60)$. We assume that the results of the strategic selection level are known and that one spot carrier with an infinite capacity is available at the operational level. The lower and the upper bounds associated with a winning bid b of a strategic carrier c for each pair (i, j) covered by b are generated as : $LB^{c,b}_{i,j} = 0.5\frac{\sum_{t\in T}\sum_{p\in P} D_{p,j,t}}{|I|}$, $UB^{c,b}_{i,j} = 1.5\frac{\sum_{t\in T}\sum_{p\in P} D_{p,j,t}}{|I|}$.

Five instances are randomly generated for each of the eight contexts. These instances differ on the values assigned to the different types of costs. For all the 40 instances, unit transportation costs for the spot carrier ($CT^{c,o}_{p,i,j}$) are generated following a uniform distribution within the interval [90,100]. The unit transportation cost of strategic carriers for each lane ($CT^{c,b}_{p,i,j}$) is generated within 70% and 75% of the corresponding spot cost. The unit penalty costs incurred by the shipper for not respecting the minimum and the maximum volumes restrictions on each lane covered by the winning bids are set to 20% of the corresponding strategic transportation cost. Unit inventory costs at warehouses and DCs are generated following a uniform distribution within the interval [4,5]. The unit backorder cost for each product p at each DC j is generated as: $CP_{p,j} = 0.8 \times \frac{\sum_i CT^{c,o}_{p,i,j}}{|I|}$.

5.2. Computational performance of the SDDP algorithm

In order to assess the computational performance of the proposed SDDP algorithm, we first evaluate its accuracy in approximating the total operational cost under demand

Table 2: Description of the contexts

Context	$\lvert J\rvert$	$\lvert I\rvert$	$\lvert P\rvert$	$\lvert CS\rvert$
1	15	10	2	3
2	10	10	2	3
3	15	5	2	3
4	25	10	2	3
5	15	10	4	3
6	15	10	4	2
7	15	10	4	4
8	15	15	2	3

uncertainty. To this end, we generate a large set of plausible demand scenarios, denoted by Ξ', such that $|\Xi'| \gg |\Xi|$ (recall that Ξ denotes the set of demand scenarios considered in the SDDP algorithm to derive average simplex multipliers used in the Benders cuts). Then, for each scenario $\xi \in \Xi'$, we compute: (i) the total operational cost, denoted by Z_ξ^{sddp}, yielded by the SDDP solution, and (ii) the optimal cost denoted by Z_ξ^*. A relatively small value of $\Delta_\xi^* = \frac{Z_\xi^{sddp} - Z_\xi^*}{Z_\xi^*}$ implies that the proposed SDDP algorithm accurately approximates the total operational cost for scenario ξ.

As described in Section 4, the SDDP algorithm does not provide a particular solution but rather an approximation of the expected operational costs over the planning horizon. In our case, we determine a SDDP solution for a scenario ξ by performing an additional forward pass on the models $M_t^{ASDP}, t = 1, \ldots, T$ obtained at the last iteration of the SDDP algorithms and by fixing the demand vector to its value in scenario ξ. These models include all the Benders cuts generated through the algorithm iterations until the stopping criteria are met. The optimal objective value Z_ξ^* is obtained by solving model M^{det} for scenario ξ.

We tested the algorithm accuracy for different parameters values that are likely to impact its performance, namely the number of trials ($N = 3, 5, 7$) and the number of scenarios ($|\Xi| = 20, 50, 100$). Table 3 reports the results obtained. More precisely, it displays for each context and each combination $(N, |\Xi|)$, the average ($Avg.$), the minimum ($Min.$)

and the maximum ($Max.$) values (in percentage) obtained for Δ_ξ^* over all the scenarios $\xi \in \Xi'$ and all the five instances of the context. Detailed results for each instance are given in Tables A.6 and A.7 in the appendix.

Table 3: Accuracy of the SDDP algorithm in approximating the total operational cost

Context	Δ_ξ^* (%)	**SDDP parameters combinations (N trials, $\|\Xi\|$ scenarios)**								
		(3,20)	**(3,50)**	**(3,100)**	**(5,20)**	**(5,50)**	**(5,100)**	**(7,20)**	**(7,50)**	**(7,100)**
1	Avg.	2.42	1.69	2.43	2.10	2.26	1.71	1.90	2.24	**1.54**
	Min.	1.13	0.42	0.54	0.54	0.83	0.37	0.75	0.62	0.00
	Max.	3.92	3.40	4.23	3.97	4.29	3.48	3.39	4.20	3.65
2	Avg.	2.78	**2.22**	2.75	2.61	2.41	2.58	2.84	2.38	2.38
	Min.	0.92	0.30	0.03	0.82	0.26	0.90	1.02	0.31	0.28
	Max.	4.84	4.59	4.94	4.66	4.15	5.09	4.72	4.80	4.55
3	Avg.	3.31	3.34	3.69	3.19	3.40	3.12	**3.10**	3.16	3.37
	Min.	1.08	1.77	1.53	0.89	0.82	1.03	0.66	0.20	1.18
	Max.	6.19	5.85	6.66	5.98	6.45	5.85	5.79	6.58	5.94
4	Avg.	3.00	3.07	2.94	3.02	2.88	3.04	**2.62**	2.81	3.00
	Min.	2.12	2.07	2.05	2.05	1.99	2.08	0.98	2.11	2.07
	Max.	4.09	4.38	3.87	4.36	3.78	4.19	3.84	3.66	4.14
5	Avg.	2.60	**2.39**	2.68	2.43	2.64	2.49	2.49	2.53	2.56
	Min.	0.74	0.59	1.08	0.71	1.47	1.33	0.45	1.11	1.24
	Max.	4.41	3.93	4.32	4.24	4.33	4.03	3.99	4.09	3.97
6	Avg.	3.06	2.82	2.96	2.92	**2.77**	2.99	2.96	3.02	2.92
	Min.	1.03	0.62	1.06	0.88	0.65	0.77	1.00	0.80	0.82
	Max.	4.65	4.66	4.79	4.69	4.71	4.69	4.59	4.71	4.71
7	Avg.	2.04	2.12	2.07	1.73	1.86	1.94	**1.38**	1.66	1.84
	Min.	0.32	0.18	0.4	0.5	0.01	0.1	0.25	0.28	0.03
	Max.	3.94	4.29	3.82	3.70	3.56	3.17	2.51	3.55	3.80
8	Avg.	2.90	3.09	2.70	2.68	2.70	2.96	**2.67**	2.8	2.76
	Min.	1.23	1.48	0.91	0.44	0.4	1.14	0.06	0.11	0.51
	Max.	4.30	5.16	4.90	4.19	4.54	4.81	4.61	4.76	4.55

The results of Table 3 show that the SSDP algorithm generally well approximates the total cost for all the instances considered and for all the SDDP parameters values. Indeed, the deviation between the SDDP cost and the optimal true cost for each demand scenario $\xi \in \Xi'$ averages 2.62% for the 40 instances and the nine combinations. To study the impact of the SDDP parameters on the algorithm accuracy, we draw in bold (in Table 3) the smallest average deviation obtained for each context. We then compute, for each context, the absolute difference between the average deviation obtained for each combination $(N, |\Xi|)$ and the minimum deviation obtained for this context (Hence, a difference of 0 for a combination $(N, |\Xi|)$ implies that this combination yielded the smallest deviation with regard to the optimal cost). Figure 2 displays for each context (x axis) this difference (y-axis) for each combination $(N, |\Xi|)$ (the value to the right of each point in Figure 2 is the value of this difference for the corresponding $(N, |\Xi|)$ combination).

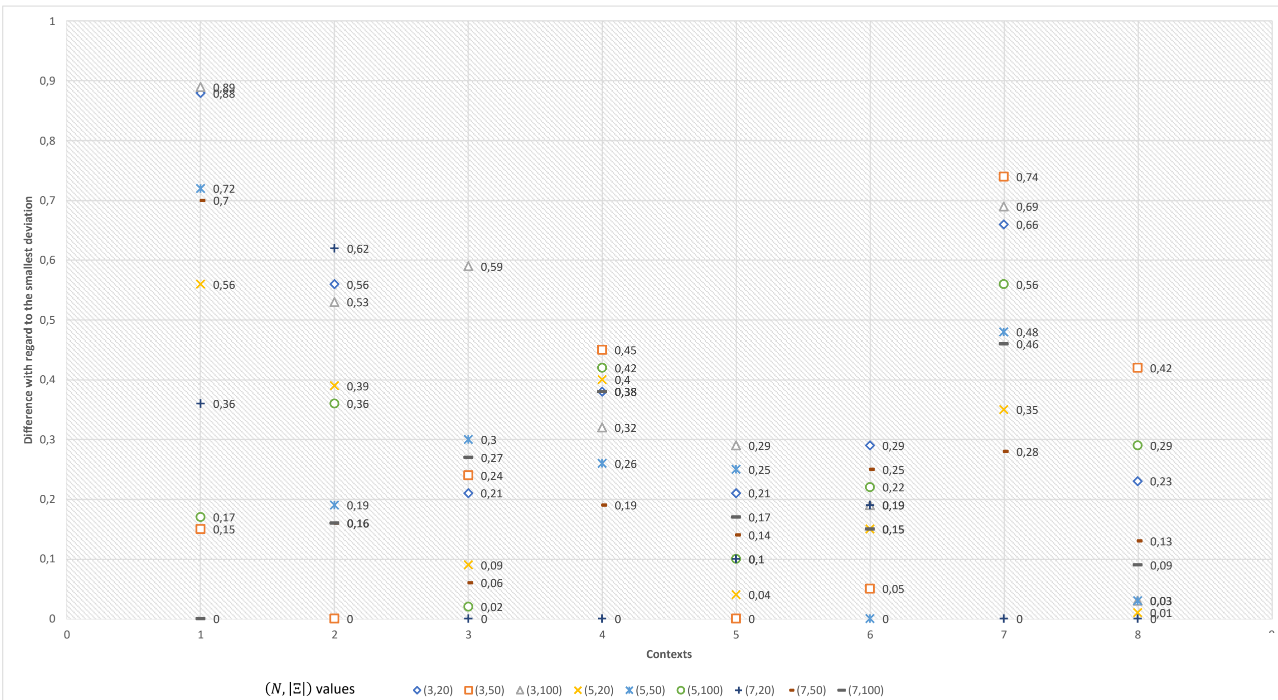

Figure 2: Comparison between the different SDDP parameters settings on the algorithm accuracy

One can observe that the impact of the increase/decrease of the number of trials and the number of scenarios on the algorithm accuracy is not substantially conclusive. Although, for the eight considered contexts, fixing the number of trial to 7 (the largest value) yielded the lowest average deviation for 5 context, the difference with regard $N = 3$

or $N = 5$ remains relatively small. Besides, for a fixed number of trials, increasing the number of scenarios not always results in a smaller average deviation. If one has to decide, overall, it is the combination $(7, 100)$ that offers, in average, the best accuracy.

Next, we evaluate the computational performance of the proposed SDDP algorithm in terms of computing times and the variability in the value of the expected total cost output by the algorithm for the different combinations of the algorithm parameters. Table 4 reports the average CPU time (in seconds) and the average expected cost (lines TC) in Millions $ obtained for each context and each combination $(N, |\Xi|)$, these averages being computed on the five instances of each context. Observe that the value reported for TC corresponds to the value of the upper bound output by the SDDP algorithm when the stopping criteria are met. Detailed results for each instance are given in Tables A.8 and A.9 in the appendix. Observe that Tables A.8 and A.9 also provide the 95% confidence interval for each instance over the set of scenarios Ξ'.

Table 4: CPU time and cost variability for different parameters values of the SDDP algorithm

Context	**Cost/CPU**	**SDDP parameters combinations (N trials, $\|\Xi\|$ scenarios)**								
		(3,20)	**(3,50)**	**(3,100)**	**(5,20)**	**(5,50)**	**(5,100)**	**(7,20)**	**(7,50)**	**(7,100)**
1	TC	17.93	17.80	17.93	17.87	17.90	17.81	17.84	17.90	17.78
	CPU	1248	1848	2460	1104	2736	2832	1788	3300	3900
2	TC	11.99	11.93	11.99	11.97	11.95	11.97	12.00	11.95	11.93
	CPU	588	1152	1680	804	2220	2424	984	1944	3204
3	TC	18.12	18.25	18.32	18.23	18.26	18.22	18.21	18.22	18.26
	CPU	504	720	744	780	972	972	504	720	1308
4	TC	29.77	29.96	29.92	29.95	29.91	29.95	29.83	29.89	18.26
	CPU	1416	1104	1668	1236	1932	2436	2028	2604	3024
5	TC	35.80	35.72	35.83	35.74	35.81	35.76	35.76	35.77	35.79
	CPU	1104	2472	2520	2148	2712	2956	3324	3720	5640
6	TC	36.09	36.01	36.05	36.04	35.99	36.06	36.06	36.08	36.04
	CPU	1812	1524	2076	2 172	2124	3036	3456	3444	4560
7	TC	35.61	35.64	35.62	35.50	35.55	35.58	35.38	35.48	35.54
	CPU	1800	2628	3240	2364	2820	3600	3012	3540	5184
8	TC	17.93	17.96	17.89	17.89	17.89	17.94	17.89	17.91	17.91
	CPU	1212	1 596	1908	1764	1692	2412	2568	2 436	4 224

The results of Table 4 prove that the SDDP algorithm requires a total CPU time that varies between 504 and 5640 seconds for all the 40 instances and the nine parameters combinations. The CPU time increases with the number of trials and the number of scenarios used in the SDDP algorithm. Indeed, the CPU time required when $|\Xi| = 100$ (for all the trial values) averages 1655 seconds compared to 2165 seconds for $|\Xi| = 50$ and 2834 seconds for $|\Xi| = 20$. It averages 1626 seconds when $N = 3$ (for all $|\Xi|$ values), 2094 seconds when $N = 5$ and 2934 seconds for $N = 7$. This was predictable since increasing the number of scenarios and the number of trails increases the number of problems that need to be solved during the forward and backward passes of the SDDP algorithm. Regarding the variability in approximating the total cost, one can observe that the difference in the reported costs is not significant when the SDDP parameters take different values.

Combining the results of Tables 3 and 4 shows that there is a clear trade off between computing time and algorithm accuracy in approximating costs that must be managed when fixing the parameters values of the SDDP algorithm. Overall, the SDDP algorithm ia relatively accurate in approximating the future costs and its computational performance relatively stable varying slightly with its parameters tuning.

5.3. Relevance of the proposed approach

In this section, we investigate the relevance of the SDDP approach to produce good-quality solutions when compared to the so called "Mean Scenario-Based" or MSB approach. For the MSB approach, a solution is obtained by simply solving model M^{det} where the demands at DCS are fixed to their average values (700 for our experiments). This solution is referred to as X^{MSB} in the following.

To ensure a fair comparison of both approaches, we determine the SDDP solutions as follows. We perform a forward pass on the final models $M_t^{ASDP}, t = 1, \ldots, T$ obtained at the last iteration of the SDDP algorithms (so that all the benders cuts generated by the SDDP algorithm to approximate the cost-to-go at each stage are considered). These models are solved for the average demand scenario (700 in our experiments). The resulting SDDP solution is denoted by X^{SDDP}.

Then, we evaluate and compare the total costs induced by both solutions for different demand scenarios realization. To this end, we generate 500 plausible demand scenarios. The set of generated scenarios is denoted by Ξ'. For each scenario $\xi \in \Xi'$, we denote by Z_ξ^{MSB} the total cost induced by X^{MSB} for scenario ξ. This cost is obtained by solving a variant of model M^{det} where transport decisions (namely $X_{p,c,i,j,t}^{o}$ and $X_{p,c,i,j,t}^{b}$ variables) are fixed to their values in X^{MSB} and the demands in constraints (6) are set to their values in scenario ξ. Given that the quality of the SDDP solution may differ with regard to the parameters tuning of the SDDP approach (as observed in Section 5.2), we compare the total cost induced by X^{MSB} in two cases of parameters values combinations (N, $|\Xi|$) for each context. The first case is referred to as "Best" and the second as "Worst". For each context, the values of N and $|\Xi|$, for the best case, correspond to the $(N, |\Xi|)$ combination for which Δ_ξ^* takes the lowest value when considering the five instances and the nine $(N, |\Xi|)$ combinations as reported in Tables A.6 and A.7. The worst case fixes N and $|\Xi|$ to the values for which Δ_ξ^* takes its largest value. Then, for each parameter combination $(N, |\Xi|)$, we determine the SDDP solution as described above. Then for each SDDP solution obtained for each combination $(N, |\Xi|)$, we evaluate the total cost it induces for scenario $\xi \in \Xi'$ by solving a variant of M^{det} as for the mean-scenario based approach.

Let $\underline{Z}_\xi^{sddp}$, respectively, $\overline{Z}_\xi^{sddp}$, denote the cost obtained by the SDDP approach for scenario ξ when the best, respectively, the worst, case combination of the SDDP parameters values are considered. Table 5 reports for each context, the average, the minimum and the maximum relative difference in costs (in %) between: (i) Z_ξ^{MSB} and $\underline{Z}_\xi^{sddp}$ (lines "Best") and, (ii) Z_ξ^{MSB} and $\overline{Z}_\xi^{sddp}$ (lines "Worst"), over the scenarios $\xi \in \Xi'$ and all the five instances of each context. Observe that these relative differences in costs are computed relatively to the MSB cost: $\frac{\underline{Z}_\xi^{sddp} - Z_\xi^{MSB}}{Z_\xi^{MSB}}$ for the difference with respect to the SDDP cost for the best case; and $\frac{\overline{Z}_\xi^{sddp} - Z_\xi^{MSB}}{Z_\xi^{MSB}}$ for the worst case. Hence, a negative value under columns "Relative difference" in Table 5 implies that the SDDP approach yields a lowest total cost than the MSB approach. Table 5 also displays the parameters combination (N,

$|\Xi|$) for the best and worst cases considered for each context.

Table 5: Comparison between the SDDP and the MSB approaches

Context	Cases	Parameters combination		Relative difference (%)		
		N	$\lvert\Xi\rvert$	Avg.	Min.	Max.
1	Best	7	100	-11.22	-11.43	-11.06
	Worst	3	100	-5.88	-6.07	-5.78
2	Best	5	50	-12.15	-12.48	-11.86
	Worst	5	100	-12.15	-12.41	-11.87
3	Best	7	50	-22.95	-23.28	-22.63
	Worst	3	100	-5.6	-5.71	-5.51
4	Best	7	20	-13.51	-13.85	-13.34
	Worst	3	50	-5.73	-5.82	-5.65
5	Best	7	20	-12.3	-12.48	-12.12
	Worst	3	20	-3.79	-4.01	-3.56
6	Best	5	50	-8.78	-9.1	-8.57
	Worst	3	100	-3.55	-3.72	-3.47
7	Best	5	50	-12.5	-12.61	-12.38
	Worst	3	50	-10.6	-10.7	-10.5
8	Best	7	20	-13.33	-13.55	-13.1
	Worst	3	50	-11.13	-11.32	-10.99

The results of Table 5 show that the proposed SDDP algorithm considerably outperforms the MSB approach even for its worst performance. Although the MSB approach has the advantage of being simple to apply, the SDDP approach is much more relevant to deal with the dynamic stochastic nature of the problem. For the instances considered, it yielded a relative saving in total costs that varies between 8.57% and 23.28% when the SDDP algorithm performs the best and between 3.47% and 12.41% when it performs the worst.

6. CONCLUSION

Our paper proposes a multi-stage stochastic carrier' selection and shipment assignment model under demand uncertainty, where a set of strategic and spot carriers are available to procure transportation services and to ship loads from warehouses to distribution centers.

To deal with the dynamic nature of the problem, a Stochastic Dual Dynamic Programming method was proposed to tackle it. An experimental study is conducted to highlight the computational performance of the proposed algorithm. The results prove that the SDDP algorithm produces good-quality solutions within a reasonable computational time. Additional simulation experiments are carried out to investigate the relevance of the SDDP method compared to an average-scenario deterministic approach. Our results prove that the SDDP method yields important savings. A first extension would be to integrate the strategic and the operational decisions for carriers' selection using the SDDP algorithm. A second research avenue would be to consider uncertainty on additional parameters such as the replenishment quantities or the carriers capacity during operations.

Acknowledgments

This project was funded by the Canadian Natural Science and Engineering Council (NSERC) under grants 2016-04482. This support is greatly acknowledged.

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Appendix A. Detailed results for Section 5.2

Tables A.6 and A.7 display for each instance of each context, the average, the minimum and the maximum deviation (in %) with regard to the optimal cost; these values being computed over the scenarios $\xi \in \Xi'$.Tables A.8 and A.9 report for each instance of each context, the average estimated cost (TC) in Million Dollars ($), its 95% confidence interval (CI) in Million Dollars ($) and the total CPU time (in seconds) required by the SDDP algorithm.

Table A.6: Accuracy of the SDDP algorithm in approximating the total cost: detailed results for contexts 1-4

Context	Instance	Δ^*_ξ (%)	SDDP parameters $(N, \|\Xi\|)$								
			(3,20)	(3,50)	(3,100)	(5,20)	(5,50)	(5,100)	(7,20)	(7,50)	(7,100)
		Avg.	1.82	1.42	1.33	1.33	1.54	1.13	1.72	1.26	0.5
	1	Min.	1.13	0.77	0.54	0.54	0.83	0.47	1.06	0.62	0.004
		Max.	2.44	1.99	1.91	1.91	2.27	1.68	2.31	1.80	1.10
		Avg.	1.83	1.42	1.80	1.92	1.61	1.23	1.30	1.40	1.44
	2	Min.	1.2	0.89	1.23	1.36	0.93	0.75	0.75	0.83	0.91
		Max.	2.33	1.90	2.33	2.42	2.13	1.74	1.82	1.94	1.89
		Avg.	1.96	1.90	1.96	2.48	1.91	2.22	1.86	2.28	1.85
1	3	Min.	1.31	1.32	1.39	1.92	1.33	1.67	1.25	1.72	1.24
		Max.	2.56	2.45	2.49	3.02	2.45	2.80	2.39	2.79	2.35
		Avg.	3.28	0.91	3.60	1.35	3.59	1.01	1.76	3.49	0.70
	4	Min.	2.77	0.42	3.09	0.74	3.04	0.37	1.25	2.96	0.24
		Max.	3.92	1.41	4.20	1.78	4.29	1.51	2.29	4.20	1.27
		Avg.	3.23	2.81	3.48	3.44	2.66	2.96	2.88	2.78	3.22
	5	Min.	2.61	2.23	2.83	2.99	1.98	2.43	2.39	2.24	2.76
		Max.	3.77	3.40	4.23	3.97	3.12	3.48	3.39	3.31	3.65
		Avg.	3.18	2.77	4.18	3.68	3.32	4.27	3.67	1.96	3.41
	1	Min.	2.33	2.13	3.38	2.93	2.55	3.45	2.88	1.38	2.70
		Max.	4.11	3.49	4.94	4.43	4.06	5.09	4.39	2.67	4.19
		Avg.	4.08	3.88	3.88	3.93	3.51	3.28	3.94	4.08	3.86
	2	Min.	3.28	3.00	3.10	3.11	2.67	2.35	3.14	3.24	3.02
		Max.	4.84	4.59	4.61	4.66	4.15	4.00	4.72	4.80	4.55
		Avg.	2.22	2.04	3.41	1.62	3.25	1.83	2.46	2.08	2.12
2	3	Min.	1.60	1.11	2.76	1.05	2.48	1.12	1.79	1.36	1.53
		Max.	2.95	2.69	4.11	2.22	3.95	2.55	3.06	2.91	2.90
		Avg.	2.78	1.35	0.89	2.36	0.87	1.61	2.34	2.63	0.99
	4	Min.	2.02	0.53	0.03	1.58	0.26	0.90	1.59	1.89	0.29
		Max.	3.78	2.20	1.62	3.41	1.75	2.48	3.48	3.71	1.90
		Avg.	1.64	1.09	1.40	1.48	1.11	1.91	1.80	1.15	1.05
	5	Min.	0.92	0.30	0.66	0.82	0.52	1.07	1.02	0.31	0.28
		Max.	2.35	1.86	2.06	2.30	1.73	2.83	2.65	1.99	1.75
		Avg.	2.68	2.72	3.05	2.61	3.01	2.38	2.91	2.75	3.01
	1	Min.	2.07	2.03	2.38	1.83	2.35	1.76	2.28	2.03	2.32
		Max.	3.41	3.46	3.70	3.21	3.71	2.91	3.54	3.53	3.74
		Avg.	2.96	2.37	2.88	2.40	2.39	2.40	2.36	1.02	2.33
	2	Min.	2.28	1.82	2.24	1.67	1.71	1.68	1.62	0.20	1.68
		Max.	3.76	3.10	3.63	3.07	3.03	3.15	3.10	1.73	3.03
		Avg.	3.79	4.20	4.41	4.13	4.44	4.04	3.70	4.25	4.23
3	3	Min.	3.09	3.50	3.72	3.38	3.69	3.35	3.00	3.53	3.51
		Max.	4.44	4.89	5.05	4.81	5.09	4.70	4.35	4.92	4.93
		Avg.	5.46	5.10	5.94	5.26	5.66	5.13	5.07	5.88	5.27
	4	Min.	4.80	4.45	5.15	4.61	4.92	4.44	4.43	5.10	4.59
		Max.	6.19	5.85	6.66	5.98	6.45	5.85	5.79	6.58	5.94
		Avg.	1.68	2.29	2.16	1.53	1.48	1.65	1.43	1.88	2.02
	5	Min.	1.08	1.77	1.53	0.89	0.82	1.03	0.66	1.23	1.18
		Max.	2.21	2.80	2.75	2.12	2.16	2.36	2.00	2.55	2.55
		Avg.	3.24	3.26	3.31	3.29	3.30	3.26	3.02	3.16	3.27
	1	Min.	2.80	2.77	2.81	2.82	2.84	2.74	2.53	2.60	2.83
		Max.	3.75	3.76	3.77	3.89	3.78	3.71	3.46	3.66	3.79
		Avg.	2.81	2.76	2.47	2.52	2.40	2.49	2.36	2.70	2.57
	2	Min.	2.46	2.30	2.05	2.18	1.99	2.08	1.97	2.31	2.21
		Max.	3.20	3.21	2.86	2.89	2.86	2.93	2.79	3.14	3.00
		Avg.	2.87	2.94	3.12	2.99	3.21	3.07	3.05	2.84	3.08
4	3	Min.	2.39	2.42	2.71	2.54	2.77	2.61	2.54	2.41	2.64
		Max.	3.31	3.46	3.59	3.46	3.61	3.50	3.52	3.29	3.54
		Avg.	2.58	2.52	2.51	2.51	2.69	2.78	1.39	2.59	2.55
	4	Min.	2.12	2.07	2.05	2.05	2.21	2.31	0.98	2.11	2.07
		Max.	3.15	3.08	3.00	3.05	3.22	3.30	1.74	3.12	3.12
		Avg.	3.51	3.88	3.31	3.78	2.82	3.61	3.28	2.77	3.53
	5	Min.	3.02	3.42	2.84	3.26	2.41	3.10	2.81	2.33	3.06
		Max.	4.09	4.38	3.87	4.36	3.33	4.19	3.84	3.36	4.14

Table A.7: Accuracy of the SDDP algorithm in approximating the total cost: detailed results for contexts 5-8

			SDDP parameters $(N, \|\Xi\|)$								
Context	Instance	Δ_ξ^* (%)	(3,20)	(3,50)	(3,100)	(5,20)	(5,50)	(5,100)	(7,20)	(7,50)	(7,100)
5	1	Avg.	1.20	1.07	1.49	1.11	1.93	1.76	0.77	1.53	1.68
		Min.	0.74	0.59	1.08	0.71	1.47	1.33	0.45	1.11	1.24
		Max.	1.57	1.47	1.90	1.49	2.32	2.19	1.12	1.96	2.09
	2	Avg.	3.25	3.31	3.29	3.26	3.55	3.39	3.16	3.54	3.40
		Min.	2.81	2.81	2.79	2.78	3.06	2.86	2.72	3.10	2.94
		Max.	3.80	3.92	3.88	3.82	4.33	4.03	3.75	4.09	3.97
	3	Avg.	1.86	1.99	2.25	1.97	2.28	2.22	1.85	2.27	2.18
		Min.	1.37	1.60	1.75	1.55	1.92	1.75	1.51	1.82	1.75
		Max.	2.87	2.41	2.72	2.49	2.69	2.78	2.31	2.75	2.65
	4	Avg.	2.73	3.28	3.70	2.01	2.99	2.15	3.36	2.69	2.94
		Min.	2.19	2.78	3.10	1.54	2.47	1.76	2.75	2.11	2.43
		Max.	3.36	3.93	4.32	2.63	3.64	2.64	3.99	3.39	3.62
	5	Avg.	3.96	2.28	2.69	3.79	2.43	2.92	3.33	2.61	2.63
		Min.	3.42	1.76	2.29	3.30	1.87	2.45	2.83	2.13	2.26
		Max.	4.41	2.78	3.13	4.24	2.86	3.38	3.85	3.04	3.27
6	1	Avg.	4.08	3.82	3.83	3.89	3.93	4.18	3.81	3.98	3.97
		Min.	3.56	3.28	3.23	3.37	3.42	3.67	3.26	3.49	3.34
		Max.	4.60	4.66	4.59	4.44	4.71	4.69	4.59	4.67	4.69
	2	Avg.	3.25	2.54	2.54	2.83	2.42	3.11	3.04	3.24	2.82
		Min.	2.79	2.02	1.99	2.33	1.94	2.72	2.61	2.48	2.35
		Max.	4.03	2.90	2.97	3.39	2.86	3.68	3.43	3.71	3.26
	3	Avg.	1.40	1.02	1.48	1.33	1.01	1.18	1.39	1.20	1.16
		Min.	1.03	0.62	1.06	0.88	0.65	0.77	1.00	0.80	0.82
		Max.	1.88	1.46	2.05	1.88	1.39	1.61	1.88	1.67	1.55
	4	Avg.	4.05	3.97	4.21	4.10	3.81	3.84	3.96	4.14	4.11
		Min.	3.54	3.46	3.58	3.61	3.26	3.24	3.41	3.55	3.51
		Max.	4.65	4.47	4.79	4.69	4.52	4.45	4.53	4.71	4.71
	5	Avg.	2.55	2.76	2.73	2.43	2.68	2.63	2.63	2.56	2.52
		Min.	2.09	2.31	2.12	1.89	2.11	2.11	2.01	2.12	2.02
		Max.	3.08	3.17	3.18	2.87	3.11	3.02	3.05	3.00	2.91
7	1	Avg.	3.37	3.76	3.38	3.17	3.09	2.69	1.98	3.10	3.30
		Min.	2.86	3.25	2.88	2.76	2.62	2.23	1.56	2.65	2.86
		Max.	3.94	4.29	3.82	3.70	3.56	3.17	2.51	3.55	3.80
	2	Avg.	2.46	2.65	2.43	1.02	1.88	2.28	1.38	0.91	2.55
		Min.	1.99	2.18	1.98	0.50	1.39	1.72	0.91	0.46	1.82
		Max.	2.98	3.14	2.88	1.48	2.40	2.91	1.97	1.36	3.45
	3	Avg.	2.05	1.55	2.01	1.71	1.92	2.56	1.89	1.90	1.27
		Min.	1.65	1.12	1.56	1.09	1.37	2.11	1.46	1.51	0.67
		Max.	2.59	2.20	2.47	2.18	2.44	3.10	2.42	2.36	1.79
	4	Avg.	0.90	0.79	1.03	1.10	0.43	0.58	0.83	0.77	0.63
		Min.	0.32	0.18	0.49	0.52	0.01	0.10	0.25	0.28	0.03
		Max.	1.32	1.24	1.45	1.62	0.93	1.02	1.28	1.21	1.10
	5	Avg.	1.39	1.86	1.51	1.62	2.01	1.59	0.82	1.63	1.48
		Min.	1.03	1.47	1.14	1.24	1.60	1.26	0.49	1.21	0.83
		Max.	1.77	2.28	1.91	1.98	2.36	2.06	1.25	2.02	1.82
8	1	Avg.	1.68	1.94	1.75	0.97	0.98	1.61	0.53	0.69	0.95
		Min.	1.23	1.48	1.26	0.44	0.40	1.14	0.06	0.11	0.51
		Max.	2.21	2.48	2.31	1.38	1.49	2.22	1.06	1.28	1.45
	2	Avg.	3.60	4.44	4.22	3.54	3.76	4.10	4.04	4.08	3.92
		Min.	3.01	3.96	3.73	3.04	3.17	3.61	3.50	3.47	3.39
		Max.	4.30	5.16	4.90	4.19	4.54	4.81	4.61	4.76	4.55
	3	Avg.	3.00	3.07	1.32	3.05	2.94	2.95	2.97	3.06	2.77
		Min.	2.45	2.51	0.91	2.48	2.39	2.36	2.43	2.55	2.24
		Max.	3.62	3.61	1.87	3.54	3.53	3.51	3.57	3.62	3.35
	4	Avg.	3.14	2.97	2.99	3.06	2.81	2.97	2.90	2.96	2.99
		Min.	2.43	2.35	2.36	2.39	2.16	2.40	2.27	2.33	2.39
		Max.	3.84	3.58	3.68	3.69	3.47	3.63	3.57	3.59	3.64
	5	Avg.	3.11	3.04	3.22	2.78	3.02	3.19	2.91	3.20	3.17
		Min.	2.61	2.52	2.75	2.27	2.52	2.60	2.47	2.68	2.63
		Max.	3.74	3.65	3.80	3.31	3.58	3.81	3.42	3.78	3.67

Table A.8: CPU time (in seconds) and cost variability: detailed results for contexts 1-4

		TC/CI	SDDP parameters $(N, \lvert\Xi\rvert)$								
Context	Instance	CPU	(3,20)	(3,50)	(3,100)	(5,20)	(5,50)	(5,100)	(7,20)	(7,50)	(7,100)
		TC	17.958	17.887	17.871	17.850	17.909	17.836	17.939	17.859	17.725
	1	CI	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004
		CPU	900	1 680	3 120	1 560	3 240	2 880	2 520	3 720	3 900
		TC	17.795	17.723	17.788	17.810	17.755	17.689	17.702	17.720	17.726
	2	CI	±0.003	±0.004	±0.004	±0.003	±0.003	±0.004	±0.004	±0.004	±0.003
		CPU	1 320	2 340	1 320	780	1 020	1 920	1 620	2 520	3 480
		TC	17.852	17.842	17.852	17.943	17.844	17.898	17.835	17.908	17.833
1	3	CI	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004
		CPU	1 500	1 260	2 880	1 020	3 300	3 180	1 320	3 780	4 260
		TC	18.075	17.660	18.130	17.737	18.129	17.677	17.808	18.111	17.624
	4	CI	±0.003	±0.005	±0.003	±0.004	±0.003	±0.004	±0.004	±0.003	±0.005
		CPU	1 320	1 080	2 280	840	2 940	3 120	1 500	2 520	3 780
		TC	18.004	17.930	18.049	18.041	17.905	17.958	17.943	17.926	18.002
	5	CI	±0.003	±0.004	±0.003	±0.004	±0.004	±0.004	±0.004	±0.004	±0.003
		CPU	1 200	2 880	2 700	1 320	3 180	3 060	1 980	3 960	4 080
		TC	12.084	12.035	12.201	12.143	12.100	12.212	12.142	11.941	12.111
	1	CI	±0.003	±0.003	±0.002	±0.003	±0.003	±0.003	±0.003	±0.004	±0.003
		CPU	660	1 320	1 980	1 200	1 920	3 120	900	3 420	4 080
		TC	12.061	12.037	12.038	12.043	11.994	11.968	12.045	12.061	12.035
	2	CI	±0.002	±0.002	±0.002	±0.002	±0.002	±0.002	±0.002	±0.002	±0.002
		CPU	720	1 920	2 820	1 320	4 920	4 080	1 500	3 180	4 140
		TC	11.759	11.739	11.896	11.691	11.878	11.715	11.787	11.743	11.748
2	3	CI	±0.003	±0.004	±0.003	±0.004	±0.003	±0.004	±0.004	±0.004	±0.004
		CPU	1 020	1 380	1 680	600	3 000	2 520	1 320	1 260	4 680
		TC	12.138	11.968	11.914	12.088	11.912	11.999	12.085	12.120	11.926
	4	CI	±0.002	±0.003	±0.003	±0.002	±0.003	±0.003	±0.004	±0.004	±0.004
		CPU	300	840	1 560	360	600	1 740	900	1 320	2 220
		TC	11.942	11.876	11.914	11.922	11.879	11.974	11.960	11.883	11.872
	5	CI	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003
		CPU	240	300	360	540	660	660	300	540	900
		TC	18.195	18.203	18.262	18.183	18.254	18.142	18.237	18.208	18.254
	1	CI	±0.003	±0.003	±0.003	±0.003	±0.003	±0.004	±0.003	±0.003	±0.003
		CPU	540	1 200	720	1 860	1 560	1 920	960	1 440	2 040
		TC	18.061	17.957	18.045	17.963	17.960	17.961	17.955	17.719	17.950
	2	CI	±0.004	±0.005	±0.004	±0.004	±0.004	±0.004	±0.004	±0.005	±0.004
		CPU	360	840	960	540	660	720	240	480	1 200
		TC	17.765	18.512	18.549	18.498	18.553	18.482	18.423	18.521	18.516
3	3	CI	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004
		CPU	240	840	540	300	1 020	720	540	900	1 140
		TC	18.428	18.365	18.512	18.392	18.463	18.369	18.359	18.501	18.394
	4	CI	±0.003	±0.003	±0.002	±0.003	±0.003	±0.003	±0.003	±0.002	±0.003
		CPU	480	300	960	960	1 140	840	480	480	1 200
		TC	18.150	18.259	18.236	18.124	18.115	18.146	18.106	18.186	18.211
	5	CI	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004
		CPU	900	420	540	240	480	660	300	300	960
		TC	29.998	30.005	30.019	30.013	30.015	30.003	29.934	29.974	30.008
	1	CI	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.005	±0.004	±0.004
		CPU	720	1 200	1 800	780	1 320	2 580	1 140	1 980	1 920
		TC	30.065	30.051	29.967	29.981	29.947	29.974	29.935	30.034	29.995
	2	CI	±0.004	±0.004	±0.004	±0.004	±0.005	±0.004	±0.005	±0.004	±0.004
		CPU	1 320	1 000	1 200	900	1 200	1 200	1 680	1 500	1 740
		TC	28.789	29.811	29.863	29.824	29.887	29.847	29.842	29.781	29.851
4	3	CI	±0.005	±0.004	±0.004	±0.005	±0.005	±0.004	±0.004	±0.004	±0.004
		CPU	1 200	1 320	1 800	1 140	1 320	1 800	1 320	1 620	2 100
		TC	29.842	29.825	29.822	29.821	29.873	29.899	29.497	29.845	29.832
	4	CI	±0.005	±0.005	±0.005	±0.004	±0.004	±0.004	±0.005	±0.005	±0.005
		CPU	1 320	1 200	1 440	2 400	2 760	2 880	1 440	3 180	3 720
		TC	30.034	30.142	29.975	30.112	29.833	30.063	29.969	29.819	30.040
	5	CI	±0.005	±0.004	±0.005	±0.004	±0.005	±0.004	±0.005	±0.005	±0.004
		CPU	2 520	1 500	2 100	960	3 060	3 720	4 560	4 740	5 640

Table A.9: CPU time (in seconds) and cost variability: detailed results for contexts 5-8

		TC/CI	SDDP parameters $(N, \lvert\Xi\rvert)$								
Context	Instance	CPU	(3,20)	(3,50)	(3,100)	(5,20)	(5,50)	(5,100)	(7,20)	(7,50)	(7,100)
5	1	TC	35.456	35.410	35.560	35.425	35.712	35.654	35.306	35.573	35.624
		CI	±0.006	±0.006	±0.006	±0.006	±0.006	±0.006	±0.006	±0.005	±0.006
		CPU	720	960	900	960	720	720	2 640	2 820	2 940
	2	TC	36.021	36.039	36.034	36.022	36.123	36.069	35.988	36.122	36.071
		CI	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005
		CPU	1 560	3 480	2 880	3 000	2 880	3 240	2 700	4 980	5 280
	3	TC	35.530	35.576	35.667	35.571	35.677	35.657	35.527	35.674	35.642
		CI	±0.006	±0.006	±0.006	±0.006	±0.006	±0.006	±0.006	±0.006	±0.006
		CPU	840	1 200	3 000	2 280	2 340	3 660	4 080	4 260	4 680
	4	TC	35.712	35.902	36.047	35.460	35.803	35.510	35.930	35.699	35.782
		CI	±0.006	±0.006	±0.005	±0.006	±0.006	±0.005	±0.005	±0.006	±0.006
		CPU	1 020	2 460	1 740	1 440	1 740	1 380	1 740	2 040	7 740
	5	TC	36.290	35.704	35.844	36.229	35.756	35.926	36.070	35.817	35.824
		CI	±0.004	±0.005	±0.005	±0.004	±0.005	±0.005	±0.005	±0.005	±0.006
		CPU	1 380	4 260	4 080	3 060	5 880	5 280	5 460	4 500	7 560
6	1	TC	36.492	36.402	36.404	36.425	36.439	36.526	36.395	36.455	36.452
		CI	±0.004	±0.005	±0.005	±0.004	±0.005	±0.004	±0.005	±0.005	±0.005
		CPU	1 360	1 800	1 800	3 300	4 140	4 800	4 440	4 320	5 040
	2	TC	36.329	36.081	36.080	36.184	36.038	36.281	36.256	36.329	36.178
		CI	±0.004	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005
		CPU	960	1 500	2 880	2 760	1 980	3 300	2 760	2 940	3 660
	3	TC	35.439	35.308	35.467	35.414	35.304	35.364	35.436	35.370	35.358
		CI	±0.005	±0.005	±0.005	±0.006	±0.005	±0.005	±0.005	±0.006	±0.005
		CPU	2 520	1 800	2 580	2 520	1 860	3 000	3 480	3 240	4 620
	4	TC	36.469	36.442	36.524	36.485	36.383	36.395	36.436	36.499	36.489
		CI	±0.004	±0.004	±0.004	±0.004	±0.005	±0.005	±0.005	±0.004	±0.004
		CPU	1 500	1 260	1 740	1 080	1 440	1 680	2 580	2 280	3 420
	5	TC	35.746	35.821	35.809	35.706	35.793	35.776	35.774	35.752	35.737
		CI	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005	±0.005
		CPU	720	1 260	1 380	1 200	1 200	2 400	4 020	4 440	6 060
7	1	TC	35.956	36.090	35.960	35.886	35.856	35.720	35.473	35.861	35.930
		CI	±0.005	±0.005	±0.005	±0.005	±0.006	±0.006	±0.006	±0.005	±0.005
		CPU	1 680	1 800	2 220	1 860	1 440	1 980	2 820	2 280	3 840
	2	TC	35.723	35.788	35.715	35.222	35.520	35.662	35.348	35.183	35.753
		CI	±0.006	±0.006	±0.006	±0.006	±0.006	±0.008	±0.007	±0.007	±0.005
		CPU	1 380	4 260	4 440	2 520	4 260	4 800	4 740	4 560	5 400
	3	TC	35.835	35.661	35.821	35.718	35.791	36.013	35.780	35.783	35.562
		CI	±0.005	±0.006	±0.005	±0.006	±0.005	±0.005	±0.005	±0.005	±0.006
		CPU	1 620	1 440	1 920	1 800	1 920	2 880	1 380	2 520	4 440
	4	TC	35.387	35.347	35.433	35.457	35.222	35.275	35.363	35.342	35.292
		CI	±0.006	±0.006	±0.006	±0.006	±0.007	±0.006	±0.006	±0.006	±0.006
		CPU	3 060	4 620	5 100	3 840	5 160	6 060	4 260	5 460	7 260
	5	TC	35.172	35.332	35.212	35.251	35.384	35.240	34.972	35.254	35.202
		CI	±0.006	±0.006	±0.006	±0.006	±0.005	±0.006	±0.007	±0.006	±0.006
		CPU	1 260	1 020	2 520	1 800	1 320	2 280	1 860	2 880	4 980
8	1	TC	17.725	17.771	17.739	17.602	17.604	17.714	17.526	17.553	17.599
		CI	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004	±0.004
		CPU	1 920	2 100	2 700	4 320	2 820	3 120	6 540	4 740	8 820
	2	TC	18.038	18.185	18.146	18.028	18.065	18.125	18.115	18.122	18.093
		CI	±0.004	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003
		CPU	1 140	1 440	1 500	1 380	1 920	3 480	1 680	2 040	4 080
	3	TC	17.791	17.803	17.501	17.800	17.781	17.782	17.786	17.802	17.751
		CI	±0.003	±0.003	±0.004	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003
		CPU	720	900	1 500	1 260	1 380	1 560	1 200	1 500	2 700
	4	TC	18.164	18.135	18.138	18.151	18.107	18.134	18.122	18.132	18.138
		CI	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003
		CPU	1 200	1 920	1 980	1 200	1 140	2 040	1 440	1 740	2 340
	5	TC	17.935	17.924	17.955	17.878	17.920	17.950	17.901	17.951	17.946
		CI	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003	±0.003
		CPU	1 080	1 620	1 860	660	1 200	1 860	1 980	2 160	3 180