The Synchronized Multi-commodity Multi-service Transshipment-Hub Location Problem with Cyclic Services and Demand

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Abstract. The synchronized multi-commodity multi-service Transshipment-Hub Location Problem is a hub location problem variant faced by a logistics service provider operating in the context of synchromodal logistics. The provider must decide where and when to locate transshipment facilities in order to manage many customers’ origin-destination shipments with release and due dates while minimizing a total cost given by location costs, transportation costs, and penalties related to unmet time constraints. The considered synchromodal network involves different transportation modes (e.g., truck, rail, river and sea navigation) to perform long-haul shipments and the freight synchronization at facilities for transshipment operations. To the best of our knowledge, this variant has never been studied before. Considering a time horizon in which both transportation services and demand follow a cyclic pattern, we propose a time-space multi-network representation of the problem and an ad-hoc embedding of the time-dependent parameters into the network topology and the arcs’ weight. This allows to model the flow synchronization required by the problem through a Mixed-Integer Linear Programming formulation with a simplified structure, similar to well-known hub location problems and avoiding complicating constraints for managing the penalties. Through an extensive experimental campaign conducted over a large set of realistic instances, we present a computational and an economic analysis. In particular, we want to assess the potential benefits of implementing synchromodal logistics operations into long-haul supply-chains managed by large service providers. Since flexibility is one of the main feature of synchromodality, we evaluate the impact on decisions and costs of different levels of flexibility regarding terminals’ operations and customers’ requirements.

Keywords: Synchromodal logistics, Transshipment-Hub Location Problem, cyclic schedules, time-space multi-network

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1 Introduction

Synchromodal logistics is becoming a more and more relevant paradigm for managing operations in complex logistics networks in which goods for many customers are shipped long-distance. These networks are usually managed by Logistics Service Providers (LSPs), relying on transport and transshipment services offered by carriers and terminals, respectively. LSPs should follow the four main pillars of synchromodal logistics: real-time information, flexibility, cooperation and coordination, and synchronization (Giusti et al., 2019b). In a synchromodal logistics network, LSPs and their partners share real-time information and keep a flexible behavior to activate re-planning procedures as a quick response to disruptions, adjusting operations schedules and scopes. Flexible services can reduce costs, facilitate the modal shift to more sustainable vehicles, diminish the impact of congestion, and provide more alternatives in case of demand changes and unexpected events (Zhang et al., 2022). Customers also should adopt a more flexible perspective, thus allowing an a-modal booking model, i.e., relaxing shipments’ requirements in terms of vehicles and routes while deciding only on necessary information such as departure, arrival, release date, and due date. Customers may hand over transportation mode and route decisions in exchange for better services or lower costs (Khakdaman et al., 2020). Synchromodal logistics also requires a so-called orchestrator that facilitates strong cooperation and coordination mechanisms by synchronizing shipment flows and operations to improve the overall service quality and minimize costs. An LSP can take the orchestrator role and implement synchromodal logistics to achieve better transportation modes and resource utilization, an improved consolidation of loads, flexibility and freedom to switch modes, and services synchronization (Steadieseifi et al., 2014).

A network based on synchromodal logistics strongly relies on the usage of multi-modal transport services. These services require, as a common procedure, the use of intermediate terminals for transshipment operations to move goods stored in containers from one mode to another. Failing to handle the flow of containers properly contributes to delays in terminal operations that can propagate through the whole logistics network causing major congestion. Therefore, it is important to correctly plan in advance the usage of such terminals and the relative resources in order to achieve sustainable supply chains. A tactical problem arising in synchromodal logistics is the problem of an LSP that must contract the terminals and plan the transshipment operations to minimize the overall cost of the logistics network. The LSP takes the role of the orchestrator required in synchromodal logistics and must plan all the transport and transshipment activities, coordinating the contracted terminals and carriers. Carriers are external providers of multi-modal transport services with fixed schedules periodically repeated with the same pattern (e.g., every week). The customers of the LSP use the a-modal booking option, and their demand follows a similar cyclic pattern as for the transport services. That allows the LSP to contract terminals for a larger planning horizon (e.g., a season, six months, an year) in which services and demand are repeated cyclically. An important factor that the LSP should consider is that missing the commodities’ release and due dates require monetary compensation to customers. The LSP must pay a cost for late collection and early delivery to compensate for the storage costs faced by customers. In contrast, the LSP must correspond a
much higher compensation in case of late delivery for causing delays and making customers unsatisfied. Those features must be addressed by considering flow synchronization mechanisms relevant to synchromodal logistics.

This problem can be modeled as a variant of the Hub Location Problem (HLP), consisting in locating hubs that work as consolidation, connecting, and switching points for flows between origins and destinations (Farahani et al., 2013). Studying the HLP problem in synchromodal logistics is particularly interesting since the design of sophisticated planning methods to enable synchronization mechanisms has been identified as a critical success factor (Pfoser et al., 2016). The lack of models integrating the time dimension (Alumur et al., 2021) requires the design of ad-hoc new models explicitly addressing time and synchronization issues. Only a few papers in the literature consider synchronization procedures in tactical planning regarding the location of transshipment terminals and the flow management throughout them. Moreover, in the literature on supply chain management, the problems studied rarely integrate tactical and operational decisions with location decisions and, when they do, the network structure is oversimplified (Melo et al., 2009). Considering complex networks is especially important in synchromodal logistics. Moreover, the mathematical modeling field still requires numerical experiments on instances representing wide logistics networks consisting of many transshipment terminals and several transport services with modality alternatives (Rentschler et al., 2022). Our goal is to fill this gap by providing a new model for the HLP that accounts for many complex features of synchromodal logistics, and that can be a valuable tool to analyze how solutions differ when considering different complex network structures, amounts of demand, and stakeholders’ behaviors.

The contribution of this work is two-fold. The first contribution is the introduction in the scientific literature of a new complex variant of the HLP. In particular, we define and study the synchronized multi-commodity multi-service Transshipment-Hub Location Problem (STHLP) in which we consider an LSP that needs to contract transshipment terminals and coordinate its partners to achieve an efficient flow synchronization. The objective of the LSP is to minimize the fixed costs for contract terminals and the costs to use the different types of services. Some relevant aspects introduced in the STHLP are multi-commodity flows, multi-modal transport services, transshipment operations within nodes, a multi-period setting, and synchronization mechanisms managed with earliness and lateness penalties. We provide a Mixed-Integer Linear Programming (MILP) formulation for the STHLP designed over a time-space multi-network representation of the problem. By doing so, the model directly integrates synchronization mechanisms in the decisions regarding how to manage the commodities flow passing through the contracted terminals. This methodology, often used in network design and other types of problems, up to our knowledge, has never been used in the context of HLPs.

The second contribution is a computational validation and an economic analysis of the problem over a large set of diversified instances representing complex networks (such as the European one) for container transportation. The analysis objective is to study how different network structures and stakeholders’ behaviors affect the computation times, terminals selection and usage, transport services usage, and the different types of costs. The network structures
in each instance change due to the different amounts of commodities, terminals, and time periods considered. Instead, with stakeholders’ behavior, we intend to address different levels of flexibility from customers and terminals. Flexibility is an essential feature of synchromodal logistics and in our analysis emerges that, in a more collaborative environment, costs can be reduced and indirectly also emissions. Finally, we believe that the contributions of the paper can also be relevant for other contexts strictly related to synchronomodality in which synchronization plays an important role, such as cross-docking (Gümiş and Bookbinder, 2004), just-in-time logistics (Hofmann and Rüsch, 2017), and the so-called Physical Internet (Ambra et al., 2019).

The rest of the paper is organized as follows. In Section 2, we highlight the novelties introduced in our study compared to other HLP and discuss the literature related to synchronization issues in similar problems. In Section 3, we describe the synchromodal logistics system considered in the paper, defining the logistics network, stakeholders, and services and then defining the characteristics of the problem more precisely. In Section 4, we present a time-space multi-network representation of the problem able to capture all its features in terms of nodes and arcs characteristics in order to formulate the problem through a simplified MILP formulation consisting in locating terminals and managing multi-commodity flows with the aim of minimizing the overall costs. The experimental campaign with the instance generation process and the computational and economic analysis are presented in Section 5, while conclusions and some possible future research lines are presented in Section 6.

2 Literature review

The problem addressed in this paper considered decisions regarding transshipment and facility location. The Transshipment Problem is a particular case of the Transportation Problem in which it must be found the cheapest routes for moving one or more commodities from a set of origin nodes to a set of destination nodes passing through intermediate facilities (Khurana, 2015). Then, transshipment and location decisions are integrated into the already mentioned Hub Location Problem (HLP). The HLP consists in locating hubs, i.e., intermediate facilities, and routing the flows from origins to destinations through them (Campbell and O’Kelly, 2012). Several HLP variants exist, depending on how some features are addressed (Farahani et al., 2013). In our variant, the STHLP, we consider a discrete set of candidate nodes that can be selected as hubs, and the selection criterion is minimizing the cost incurred for locating hubs and managing flows. Multiple hubs can be located, and the number of hubs to locate is endogenous, i.e., it is not decided a priori but is determined as part of the solution. Each hub has a precise capacity and a fixed location cost. The other nodes in the networks can be connected with more than one hub, and a cost is paid for each commodity unit. Besides these classical features, other characteristics can be considered in the problem as the ones relevant to synchronomodal logistics.

In the literature, many contributions regard the HLP. Here we focus on the most recent
ones with an application regarding a similar context to the problem studied in our work. The article discussing HLP in synchron-modal logistics are very few, so we integrate the discussion with the contributions regarding multimodal transportation, which has become a main topic of interest for facility location problems in the last few years. The HLP in this context has many applications. Dukkanci and Kara (2017); Alumur et al. (2012) study the problem of one cargo company operating in Turkey using ground and airport hubs to ensure deliveries within 24 hours between each origin and destination of the network. Teye et al. (2017) consider an urban intermodal transport system, also referred to as an import/export intermodal transport system, in which goods are moved between a port and the neighboring urban area. The aim is to locate a precise number of intermodal terminals from a set of candidate sites to shift as many cargo volumes away from the road to intermodal transport. Regarding transport for people, Yuan and Yu (2018) proposes a model to assist the local government of China-Singapore Suzhou Industrial Park in developing transit hub location plans for different budget levels. Only a few works are related to the HLP in synchron-modal logistics. Giusti et al. (2021) proposed an extension to Tadei et al. (2012) in which a multi-period approach and uncertainty on transshipment capacities and utilities are considered to locate the terminals. In this variant, transshipment terminals are located by considering how flows can be synchronized during operations to respond to the loss of capacity. Crainic et al. (2021) also consider a multi-period setting and the location of transshipment terminals is done through synchronization of flows based on time-dependent costs for storage and early and late deliveries.

Facility location problems in transportation still require an effort to include multi-commodity flows, different transport modes, and time-dependent decisions as the ones regarding depots that can be located over time (da Gama, 2022). Also considering the HLP more in general there are still many lacks. The more relevant ones regard the lack of models integrating the time dimension and related to real-world problems, and the absence of insights obtained from results not only concerning the objective function and computation times (Alumur et al., 2021). Our contribution aims to deal with these lacks found in HLP and facility location problems in transportation. Similarly to Crainic et al. (2021); Giusti et al. (2021), we consider a multi-period setting in which the term location has a double meaning. The location decision corresponds to contracting a terminal and defining when to use them precisely. In contrast, we consider multi-commodity flows and multimodal transport services in the STHLP, and we test the model over realistic instances of large networks such as the ones of the main logistics companies operating in Europe. Moreover, our experimental campaign also includes an analysis of terminal and service utilization.

An important distinction is required to clarify what we mean by the term synchronization compared to how it usually is considered in other problems. For instance, Mirhedayatian et al. (2021) considers a two-echelon location-routing problem in which the vehicles are synchronized to arrive at the right time at the intermediate terminals connecting the two echelons. Here, the terminals to locate are decided by considering how transport operations can be synchronized to ensure that vehicles are well coordinated. Qu et al. (2019) consider a problem in which shipments must be assigned to services. The assignment is done by considering possible future synchronization operations consisting of re-scheduling services and re-routing
shipments. In our work, instead, we consider a synchronization related to flow, aiming to select the terminals that help to avoid missing the release and due dates, unnecessary storage costs, and the most expensive transport services. We contribute to the HLP problems by adding these synchronization aspects that, up to our knowledge, are never considered in the literature altogether.

All the characteristics considered in our problem may cause the model to become excessively complex to solve. The literature on HLP with a multi-period setting and flow synchronization is scarce, so we want to rely on a few techniques commonly used in service network design (SND) problems to model time and make the model simpler. An SND problem aims to select the services that will execute the time-sensitive shipments of the customer demand (Crainic et al., 2018). On a very generic level, we can assume that HLP and SND problems have similar characteristics that can be represented with the same network structure. The main difference is that HLP decisions regard the selection of nodes, whereas SND problems decisions regard the selection of arcs. This allows us to integrate into our STHLP a network structure commonly used in SND problems where nodes represent a physical point at a precise time and arcs a movement in both space and time. This type of network is commonly called a time-space network, and it can be used to represent the cyclic repetition of schedule length over a longer planning horizon. For interested readers, a few examples can be found in Pedersen et al. (2009) and Crainic et al. (2016). However, we will deeply discuss how to build a time-space network for the STHLP in Section 4.

3 Logistics context description and definition of the optimization problem

In this section, we describe the characteristics of a synchromodal logistics network, presenting the main stakeholders involved and the HLP faced by an LSP in that context (Section 3.1), and mathematically formalize all the STHLP characteristics (Section 3.2).

3.1 The Hub Location Problem in synchromodal logistics

In synchromodal logistics, stakeholders carry out various operations to fulfill the origin-destination demand of customers requiring logistics services to transport containers on long-haul journeys. The network used for this purpose is divided into two layers for short-haul and long-haul transport. In the short-haul layer, commodities are collected from their origins and moved to one or more terminals or delivered from one or more terminals to their destinations. Terminals such as ports, rail stations, and truck terminals must handle the freight delivered, mainly executing storage and transshipment operations, which are the movement of containers from one transport mode to another performed in intermediate terminals. In the long-haul layer, commodities
are moved between terminals on routes connecting them by roads, railways, maritime routes, and river routes. Carriers offer short-haul and long-haul transport services operated with different modes, named trucks, rail, ships, and barges. Carriers and terminals cooperate under the supervision of a logistics service provider so that customers can rely on more efficient services.

In this work, we study the tactical problem of an LSP taking the orchestrator role of a synchromodal logistics network that must decide the contracts to secure with transshipment terminals for a long-term planning horizon (e.g., a season, six months, a year). The contract should also include details on how transshipment and storage capacities will be used in each period, corresponding to a single day. The storage and transshipment services offered by the contracted terminals are combined with the transport services of the carriers operating in these terminals to offer complex synchromodal solutions to manage the origin-destination flows of many commodities. All the transport services available to the LSP have a fixed schedule and capacity, so the availability of those services at terminals is an essential aspect to consider. Another crucial factor that the LSP must consider is that services and demand follow a cyclic structure repeated multiple times (e.g., every week) during the whole planning horizon. Each recurrent short-term time horizon is called schedule length. Note that each schedule length can be related to the previous and the following ones since services and demand can overlap on two schedule lengths. For instance, with a schedule length of a week, a service may start on Friday and arrive on Tuesday, and the same may also apply to the release and due dates of commodities.

The LSP must fulfill the demand of many customers with the collection and delivery points distributed over a large area (e.g., all over Europe). Each shipment order is considered a commodity with many freight units of a single type (e.g., a twenty-foot equivalent unit, pallet) with a specific origin, destination, release time, and due time. Besides those basic constraints for the shipments, customers must agree to an a-modal booking and allow the LSP to use shipment splitting, moving units of a single order separately with different transport services, and consolidation mechanisms, shipping containers of various customers on a single vehicle. That allows the LSP to decide which routes, modes, and services will be used to fulfill the demand and if shipment splitting and consolidation are required.

The objective of the LSP is to minimize the costs for contracts and services, ensuring that customers are satisfied. Performing any activity in terminals requires paying the contract cost and a fee for each period the terminal is used, plus a unit cost to transship and store commodities. Using any transport services requires only paying a unit cost. The LSP has to deal with multi-commodity flows that, if not properly managed, can lead to poor service quality, making customers unsatisfied for failing to comply with agreements, i.e., the deadlines are not respected. In that case, the LSP must pay extra costs representing the compensation to customers for collecting commodities later than expected or delivering them ahead or after their due dates. Compensations aim to mitigate the drawbacks for customers of having more units stored to handle or missing some important ones for their business. Earliness and lateness fines can provide priority mechanisms based on how much each shipment is time sensitive. Another critical factor regards the synchronization of commodity flows passing through transshipment
terminals. Arriving too early or late in a transshipment terminal may require paying for storing freight units or shipping with more expensive transport modes. Considering these aspects contribute to activating synchronization mechanisms that ensure a more efficient location of transshipment terminals and shipments allocation to the available services, considering the limited transshipment, storage, and transport capacities.

In this work, we also consider different behaviors that terminals and customers can have. We define strict and flexible behaviors for both stakeholders that will impact some costs. When stakeholders are flexible, we are in an ideal situation for synchromodal logistics implementation since terminals would require to spend less to book them in advance, and customers would allow deviating more from the original plan. We consider strict terminals with high fixed usage costs but lower operational costs and flexible terminals that, in contrast, have lower fixed usage costs but higher operational costs. Moreover, we consider strict customers with very rigid release and due dates with high penalties for deviating from the original plan and flexible customers with lower penalties. Note that since the different behaviors only affect costs, we do not need to address this aspect while designing the model, but we have to consider it only when preparing the test instances.

3.2 Mathematical definition of the STHLP

Let us consider a set $K$ of commodities to be transported, a set $O^{ph}$ of commodities’ origins, a set $D^{ph}$ of commodities’ destinations, a set $I^{ph}$ of transshipment terminals, and a set $M$ of freight transport modes (e.g., trucks, rail, barges, ships). The LSP wants to optimize its business over a long-term planning horizon (e.g., a few months). Within the planning horizon, the demand and the services follow a cyclic pattern repeated in every schedule length (e.g., a week) defined as a sequence $T = \{1, 2, \ldots, T\}$ composed of periods of the same length (e.g., a day). For each commodity $k \in K$, a demand $w^k$ must be collected from origin $o^k \in O^{ph}$ at release time $\alpha^k \in T$ and delivered to destination $d^k \in D^{ph}$ at due time $\omega^k \in T$ after a planned delivery time $\tau^k$. However, it is possible to collect any commodity $k \in K$ at its origin after its exact release time by paying a unit penalty $g^k$ for each time period of lateness as well as deliver $k$ at its destination before or after its exact due time by paying a unit penalty $e^k$ and $b^k$ for each time period of earliness and lateness, respectively. Note that the units of a commodity may travel on different routes since shipment splitting is allowed, and the penalties are paid separately for each unit for which the release or due times are not respected.

Managing commodities during their journeys from their origins to their destinations requires a set $S$ of services, which can relate both to transport and storage, and possibly be operated by different providers. Each service $s \in S$ is associated with a unit cost $c^s$ for each commodity $k \in K$, a capacity $u^s$ (i.e., the maximum units of commodities that can be stored or moved), a departure node $i^s \in I^{ph} \cup O^{ph} \cup D^{ph}$, an arrival node $j^s \in I^{ph} \cup O^{ph} \cup D^{ph}$, a starting time $\alpha^s \in T$, an ending time $\omega^s$, and a service time $\tau^s \in N$. Let us further define $S^{sh}$ as the set of short-haul transport services operated between origins/destinations and
transshipment terminals (i.e., \( S^\text{th} := \{ s \in S \mid (i_s \in O^\text{ph}, j_s \in D^\text{ph}) \lor (i_s \in I^\text{ph}, j_s \in D^\text{ph}) \} \), \( S^\text{sh} \) as the set of long-haul transport services operated between transshipment terminals (i.e., \( S^\text{sh} := \{ s \in S \mid i_s, j_s \in I^\text{ph} \} \)), and \( S^\text{st} \) as the set of storage services within transshipment terminal (i.e., \( S^\text{st} := \{ s \in S \mid i_s = j_s \in I^\text{ph} \} \)). For each storage service \( s \in S^\text{st} \), the service time is \( \tau_s = 1 \). Finally, a short-haul and long-haul transport service \( s \in S^\text{th} \cup S^\text{sh} \) also relates to a specific transport mode \( m_s \in M \).

In order to use a service of a transshipment terminal, the LSP must first secure a contract with the terminal itself and then decide in which periods it will be used. Let \( l_i \) be the fixed cost to contract terminal \( i \in I^\text{ph} \), \( q_{it} \) be the fixed cost for using the terminal \( i \in I^\text{ph} \) in period \( t \in T \), and \( h^k_{it} \) be the unit transshipment cost to manage commodity \( k \in K \) at terminal \( i \in I^\text{ph} \) in period \( t \in T \). Finally, any terminal \( i \in I^\text{ph} \) has maximum transshipment capacity \( v_{it} \) limiting the commodities’ units that can be prepared for shipping in each period \( t \in T \). Anything exceeding the transshipment capacity must be stored in the terminal until the next period.

The objective of our STHLP is to minimize a cost function composed of several components, namely, the fixed cost to contract terminals, the fixed cost for using such terminals, the unit transshipment cost to manage commodities, the unit cost for storage and transport services, and the penalty costs related to the earliness and lateness with respect to expected release or due periods. Such a minimization will be done over a single schedule length. However, since operations are executed continuously over the overall planning horizon, we need to take care of those services starting within a schedule length but ending within the next one as well as commodities released and due within two different schedule lengths. This will be addressed in the next section.

4 Mathematical formulation based on a time-space multi-network

In Section 4.1, we introduce a time-space network to easily manage the time-dependent parameters of the problem. We also discuss how to embed into the time-space network all the requirements for a correct management of the cyclic schedules and of the penalties for late collection and early/late delivery. The Mixed-Integer Linear Programming formulation of the STHLP is then presented in Section 4.2.

4.1 Time-space multi-network and management of cyclic schedules

As commonly done in the literature (see, e.g., Crainic and Hewitt, 2021), we model the above problem on a time-space \( \mathcal{G} = (\mathcal{N}, \mathcal{S}) \), in which nodes and arcs are time-dependent. The set of nodes is defined as \( \mathcal{N} = O \cup D \cup I \), where \( O = \{ (i, t) \mid i = o^k, t = \alpha^k, k \in K \} \) are origin nodes,
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\[ \mathcal{D} = \{(i, t) | i = d^k, t = \omega^k, k \in \mathcal{K}\} \] are destination nodes, and \( \mathcal{I} = \{(i, t) | i \in \mathcal{I}^{\text{sh}}, t \in \mathcal{T}\} \) are transshipment terminal nodes. We also define \( \mathcal{I}_i \subset \mathcal{I} \) as the subsets of nodes of network \( \mathcal{G} \) associated with each terminal \( i \in \mathcal{I}^{\text{sh}} \). The set of arcs corresponds to the set \( \mathcal{S} \) of services already defined above. In fact, each service \( s \in \mathcal{S} \) can be represented by an arc going from node \((i, t')\) such that \( i = i_s \) and \( t' = \alpha_s \) to node \((j, t'')\) such that \( j = j_s \) and \( t'' = \omega_s \). In the rest of the discussion, we will indicate with \((i, t)_s\) the departure node and with \((j, t)_s\) the arrival node of service \( s \in \mathcal{S} \). Note that \( \mathcal{G} \) is a multi-network since there might be parallel arcs linking the same two nodes but referring to different modes.

To deal with services and demand exceeding the schedule length, we followed a common practice used in service network design (see, e.g., Crainic et al., 2016), where arcs wraps around to represent services starting and ending in different schedule lengths. In such a network, to avoid loops and other types of misbehavior, we also need to ensure that the path length of a unit related to a certain commodity does not exceed \( T \). To this aim, we will allow assigning flow of a commodity \( k \in \mathcal{K} \) to a time-space arc only if it belongs to a set \( \mathcal{S}^k = \{s \in \mathcal{S}^{\text{sh}}| i_s = o^k \lor j_s = d^k\} \cup \{s \in \mathcal{S}^{\text{sh}} \cup \mathcal{S}^{\text{st}} | (\alpha^k \leq \alpha_s \leq \omega_s) \lor (\omega_s < \alpha^k \leq \alpha_s) \lor (\alpha_s \leq \omega_s < \alpha^k)\} \). In \( \mathcal{S}^k \), the first set indicates that only short-haul transport arcs related to commodity \( k \) are considered, whereas the second set includes only long-haul transport and storage arcs excluding those related to time-infeasible services (the three specified cases).

Finally, the penalties concerning the late collection and early or late delivery of commodities can be integrated directly into the cost of short-haul arcs. Such a non-trivial procedure works as follows. Given a commodity \( k \in \mathcal{K} \), a short-haul arc \( s \in \mathcal{S}^{\text{sh}} \) such that \((i, t)_s = (o^k, \alpha^k)\) is associated with a cost \( C^k_s = c^k_s + \delta g^k \), where \( \delta = (T + \omega_s - \tau_s - \alpha^k) \mod T \) is the number of periods of lateness for which we must pay the unit penalty \( g^k \). Instead, before assigning the penalties to short-haul arc \( s \in \mathcal{S}^{\text{sh}} \) such that \((j, t)_s = (d^k, \omega^k)\) representing the delivery of commodity \( k \in \mathcal{K} \), we must identify if that arc represent an early or a late delivery for the two cases in which \( \alpha^k < \omega^k \) and \( \omega^k > \alpha^k \). In the first case, an arc \( s \) such that \( \alpha^k < \alpha_s + \tau_s < \omega^k \) represents an early delivery, and a late delivery otherwise. In the second case, an arc \( s \) such that \( \alpha^k > \alpha_s + \tau_s > \omega^k \) represents a late delivery, and an early delivery otherwise. An arc representing a late delivery is associated with a cost \( C^k_s = c^k_s + \delta b^k \), where \( \delta = (T + \alpha_s + \tau_s - \omega^k) \mod T \) is the number of periods of lateness for which we must pay the unit penalty \( b^k \). Instead, an arc representing an early delivery is associated with a cost \( C^k_s = c^k_s + \delta e^k \), where \( \delta = (T + \omega^k - \alpha_s - \tau_s) \mod T \) is the number of periods of earliness for which we must pay the unit penalty \( e^k \). Note that, the cost of any short arc departing from an origin for which \( \omega_s - \tau_s = \alpha^k \) and any arc arriving in a destination for which \( \alpha_s + \tau_s = \omega^k \) includes only the unit transport cost since those arcs respectively represent the collection and delivery of commodity \( k \in \mathcal{K} \) on time. For each arc \( s \not\in \mathcal{S}^{\text{sh}}, C^k_s = c^k_s \). To clarify the just described labeling mechanism and how penalties work, we report the graphical representation of a simple time-space network related to a single commodity in Example 1.

**Example 1** Let us consider, without loss of generality, a single-commodity single-mode network that operates over 7 periods. Such a network, shown in Figure 1, is composed of an
origin \(o_3\) (i.e., the commodity is released in period 3), a destination \(d_6\) (i.e., the commodity is due at period 6), and two transshipment terminals \(i\) and \(j\) (7 nodes appear for each terminal). The network includes short-haul transport arcs for collecting the commodity in \(o_3\), delivering the commodity in \(d_6\), storage arcs, and long-haul transport arcs. The dotted arcs represent services arriving in the following schedule length and services departing from the previous one. For simplicity, each arc is considered with a null cost, except for the short-haul arcs, which are labeled with the earliness and lateness penalties. The commodity is released in period 3, and there will not be any penalty if collected immediately. Otherwise, we must pay a lateness penalty \(q\) proportional to each period of delay. Then, the commodity must be stored in node \(i\) or sent to node \(j\), where it is prepared for delivery at its destination. If the commodity reaches the destination exactly in period 6, only the transport cost is paid. Otherwise, penalties \(e\) and \(b\) are also paid for each period of earliness and lateness. In any case, note that this network forces to fulfill the demand at most in period 2, ensuring that the commodity travel time will not exceed the schedule length. Finally, given the above assumptions, note that nodes \(i_2\) and \(j_3\) can be dropped from the network since they would never be traversed in practice.

4.2 Mathematical formulation

The LSP decisions are now related to the time-space multi-network through the selection of nodes and the commodities assignment to services. In particular, deciding to use a terminal in a specific period corresponds to selecting the corresponding node, while the management of commodities is done by creating flow paths over arcs. So, let us define the following decision variables:

- \(x_i \in \{0, 1\}\): binary variables equal to 1 if terminal \(i \in I^{ph}\) is contracted;
• $y_{(i,t)} \in \{0,1\}$: binary variables equal to 1 if node $(i,t) \in I$ is selected, i.e., the terminal $i \in \mathcal{I}^h$ is used in period $t \in T$;

• $z^k_s \geq 0$: continuous variables representing the flow of commodity $k \in \mathcal{K}$ on arc $s \in \mathcal{S}^k$.

Then, a Mixed-Integer Linear Programming formulation for our STHLP is as follows (given the great amount of notation introduced in the previous section, we provide in Table 1 a summary of what is needed to understand the formulation):

$$\min_{x,y,z} \sum_{i \in \mathcal{I}^h} l_i x_i + \sum_{(i,t) \in I} q_{it} y_{(i,t)} + \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}^k} C^k_s z^k_s + \sum_{k \in \mathcal{K}} \sum_{(i,t) \in I} \sum_{s \in \mathcal{S}^k \setminus \mathcal{S}^u} h^k_{it} z^k_s$$

subject to

$$\sum_{s \in \mathcal{S}^k \setminus \mathcal{S}^u} (i,t)_s = (i,t), \quad z^k_s = \begin{cases} w^k & \text{if } (i,t) = (o^k, a^k), \\ 0 & \text{if } (i,t) \in \mathcal{N} \setminus \{(o^k, a^k), (d^k, \omega^k)\}, \\ -w^k & \text{if } (i,t) = (d^k, \omega^k), \end{cases} \quad k \in \mathcal{K} \quad (2)$$

$$\sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}^k \setminus \mathcal{S}^u} z^k_s \leq v_{it} y_{(i,t)}, \quad (i,t) \in I \quad (3)$$

$$\sum_{k \in \mathcal{K}} z^k_s \leq u_s y_{(i,t)}, \quad s \in \mathcal{S} \quad (4)$$

$$\sum_{k \in \mathcal{K}} z^k_s \leq u_s y_{(j,t)}, \quad s \in \mathcal{S} \quad (5)$$

$$y_{(i,t)} \leq x_i, \quad (i,t) \in I \quad (6)$$

$$x_i \in \{0,1\}, \quad i \in \mathcal{I}^h \quad (7)$$

$$y_{(i,t)} \in \{0,1\}, \quad (i,t) \in I \quad (8)$$

$$z^k_s \geq 0, \quad k \in \mathcal{K}, s \in \mathcal{S}^k \quad (9)$$

The objective function (1) minimizes the fixed costs for securing a contract and the fee to use a terminal in a period plus the unit costs for transshipping and transporting commodities. Constraints (2) ensure the flow conservation for each commodity at each node. Constraints (3) limit flows on transport arcs to the maximum transshipment capacity of the transshipment node if selected, and to 0 otherwise. Constraints (4) and (5) guarantee that if any flow uses arcs passing through transshipment nodes, then those nodes are selected. Constraints (6) ensure that if a transshipment node is selected in at least one period, then a contract with the corresponding terminal must be secured. Finally, constraints (7), (8), and (9) represent the decision variables’ domains.

Note that reformulating the model over the time-space multi-network and embedding the penalties into the short-haul arcs cost has largely simplified the model. The proposed STHLP
Table 1: Summary of sets and parameters used in the model

<table>
<thead>
<tr>
<th>Sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{K}$</td>
<td>set of commodities</td>
</tr>
<tr>
<td>$\mathcal{I}^{ph}$</td>
<td>set of transshipment terminals</td>
</tr>
<tr>
<td>$\mathcal{I}, \mathcal{O}, \mathcal{D}$</td>
<td>set of transshipment nodes, origin nodes, and destination nodes</td>
</tr>
<tr>
<td>$\mathcal{I}_i$</td>
<td>set of nodes related to terminal $i \in \mathcal{I}^{ph}$</td>
</tr>
<tr>
<td>$\mathcal{N}$</td>
<td>set of nodes of the time-space multi-network defined as $\mathcal{I} \cup \mathcal{O} \cup \mathcal{D}$</td>
</tr>
<tr>
<td>$\mathcal{S}^{sh}, \mathcal{S}^{lh}, \mathcal{S}^{st}$</td>
<td>set of short-haul transport services, long-haul transport services, and storage services</td>
</tr>
<tr>
<td>$\mathcal{S}$</td>
<td>set of services corresponding to the arcs of the time-space multi-network defined as $\mathcal{S}^{sh} \cup \mathcal{S}^{lh} \cup \mathcal{S}^{st}$</td>
</tr>
<tr>
<td>$\mathcal{S}^k$</td>
<td>subset of $\mathcal{S}$ containing only the arcs that commodity $k \in \mathcal{K}$ can use</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_i$</td>
<td>fixed cost for securing a contract with terminal $i \in \mathcal{I}^{ph}$</td>
</tr>
<tr>
<td>$q_{it}$</td>
<td>fixed cost for selecting node $(i, t) \in \mathcal{I}$</td>
</tr>
<tr>
<td>$h_{kt}$</td>
<td>unit transshipment cost for commodity $k \in \mathcal{K}$ in node $(i, t) \in \mathcal{I}$</td>
</tr>
<tr>
<td>$C_{ks}$</td>
<td>unit cost for commodity $k \in \mathcal{K}$ to use arc $s \in \mathcal{S}^k$</td>
</tr>
<tr>
<td>$w^k$</td>
<td>demand of commodity $k \in \mathcal{K}$</td>
</tr>
<tr>
<td>$v_{it}$</td>
<td>transshipment capacity of node $(i, t) \in \mathcal{I}$</td>
</tr>
<tr>
<td>$u_s$</td>
<td>service capacity of arc $s \in \mathcal{S}$</td>
</tr>
</tbody>
</table>

The Synchronized Multi-commodity Multi-service Transshipment-Hub Location Problem with Cyclic Services and Demand formulation, in fact, boils down to a more classical HLP with multi-commodity flow constraints (as those studied by Meraklı and Yaman, 2017; Ebery et al., 2000) and some additional big-$M$ constraints.

5 Experimental campaign

Section 5.1 is devoted to discussing in detail the methodology used to generate instances representing realistic logistics networks. The set of instances considered accounts for different network structures and stakeholders’ behaviors. The set of instances used in the experimental campaign and the results of the computational and economic analysis are described in Section 5.2. All the managerial insights discussed while presenting the results are summarized in Section 5.3.

5.1 Instance generation

5.1.1 Terminals, modes, and commodities

We want to simulate logistics networks covering large areas (e.g., Europe) and divided into a set $\mathcal{R}$ of smaller geographical regions (e.g., Netherlands, North Italy, Ireland) in which internal distances allow short-haul movements within a few hours. For each region $r \in \mathcal{R}$, we generate
a centroid \( P_r = (x_r, y_r) \) where coordinates \( x_r \) and \( y_r \) are randomly drawn in a square with 4200km of diagonal, i.e., more or less the size of a square containing all European countries. We also ensure that each region centroid is at least 500Km far from another region centroid. We associate with each region \( r \in \mathcal{R} \) a set \( \mathcal{D}_r \) of transshipment terminals composed of 5 small, 3 medium, and 2 large terminals in terms of transshipment capacity, assuming that terminals with higher capacity are less present compared to terminals with smaller capacity. In order to proportion our networks with respect to the real-world container’s movements (World Shipping Council, 2021), we assume a daily transshipment capacity equal to 1, 2.5, and 5 thousand containers for small, medium, and large terminals, respectively. Then, the long-haul travel distance between all transshipment terminals belonging to a region \( r' \in \mathcal{R} \) and all those belonging to region \( r'' \in \mathcal{R} \) is calculated as the Euclidean distance \( d(P_{r'}, P_{r''}) \) between the relative centroids. Since a time period corresponds to a single day thus making short-haul distances negligible with respect to the long-haul ones, we assume a null distance for movements internal to the same region.

Trucks, rail, sea ships, and river barges are considered possible modes for long-haul services between regions. For each mode \( m \in \mathcal{M} \), we assume an average speed \( \nu_m \), equal to 62.5, 50, 29, and 33 km/h for truck, rail, sea ship, and river barge, respectively. This estimation has been done according to the available schedules of some leading European companies such as DHL, COSCO Shipping Lines, Kuehne Nagel. We assume that large terminals always allow trucks, rail, and sea ships, while river barges have 20% probability to be available, medium terminals always allow trucks and rail, while sea ships and river barges have 20% probability to be available, and small terminals always allow trucks, never allow sea ships, while rail and river barges have 80% and 20% probability to be available, respectively.

The demand for each commodity is generated by ensuring that the total demand to manage in each region \( r \in \mathcal{R} \) equals a certain proportion \( \lambda > 0 \) of the total transshipment capacity of that region, i.e., \( \sum_{k \in \mathcal{K}_r} w^k = \lambda \min \left\{ \frac{|\mathcal{K}_r| \sum_{t \in \mathcal{T}_r} \sum_{i \in \mathcal{I}_r} v_{it}}{T}, \sum_{t \in \mathcal{T}_r} \sum_{i \in \mathcal{I}_r} v_{it} \right\} \), where \( \mathcal{K}_r \) is the set of commodities with an origin or a destination in \( r \). Moreover, we also ensure that the origin and the destination of each commodity are associated with different regions \( r' \) and \( r'' \) (i.e., at least a long-haul service is required in between). Finally, the release time \( \alpha^k \) is randomly drawn from \([1, T]\) while the due time \( \omega^k \) is drawn from a Normal distribution with mean \( \alpha^k + \frac{|\mathcal{M}| d(P_{r'}, P_{r''})}{\sum_{m \in \mathcal{M}} \nu_m} \), stdev \( \frac{T}{4} \), and truncated in \( \left[ \alpha^k + \frac{d(P_{r'}, P_{r''})}{\max_{m \in \mathcal{M}} \nu_m}, \alpha^k + T - 1 \right] \). To any due time exceeding the schedule length, we apply a \( \mod T \) as explained in Section 4.1.

### 5.1.2 Services and schedules

Short-haul and storage services are scheduled in each time period. Moreover, a short-haul has a capacity large enough to manage all the demand released at the origin where the service departs or due at the destination where the service arrives, while a storage service has a capacity equal to 10 times the transshipment capacity of the terminal in which it is executed. Long-
haul services, instead, have precise schedules to be generated. Such schedules are generated according to specific probabilities and characteristics of the modes, and only between terminals in which such modes are available. More precisely:

- truck services are scheduled between all terminals in each time period. The capacity of each service, which differently from the other modes can be split into many vehicles, is equal to half of the total transshipment capacity of the departure terminal.

- rail services are ensured between each pair of terminals with probability 85%. Services are repeated every 3 periods and can carry 100 containers.

- sea ship services are ensured between each pair of terminals with probability 60%. Services are repeated every 7 periods and can carry 1500 containers.

- river barge services are scheduled between all terminals. Services are repeated every 2 periods and can carry 80 containers.

Note that the starting period of each service is generated randomly within the schedule length.

Finally, we assume that short-haul services have null travel times (given the null distances assumed within the same region), storage services have a unit service time, while a long-haul service \( s \in S^{\text{lh}} \) between regions \( r' \) and \( r'' \) and operated by mode \( m \) have a travel time equal to

\[
\tau_s = \frac{d(P_{r'}, P_{r''})}{\nu_m}.
\]

5.1.3 Costs and penalties

It is arduous to obtain precise costs regarding logistics operations of a complex network as the one we want to simulate since private companies do not often share this information for many reasons. So, to ensure that the results obtained for our problem were reasonably realistic, we calibrated the generation process of the costs with respect to the revenues of the world’s fifty largest LSPs (such data are publicly available, see Armstrong & Associates, 2022). Moreover, to increase the accuracy of the cost generation, we also relied on the information collected in collaboration with important logistics companies during the SYNCHRO-NET Horizon 2020 project (Giusti et al., 2018, 2019a).

The costs related to terminals depend on their sizes and behaviors. Terminals follow the rules of the economy of scale, so larger terminal has lower storage and transshipment costs compared to the smaller ones. Regarding the terminal behaviors, we assume that a strict terminals prefer to ensure revenues in advance, therefore, they tend to have higher fixed costs but lower operational costs, whereas the contrary happens for flexible terminals. More precisely, for each \((i, t) \in T\):
• the fixed cost to secure a contract with the terminal is \( l_i = 2 \sum_{t \in T} v_{it} \);

• the fixed cost for using the terminal in period \( t \in T \) is \( q_{it} = 8 v_{it} \mu_t \), where \( 0 \leq \mu_t \leq 1 \) is a parameter representing the terminal flexibility (the higher the value, the stricter the terminal);

• for each commodity \( k \in K \) in period \( t \in T \), the unit transshipment cost is \( h_{it}^k = \bar{h} + (1 - \mu_t) \frac{q_{it}}{v_{it}} \), where \( \bar{h} \) is an average transshipment cost per container equal to 50, 40, and 30 for small, medium, and large terminals, respectively;

• for each commodity \( k \in K \) and storage service \( s \in S_{st} \), the unit storage cost is \( C_{ks} = 0.6 \frac{h_{it}^k}{\mu_c} \).

Finally, we generate long-haul service costs proportionally to distances and modes. For a commodity \( k \in K \), the unit cost for a long-haul service \( s \in S_{lh} \) departing from region \( r' \) and arriving at region \( r'' \) is \( C_{ks} = \bar{C} d(P_{r'}, P_{r''}) \), where \( \bar{C} \) is an average cost parameter per km equal to 0.8, 0.5, 0.25, and 0.3 for truck, rail, ship, and barge, respectively. Instead, short-haul services do not have transport costs, but only earliness or lateness penalties. We generate such penalties proportionally to the flexibility of the customers requiring the specific commodity. In particular, stricter customers tend to exhibit higher penalty costs for early or late arrivals. For a commodity \( k \in K \), the unit penalties are \( b_k = \bar{d} \mu_c \) and \( g_k = e_k = 0.7 b_k \), where \( \bar{d} \) is the average Euclidean distance between the centroids of all the regions and \( \mu_c > 0 \) is a parameter representing the customer flexibility (the higher the value, the stricter the customer).

5.2 Numerical experiments

The aim of our experimental campaign is to study how computation times (Section 5.2.1), costs (Section 5.2.2), and terminals and services utilization (Section 5.2.3) vary depending on the generation parameters. For the numerical tests, we generated 480 instances based on 5 repetitions of the 96 possible combinations of \( |K| = \{100, 200\} \) number of commodities, \( |R| = \{4, 6\} \) number of regions, \( T = \{7, 14\} \) number of time periods, 3 levels of demand (low, medium, and high) corresponding to \( \lambda = \{0.05, 0.1, 0.2\} \), 2 levels of terminals flexibility (flexible or strict) corresponding to \( \mu_t = \{0, 1\} \), and 2 levels of customers flexibility (flexible or strict) corresponding to \( \mu_c = \{0.2, 5\} \). Note that considering flexible terminals implies that the fixed costs for using each terminal in every period are always equal to 0.

All the instances are solved with CPLEX v12.9 via its Java Concert Technology APIs with a maximum solving time per instance of 4 hours (14400 seconds). All the experiments were done on an Intel(R) Core(TM) i7-6700 CPU @ 3.40GHz machine with 16GB RAM and running Windows 7 Professional 64-bit.

In all the following analyses, we group the instances in three different ways, namely, their network structures, stakeholders’ behaviors, and levels of demand. The network structure is
determined by $|\mathcal{K}|$, $|\mathcal{R}|$, and $T$ since those parameters directly affect the number of nodes and arcs of the network. Stakeholders’ behaviors regard the parameters $\mu_t$, and $\mu_c$, used to represent if stakeholders are flexible or strict, affecting some of the costs. The demand level is affected by the parameter $\lambda$ varying the total number of units to ship. The values presented in all the analyses are the averages over the 5 repetitions of each specific combination of the generation parameters.

5.2.1 Computational analysis

In this computational analysis, we present the solving time in seconds $t(s)$, the time-to-best in seconds $ttb(s)$, and the percentage MIP Gap $\text{gap}(\%)$. The $ttb(s)$ indicates the time in which the best solution is found, while the $\text{gap}(\%)$ indicates the percentage gap between the best feasible solution and the best lower bound found.

The results regarding the different stakeholders’ behaviors are presented in Figure 2 and Table 2. Figure 2 shows box plots for the $t(s)$ and the $ttb(s)$, while Table 2 displays the disaggregated data, the average, the best, and the worst $t(s)$, $ttb(s)$ and $\text{gap}(\%)$. It is evident that a more flexible behavior, especially from terminals, implies a considerable reduction of $t(s)$ and $ttb(s)$. The optimal solution when flexible terminals are considered is always found much earlier than the available 4 hours, in about 8 minutes on average. The number of binary variables is less with flexible terminals since the contract cost is the only fixed cost that must be paid in that situation, so those instances are easier to solve. Instead, many instances with strict terminals are not solved to the optimum, but the $\text{gap}(\%)$ is low and often lower than 1, excluding a few cases in which the $\text{gap}(\%)$ is very high, and so it is not guaranteed that the best solution found is close to the optimum. On average, the $ttb(s)$ is much less than the $t(s)$, especially for instances with strict customers.

Figure 3 and Table 3 show the box plot and the disaggregated table of the computation performance for different network structures, respectively. In general, increasing the number of periods makes the problem harder to solve, so considering longer schedule lengths or partitioning the schedule lengths into more periods requires a higher computational effort. The number
of regions and commodities have a larger impact on the \( t(s) \) when the number of periods is lower (\( T=7 \)). The number of regions affects the \( t(s) \) more than the number of commodities, but increasing the number of commodities affects the \( ttb(s) \) more. The solutions are, on average, always very close to the optimum except in a few cases when \( t = 14 \) and \( |K| = 200 \) are considered.

Regarding the levels of demand, the box plot and the disaggregated data are displayed in Figure 4 and Table 4, respectively. The most difficult instances to solve are the ones with low demand. It is reasonable to assume that, when the level of demand is higher, some services become essential for fulfilling the demand thus the decision process becomes easier.

We can conclude that CPLEX performs quite well with small networks with high demand and flexible stakeholders. Instead, it can be interesting to develop a specific solution method if the specific problem of an LSP requires considering larger networks, especially when the number of periods is higher, a very low demand compared to the available services/transshipment...
Finally, penalties include all types of earliness and lateness penalties, i.e.,

Table 3: $t(s)$, $ttb(s)$, and gap(%) for different numbers of regions, commodities, and periods

<table>
<thead>
<tr>
<th>demand</th>
<th>terminals</th>
<th>customers</th>
<th>$T=4$</th>
<th>$T=6$</th>
<th>$T=4$</th>
<th>$T=6$</th>
<th>$T=4$</th>
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<tbody>
<tr>
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<td>8188</td>
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<td>0.03</td>
<td>11716</td>
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<td>0.37</td>
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<td>4189</td>
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<td>0.01</td>
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<td>67</td>
<td>64</td>
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</table>

Figure 4: $t(s)$ and $ttb(s)$ for different levels of demand

5.2.2 Economic analysis

In this analysis, we divide the costs into four different categories: transport costs, terminals’ operational costs, terminals’ fixed costs, and penalties. Transport costs include all the costs for long-haul transport operations, i.e., $\sum_{k \in K} \sum_{s \in S} C_{s}^{k} x_{s}$. Terminals’ operational costs are the sum of storage and transshipment costs, i.e., $\sum_{k \in K} \sum_{s \in S} C_{s}^{k} + \sum_{k \in K} \sum_{i, t} \sum_{q \in T} \sum_{q(t) \in I} h_{q(t)}^{k}$. While terminals’ fixed costs comprise the contract and usage costs, i.e., $\sum_{k \in K} \sum_{s \in S} C_{s}^{k} \gamma_{s}^{k} x_{s}$.

Finally, penalties include all types of earliness and lateness penalties, i.e., $\sum_{k \in K} \sum_{s \in S} C_{s}^{k} \gamma_{s}^{k} x_{s}$. capacities, and strict stakeholders.
In all the following figures, we show the average cost to ship a single container (i.e., all costs are divided by the total demand $w_{tot} := \sum_{k \in K} w^k$) and the contribution of the previously mentioned types of costs. Using unit costs instead of total costs is required to compare instances with very different total costs due to their network structure or units to ship. Figure 5 shows the unit cost composition for different levels of demand: The total unit cost when terminals and customers are flexible is the lowest one. The terminals’ behaviors impact the total unit cost the most, mainly for the higher fixed costs paid. Flexible customers mainly contribute to reducing transport costs since it becomes preferable to miss deadlines and take cheaper transport modes. The penalties paid in that case can be intended as a possible discount the LSP can offer to customers for convincing them to be more flexible in their requests. Instead, the unit costs for different network structures are displayed in Figure 6. Considering more periods contributes the most to reducing the unit cost, especially for the fewer penalties and transport costs paid. This happens because the demand is distributed over a larger time horizon, and thus it is possible to select better services and avoid more penalties. Increasing the number of regions also contributes to the rise of all types of costs. In this case, the demand is distributed over a larger area, and thus it requires using more terminals and transport services since the regions involved are more. Instead, considering more commodities always result in a lower unit cost, since the demand is better distributed in space and time. Finally, Figure 7 shows the impact on costs of different demand levels. Increasing the units to ship reduces the terminals’ fixed and operational costs since the terminals are used more efficiently. Penalties are also fewer due to the best proportion between demand fulfilled on time and the one in which...
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Figure 5: Unit costs for flexible and strict stakeholders

Figure 6: Unit costs for different numbers of regions, commodities, and periods

Figure 7: Unit costs for different levels of demand
deadlines are missed. In contrast, transport costs tend to increase when the demand is higher because cheaper services become fully booked, and thus the most expensive ones are required. The total unit cost is less with medium demand. The reason is that with medium demand is possible to better balance the different types of costs by utilizing better the terminals’ capacity and having enough cheaper transport services.

5.2.3 Terminals analysis

In this section, we evaluate how much the different types of terminals for each size are contracted and used when we consider the different stakeholders’ behavior, network structure, and levels of demand. Figures 8–10 show the percentage of contracted terminals for each size type for stakeholders’ behavior, network structure, and levels of demand, respectively.

![Figure 8: Percentage of contracted terminals for each terminal size for flexible and strict stakeholders](image)

Figure 8: Percentage of contracted terminals for each terminal size for flexible and strict stakeholders

![Figure 9: Percentage of contracted terminals for each terminal size for different numbers of regions, commodities, and periods](image)

Figure 9: Percentage of contracted terminals for each terminal size for different numbers of regions, commodities, and periods
From Figure 8 it can be seen that, when terminals are strict, an LSP should contract fewer medium and large terminals, but it is the opposite for small terminals as the lower fixed costs make them more convenient. When flexible customers are considered, larger terminals are contracted more compared to when customers are strict. The opposite happens with small and medium terminals which are contracted more when customers are strict. Flexible customers allow LSPs to wait for cheaper transport options, and that’s why large terminals offering more services are contracted more often.

Figure 9 instead shows that, when $T = 14$ is considered, the number of large terminals contracted are more than when we consider $T = 7$, except in the combination with $|K| = 200, |R| = 6$ in which the percentage is almost the same. The opposite happens for small and medium terminals. An LSP should rely more on large terminals than small and medium ones when a larger schedule length is considered. A higher value of $|K|$ and a lower value of $|R|$ always contribute to increasing the number of large terminals contracted. Having more commodities contribute to spreading origins/destinations and release/due times more in space and time, so large terminals and the more services available there are more convenient and with longer schedule length can be used to transshipped more units thus fewer small and medium terminals are needed. With more regions, there are more large terminals available, so a lower percentage of them is required. Small and medium terminals do not seem to follow a precise pattern regarding the different values of $|K|$ and $|R|$.

Finally, in Figure 10, we can see that all the types of terminals are contracted more when the demand increase. However, the percentage of large terminals contracted tend to increase more when the demand pass from low to medium, followed by the medium terminals. Instead, from medium to high demand the contract with medium terminals is increasing the most. Large terminals are convenient in terms of transshipment costs and thus are contracted more often than the others types. Small terminals are used more to compensate for the exceeding containers because they have less fixed costs than medium terminals, which become more convenient when the demand is high.
Since the terminals contracted are not used in all periods, in the following we want to assess their usage during the whole schedule length in terms of the percentage of periods among the available ones in which contracted terminals are used. Figures 11–13 show the terminals’ usage for each size type for stakeholders’ behavior, network structure, and levels of demand, respectively. The usage of all types of terminals is higher when flexible terminals are considered.

Figure 11: Percentage of terminals usage for each terminal size for flexible and strict stakeholders

Figure 12: Percentage of terminals usage for each terminal size for different numbers of regions, commodities, and periods

while the usage decreases a little for small terminals and substantially for medium and large terminals when strict terminals are considered. For an LSP is more profitable to reduce fixed costs by concentrating the transshipment operations in fewer periods, especially for medium and large terminals, which have higher costs than small terminals. Convince terminals to be more flexible can be beneficial to reduce congestion in terminals as transshipment operations are distributed over more periods. Larger terminals are used more often when strict customers are considered, while small and medium terminals usage is more or less the same with both customers’ behaviors. The usage of large terminals is higher in general and their capacity is the larger one, so convincing customers to be flexible can be beneficial as well in reducing the workload of each period. Moreover, when $T = 14$, small and medium terminals are used
less often, except medium terminals when $|R| = 6$ and $|K| = 200$, while large terminals are used more often. The cheaper ship services are offered more often in large terminals and thus a longer schedule length allows the LSP to use more of those services. Considering more regions reduces the usage of all terminals as the commodities’ origins/destinations are disseminated in more regions. That requires contracting more terminals that nevertheless are used less. Regarding the variation of $|K|$, it does not seem to be any pattern. For all the levels of demand, the large and then the medium terminals contracted are used more often. However, increasing demand increments the usage of all terminals, especially the usage of small terminals.

The capacity used in the different terminals is show in Figures 14-16, which display the percentage of containers handled in terminals with different sizes for stakeholders’ behavior, network structure, and levels of demand, respectively. In general, large terminals are used more often, then medium terminals, and lastly small terminals. That is due to the high volumes of containers that an LSP must ship, requiring large volumes of transshipment capacities. The percentage of containers transshipped in large terminals is higher when flexible terminals are
considered, while for small and medium terminals is higher when terminals are strict. Fewer containers are handled in small terminals when flexible customers are considered, while it is the opposite for large terminals. More or less the same percentage of containers is handled in medium terminals in all cases. When $T = 14$ is considered, a higher volume of containers is transshipped in large terminals, while fewer containers are transshipped in small and medium terminals. With more regions, the percentage of containers transshipped in small and large terminals is higher, while it is lower in medium terminals. When $|K|$ is higher, large terminals transship more containers, while small and medium terminals have a smaller percentage. Regarding demand, the containers transshipped in large terminals are higher with medium demand and are lower when the demand is high since the LSP must rely more on small and medium terminals to fulfill all demand.

Finally, Tables 5–7 contain information on the transshipment capacity used in each type of terminal, the total capacity used, the total demand $w_{\text{tot}}$, and the percentage of capacity used for transshipping units exceeding the minimum operations required. This last characteristic is grasped through a performance indicator that we called Extra Transshipment Index (ETI). The ETI counts the percentage of transshipment operations exceeding the two minimum required
and is calculated as follows:

$$ETI := 100 \frac{\sum_{k \in K} \sum_{(i,t) \in I} \sum_{s \in S^k \setminus S^i} \lambda^k_{s} \cdot z^k_{s} - 2 \sum_{k \in K} w^k}{\sum_{k \in K} \sum_{(i,t) \in I} \sum_{s \in S^k \setminus S^i} \lambda^k_{s} \cdot z^k_{s}}$$ \hspace{1cm} (10)

In fact, each unit is managed by at least two transshipment operations, one to move the container from the short-haul service in the origin region to the long-haul service going to the destination region, and another one from the long-haul service to the short-haul service in the destination region. In case the unit uses more than one long-haul service, an additional transshipment operation is required for each additional long-haul service. For instance, a shipment done with two long-haul services required three transshipment operations, a shipment done with three long-haul services required four transshipment operations, and so on.

Table 5: Transshipment capacity (KTEU) for each terminal size, units to ship (KTEU), and the ETI for flexible and strict stakeholders

<table>
<thead>
<tr>
<th>terminals customers</th>
<th>transshipment capacity</th>
<th>$w_{tot}$</th>
<th>ETI</th>
</tr>
</thead>
<tbody>
<tr>
<td>flexible flexible</td>
<td>small: 16.77 medium: 23.80 large: 71.35 total: 111.92</td>
<td>53.91</td>
<td>3.66%</td>
</tr>
<tr>
<td>flexible strict</td>
<td>small: 17.96 medium: 23.75 large: 70.66 total: 112.37</td>
<td>53.91</td>
<td>4.05%</td>
</tr>
<tr>
<td>strict flexible</td>
<td>small: 22.47 medium: 23.79 large: 65.78 total: 112.04</td>
<td>53.80</td>
<td>3.97%</td>
</tr>
<tr>
<td>strict strict</td>
<td>small: 26.05 medium: 23.91 large: 62.94 total: 112.90</td>
<td>53.91</td>
<td>4.50%</td>
</tr>
</tbody>
</table>

Table 6: Transshipment capacity (KTEU) for each terminal size, units to ship (KTEU), and the ETI for different numbers of regions, commodities, and periods

| $T$ | $|I|$ | $|K|$ | transshipment capacity | $w_{tot}$ | ETI |
|-----|-----|-----|------------------------|----------|-----|
| 7   | 4   | 100 | small: 14.90 medium: 16.33 large: 31.31 total: 62.54 | 30.43 | 2.69% |
| 7   | 4   | 200 | small: 16.03 medium: 17.78 large: 40.23 total: 74.04 | 36.18 | 2.27% |
| 7   | 6   | 100 | small: 20.80 medium: 19.30 large: 41.81 total: 81.91 | 39.35 | 3.91% |
| 7   | 6   | 200 | small: 21.67 medium: 21.58 large: 55.01 total: 98.26 | 46.94 | 4.45% |
| 14  | 4   | 100 | small: 19.66 medium: 27.07 large: 76.49 total: 123.22 | 59.83 | 2.89% |
| 14  | 4   | 200 | small: 21.93 medium: 29.61 large: 91.02 total: 142.56 | 69.26 | 2.83% |
| 14  | 6   | 100 | small: 24.89 medium: 29.52 large: 91.49 total: 145.90 | 69.01 | 5.40% |
| 14  | 6   | 200 | small: 26.63 medium: 29.31 large: 114.10 total: 170.04 | 80.06 | 5.83% |

Table 7: Transshipment capacity (KTEU) for each terminal size, units to ship (KTEU), and the ETI for different levels of demand

<table>
<thead>
<tr>
<th>demand</th>
<th>transshipment capacity</th>
<th>$w_{tot}$</th>
<th>ETI</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>small: 9.02 medium: 8.55 large: 29.72 total: 47.29</td>
<td>23.07</td>
<td>2.45%</td>
</tr>
<tr>
<td>medium</td>
<td>small: 16.56 medium: 18.03 large: 61.04 total: 95.63</td>
<td>46.19</td>
<td>3.39%</td>
</tr>
<tr>
<td>high</td>
<td>small: 36.86 medium: 44.86 large: 112.28 total: 194.00</td>
<td>92.39</td>
<td>4.75%</td>
</tr>
</tbody>
</table>

results show that, when stakeholders are strict, containers use more transshipment operations and long-haul services to reach their destinations, thus terminals in some regions are used more as intermediate points before the container reaches its destination region. Fewer large terminals are contracted and all terminals are used in fewer periods when terminals are strict, so the long-haul services that can be used are also fewer forcing more containers to pass through
intermediate regions. Strict customers force containers to arrive on time and thus combining two different long-haul services become more profitable than waiting for a cheap direct transfer between the origin and destination regions. Instead, if we consider different network structures, the number of regions contributes the most to increasing the ETI as it adds more connections and a higher probability that a region is in between the other two. A larger number of commodities and periods contributes more to increasing the ETI when there are more regions. However, the ETI is lower when $|K| = 200$ and $|R| = 4$ are considered. Increasing the demand always leads to a larger ETI since there are more units to ship and thus combining more long-haul services becomes more often the cheaper option available compared to using direct truck services.

5.2.4 Services analysis

Our last analysis regards the usage of long-haul services. Figures 17–19 show the percentage of long-haul services usage for stakeholders’ behavior, network structure, and levels of demand, respectively.

First, the very low truck usage percentages in the results (which could seem unrealistic) must be justified. The LSP has total freedom to decide the long-haul services as we assumed that customers use the a-modal booking option, thus the percentage of truck utilization is very low compared to what happens very often in reality, as customers trust more truck services and force LSPs to use those services. In general, the results show that relaxing this constraint increase the utilization of less costly and polluting modes, improving the efficiency and sustainability of the logistics network.

Strict behaviors, especially for customers, always contribute to increasing the usage of trucks that are not only more expensive but also less sustainable compared to the other transport modes. In contrast, ship services are always used less frequently when stakeholders are strict as those services are slower and less frequent, making it less convenient to wait for them when customers are strict. In that case, rail services contribute the most to substituting the ship services used when customers are flexible. Considering strict terminals increase the percentage of barge services, while strict customers decrease it. Regarding the network structure, increasing $|T|$ always corresponds to higher usage of ship services and lower usage of truck, rail and
Figure 18: Long-haul services usage by mode for different numbers of regions, commodities, and periods

Figure 19: Long-haul services usage by mode for different levels of demand

barge services. The truck and barge usage are higher when $|R| = 6$ as more barge connections are available and more containers are probably shipped with a combination of trucks and long-haul services using other modes. The only exception regard barge services when $T = 7$ and $|K| = 200$. When more commodities are considered, ship services are used more, and rail services are used less. In that case, the units to ship are divided among more commodities with different deadlines, and thus it is easier to find available ship services. Increasing the demand always reduces the percentage of barge services usage, as the barge services are less respect to the other services, while truck and rail services usage increases. Ship services usage is higher with medium demand and lower with high demand. Until a certain amount of units must be shipped, the most profitable services, the ship ones, are available. After a certain number of containers is reached, those services get saturated in favor of the others.
5.3 Managerial insights derived

In this section, we summarize the main managerial insights discovered during the analysis of the results regarding costs, terminals, and services. The benefits of adopting a more flexible behavior are evident. The difference in costs related to different behaviors can be used as an indicator of how much an LSP can invest for convincing stakeholders to increase their flexibility. An LSP can propose higher costs per unit to mitigate fixed costs of terminals and discounts to customers for adopting an a-modal booking with more relaxed time constraints. Even if the LSP has to spend all the money that it would gain by dealing with flexible stakeholders’, there are also some indirect benefits that would make it convenient. In fact, less usage of trucks would reduce emissions, making the whole process more sustainable. Customers may also benefit from a cheaper service that, if well organized, can still provide good service quality.

An LSP may also use the insights regarding costs depending on the network structure, to decide how to design its network and better understand the most critical aspects impacting the costs. A larger schedule length has the demand distributed over a larger time horizon and makes it possible to rely on cheaper services, usually the ship ones, and avoid more penalties, resulting in a lower unit cost as fewer penalties and transport costs are paid. Instead, operating in more regions results in higher costs since more terminals are required and truck services are used more often to transport commodities to intermediate regions. If an LSP has to deal with more commodities, costs can be reduced since the utilization of more maritime services become a good solution to mitigate transport costs. An LSP should also try to accept an amount of demand to have the right balance between distributing the terminals’ fixed costs among the units to ship, using more expensive transport services, and paying penalties. Cheaper services become fully booked with higher demand, and thus the most expensive ones are required.

Regarding terminals, it seems that an LSP should rely more on large terminals since those are more convenient regardless of the stakeholders’ behaviors, network structures, and levels of demand. The majority of the containers, above 50%, should be handled in large terminals, but small and medium terminals are also important to avoid higher fixed costs, rely on more services with different schedules, and deal with a high amount of demand. Convincing stakeholders to be more flexible is not only a possible way to decrease the overall costs but also a way to reduce congestion in terminals as transshipment operations are distributed over more periods. Instead, strict stakeholders are forced to use terminals more often as intermediate points due to the less usage of terminals and the need to arrive on time to avoid higher penalties.

6 Conclusions

We have presented the STHLP, a new variant of the HLP in which we addressed complex aspects of synchromodal logistics, such as multi-commodity flows, multimodal transport ser-
vices, storage and transshipment operations, and synchronization mechanisms based on penalties. The STHLP contributes to the literature on HLP by filling the gaps regarding considering the time dimension, integrating tactical and operational decisions with location decisions, and study HLP into complex logistics networks. First, we provide a time-space multi-network representation of the problem and of all its characteristics useful to define a handy MILP formulation. Such a model, showing a typical structure in HLP, makes it easy to solve the problem through commercial solvers and to leverage specific HLP solutions methods existing in the literature. We performed a computational analysis, showing that for certain instances CPLEX could obtain optimal solutions in a reasonable time, especially when stakeholders behave in a more flexible way. The computational analysis also showed the parameters that can impact more on the solving time.

Given the novelty of the setting, we also presented an economic analysis in which we discuss how costs vary for different types of instances. The insights of this analysis can help LSPs to define discount mechanisms to convince customers to be more flexible. We evaluated different results regarding contracts and how terminals are used for terminals of each size. This analysis can help LSPs to decide how many terminals of each size to contract and how to allocate the transshipment operations to those terminals. Finally, we performed an analysis of the long-haul services usage, showing that customers relying on a-modal booking and with flexible behavior can contribute to using cheaper and more eco-friendly solutions.

The present work opens several research directions. In particular, in the future, we are interested in studying different policies regarding penalties. It could be worth to consider two different cases in which customers have requests to collect and deliver commodity units together or separately, and two cases in which penalties are paid for each unit or for the whole commodity at once when the last unit arrives. By combining those collection/delivery and penalty policies, four different case studies can be compared both from a computational and an economic point of view. Another idea is to consider a stochastic setting instead of a deterministic one. This would be a better way to study this type of tactical problem and integrate other synchronization mechanisms based on possible recourse actions to deal with the uncertainties. In literature, that has been done successfully for different uncertain parameters such as travel times (Lanza et al., 2021), transshipment capacity in terminals (Giusti et al., 2021), and demand (Hewitt et al., 2019).

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References


The Synchronized Multi-commodity Multi-service Transshipment-Hub Location Problem with Cyclic Services and Demand


