The Effect of Visibility on Forecast and Inventory Management Performance during the COVID-19 Pandemic

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Abstract. During the COVID-19 pandemic, healthcare organizations suffered a shortage of essential medical supplies, such as personal protective equipment, which resulted in severe consequences. This study aims to assess the impact of one potential factor for this shortage, i.e., the lack of visibility over the consumption of personal protective equipment. To do so, different forecasting methods combined with a periodic review inventory system are tested on semi-simulated data that include various visibility issues. The forecasting methods are categorized based on the data used. The Holt and naïve methods are selected as demand-based forecasting methods, and a modified compartmental epidemiological model is explored for its use of pandemic data to forecast demand. This paper studies three of the most common data visibility problems. Specific scenarios have been developed to analyze the impact of (1) delayed data, (2) temporally aggregated data, and (3) erroneous data on the performance of the system. Our findings indicate that, in most cases, data visibility issues directly influence the healthcare supply chain and diminish the performance of the system. However, when these visibility issues result in exponentially large over-forecasts, we observe a performance improvement in the system. This phenomenon is particularly true for a system that uses the epidemiological compartmental model as its forecasting method while using lagged data.

Keywords: inventory management, forecasting, visibility, healthcare, disruption, pandemic.

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1 Introduction

In March 2020, the coronavirus disease 2019 (COVID-19) was declared a global pandemic by the World Health Organization (WHO) \[1\]. With over 5 million deaths as of February 2022 \[2\], COVID-19 has been one of the deadliest events in recent human history. Its economic impact is nothing short of a catastrophe. Not only has bankruptcy become a constant threat, but it also brought the world governments to the brink of an economic collapse comparable to the economic shock of the 2008 financial crisis \[3\]. Moreover, the global supply chain has experienced significant disruptions in almost every industry \[4–8\]. Factory shutdowns \[9, 10\], uncertain and lengthy lead times \[11, 12\], export restrictions \[13, 14\], and fluctuating demand \[15, 16\] are just a few contributing factors to the inadequacy of the supply chain during the COVID-19 pandemic era, and healthcare supply chains (HSCs) are no exception. During the initial stages of the pandemic and in the absence of a viable vaccine, a sudden increase in hospitalizations pushed the healthcare facilities to their limits \[17\]. Since this virus spreads mostly via airborne particles and droplets, the most effective prevention methods of transmission are social distancing and the use of personal protective equipment (PPE) \[18\]. Protecting the frontline health workers was the obvious and utmost priority. The skyrocketing demand for PPE resulted in severe shortages within healthcare facilities. Reports of PPE shortages \[19\] were alarming and led to the implementation of “crisis capacity strategies” where extreme measures such as the reuse of N95 masks were suggested \[20\].

The COVID-19 pandemic has exposed deficiencies in current HSCs. Warnings about the upcoming pandemics \[21\] following the 2002-2004 SARS outbreak \[22\] have largely been ignored, which left HSCs unprepared to face such an event, failing to perform adequately when it was needed the most. There is thus an urgent need to revitalize the existing supply chain, at least within the healthcare industry. Generally, the supply chain can be divided into two main sections: upstream and downstream. In this paper, we focus on the downstream processes, particularly the factors that can directly impact the flow of products. Supply chain managers, hereon managers, generally have little to no control over the production line of their suppliers, and it was even less the case during the pandemic. Therefore, their focus must be on downstream activities such as inventory and data management. Most importantly, they should consider the impact of supply chain visibility, or lack thereof, on the performance of the system should it require further enhancement.

Barratt and Oke \[23\] define supply chain visibility as “the extent to which actors within a supply chain have access to or share the information which they consider as key or useful to their operations and which they consider will be of mutual benefit”. The ability to track demand, replenishment, and inventory within the system could potentially be vital to the system’s performance. In Canada, the operations and processes of the healthcare supply chain fall under the provincial jurisdictions \[24\] with highly diverse strategies about their inventory and data management systems, and with minimal visibility on the various segments of the supply chain \[25\], which makes it fragmented and inefficient \[26\]. The lack of visibility in the system is further intensified when encountering a
crisis as critical as the COVID-19 pandemic \cite{27, 28}. In the absence of proper data management infrastructures that can provide timely and reliable reports on the status of the supply chain, managers are forced to rely on their intuitions, which could negatively impact the overall system performance.

Demand forecasting is an integral part of any inventory system. Understanding the limitations and capabilities of forecasting methods becomes even more crucial for a successful manager considering that the potential data visibility issues (DVIs) within the system might have been amplified due to the pandemic. Therefore, there exists an immediate need to examine the functionality of common forecasting methods in the presence of DVIs. Our analyses focus on the impact of various DVIs on the performance of a system that employs different forecasting methods in the context of a pandemic.

The contributions of this paper are as follows. First, we compare two widely used forecasting methods within professional communities (i.e., the naïve and Holt methods) to an epidemiological compartmental model. This comparison is made both on the forecasting performance and the performance of the resulting inventory management system. Second, we investigate common issues associated with the visibility or lack thereof in the HSCs and analyze their impacts on the system’s performance in the specific context of a pandemic. In particular, the examined DVIs are data delay, temporally aggregated data and erroneous data. We present a separate scenario for each DVI and assess its direct impact on the performance of the system. In addition, to replicate real-world situations, we analyze a randomized delay data scenario as well as a randomized temporally aggregated data scenario. In the scenario that analyzes the erroneous data, both under- and over-reporting within the data are considered, and their impacts on the performance of the system are analyzed.

This paper is organized as follows. The related literature is presented in Section 2. Section 3 describes the problem. Section 4 describes the general solution approaches for this problem. Section 5 presents the numerical study and the associated results. Finally, Section 6 provides our conclusions.

2 Literature review

In this section, we review two research streams relevant to this paper, i.e., (1) demand forecasting and (2) supply chain visibility, with a specific focus on the context of a pandemic. Then, our paper is positioned with respect to this literature.

2.1 Demand forecasting

Managing the inventory of PPE during a pandemic is a challenging task. Most inventory management systems perform as expected when the demand is stable. However, when the demand expe-
periences high levels of volatility, such as in the context of a pandemic, the impact of the forecasting process on the system’s performance becomes more prominent since it directly affects decision-making. Forecasting the demand for PPE during a pandemic is complicated. First, the new demand is often drastically different from past demand patterns. Second, demand patterns are challenging to predict as they are often linked to many factors (e.g., panic buying and hoarding behaviours observed during the COVID-19 pandemic [29, 30]), which are, in turn, difficult to anticipate.

Several forecasting methods exist to predict demand during a pandemic. These methods can be grouped into two categories. In the first one, the forecast is directly based on the demand data. These methods include classical statistical methods that are used extensively within the scientific communities and the industry. Forecasting methods such as naïve forecast [31], simple exponential smoothing [32], Holt-Winters [33], regression models [34], and autoregressive integrated moving average (ARIMA) models [35-37], to name a few, are mainly based on the historical data of the time series that is being predicted.

In the second category, the demand is predicted through the utilization of epidemiological data along with the pandemic’s behaviour. This is a two-tier method, where forecasting the pandemic’s consequences (e.g., infected population, hospitalizations) are considered as trigger parameters in the forecast of excess demand (i.e., demand above the average due to the pandemic) for medical supplies such as PPE. In this case, the same methods mentioned in the first category can also be employed to predict the pandemic behaviour [38]. As an example, Sun [39] proposes a modification to the ARIMA model to forecast the dynamics of the pandemic. Swapnarekha, Behera, Nayak, Naik, and Kumar [40] rather use the multiplicative Holt-Winters model and observe that it produces good forecasts of the number of confirmed infected cases. However, a more detailed interpretation of the pandemic behaviour can be produced using the well-established compartmental epidemiological model, first introduced by Kermack, McKendrick, and Walker [41]. In its simplest form, the model places each member of the population in different compartments (i.e., susceptible, infected, and removed) based on their status and uses a series of differential equations to explain the interactions between them. Hence, the name SIR is appointed for the proposed model. Extensive studies using a compartmental model have been done on past [42, 43] and current pandemics [44-47].

The SIR model can also be extended to study the pandemic under external factors such as social distancing. Gounane, Barkouch, Atlas, Bendahmane, Karami, and Meskine [48] develop a nonlinear SIR model to incorporate the effect of social distancing. To study the effect of lockdown on the pandemic, Ianni and Rossi [49] propose a time-dependent SIR model. Furthermore, researchers have modified the SIR model to include additional compartments that represent specific population segments. The exposed compartment is the most common addition to the original model, hence “E” in SEIR. It represents the latency between the contraction of disease and the ability to transmit the infection by an individual [50]. Moreover, to investigate the population at healthcare facilities at any given time, the “Hospitalized” compartment can be added to the model [51, 52]. Once the required pandemic parameters are predicted, the data is used to forecast the
excess demand (again, the demand above the average). In the case of HSCs, the number of required units per patient can represent the excess demand. Lum, Johndrow, Cardone, et al. [53] propose a mathematical model that employs the daily average number of contacts between infected patients and healthcare workers as a coefficient which is multiplied by the projection of hospitalization to forecast the PPE demand. Several proposals have transformed daily pandemic data, such as daily infections and hospitalizations, into PPE demand in any region. Furman, Cressman, Shin, et al. [54] propose using a queueing model to predict the required PPE during the COVID-19 pandemic. Nikolopoulos, Punia, Schäfers, Tsinopoulos, and Vasilakis [31] employ the growth rate of COVID-19 incidents in conjunction with a parameter that can capture the effect of the pandemic. Yom-Tov and Mandelbaum [55] propose a time-varying queueing model to determine the required unit per patient.

2.2 Supply chain visibility

Supply chain visibility is a critical component in a system’s performance, especially within the healthcare industry, where any deficiency might result in grave consequences. The COVID-19 pandemic has again shown a lack of resiliency and robustness in the supply chain industry. Data visibility remains one of the biggest challenges for managers, as shown during the initial stages of the pandemic [56, 57]. With increasing capabilities in the information-sharing systems provided by modern information technologies, the effect of data visibility on the supply chain industry has been investigated more prominently [58–64]. There are numerous factors contributing to data visibility problems within the supply chain industry. In general, and even more during a pandemic, three of the most common issues regarding data visibility are: (1) erroneous data, (2) delay in data, and (3) temporally aggregated data.

Since the late 1950s, the bullwhip effect has been associated with a lack of data visibility. Many scholars point out the importance of information sharing and its impact on reducing the amplified demand throughout the value chain [65, 67]. Information inaccuracies and errors are among the contributing factors to the bullwhip effect, which might result in under- or over-reporting demand within a system. Lu, Feng, Lai, and Wang [68] provide two primary sources of data inaccuracy and their impact on the system’s performance with regard to the bullwhip effect. They mention that the errors might occur either during the information delivery to the next level (from downstream to upstream) or during the collection of data from the customers. Their study concludes that data sharing has contrasting beneficial values for the manufacturers depending on the source of the error. Kwak and Gavirneni [69] further outline the negative impact of errors on the value of information sharing, where it is best to assume the information is not available if the variance of information errors outweighs that of the end-customer demands. Under- and over-reporting are also potential sources of errors within the data. Multiple studies have been conducted on under-reporting of the number of infected cases and how it might lead to ineffective preventive policies during the COVID-19 pandemic [70, 71]. In addition, threats of shortages could result in a significant over-reporting
of demand which is a major problem for a supply chain manager.

Another major contributing factor to DVIs is the information delay [72]. It is possible to quantify the cost of information delay and the value of the most recent demand data. Munoz and Clements [73] find that the disruption in the flow of information has a more obstructive effect on revenue than the product delay. Moreover, Chen [74] provides a comparative analysis of production lead times and information delays within different stages of a supply chain that entirely belongs to a single firm. He concludes that within such settings, data lags are less costly than production lead times. The results also show that, data delays in the upstream supply chain are less detrimental than in the downstream supply chain. Hoberg and Thonemann [75] analyze the effect of the information delay on echelon stock policies. They conclude that the presence of information delay deteriorates the system’s performance. However, increasing the length of delay does not automatically translate into a further decline in performance. Hosoda and Disney [76] explore a similar problem on a linked two-level supply chain and find that not all levels benefit from shorter delays. In the context of the COVID-19 pandemic, Sarnaglia, Zamprogno, Fajardo Molinares, Godoi, and Jiménez Monroy [77] recognize the existence of data delay and propose a methodology for a forecasting model to correct the notification delay. Closer to our study, Tucker and Wang [78] analyze the impacts of homogeneous and heterogeneous delay in data on preventive policies in the United States. Their results indicate that data delay could lead decision-makers to misinterpret these policies.

Temporal aggregation of data is another potential supply chain visibility issue. The manager might receive the demand data aggregated and transmitted at lower frequencies than initially collected. Temporal aggregation is defined as the process of transforming high-frequency time series (e.g., daily) into a low-frequency time series (e.g., weekly) [79]. It is established that data aggregation results in information loss [80] and variance reduction [81]. In the supply chain context, the aggregation of data is associated with “risk-pooling” to reduce the demand uncertainty and improve the planning and forecasting [82]. Rostami-Tabar, Babai, Syntetos, and Ducq [83] conclude that the performance improvements through data aggregation are a function of the aggregation level, among others. Yet, the aggregated data may not always be beneficial, as shown by Gfrerer and Zäpfel [84], where robust production planning requires the disaggregation of the aggregated production plan into a feasible detailed plan. However, in most inventory management systems, the managers do not have access to the detailed demand; hence, the evolution of the demand is unknown to the system. Jin, Williams, Tokar, and Waller [85] observe that beneficial impacts of the data aggregation on the forecast depend on the demand signal’s autocorrelation, which does not necessarily hold for all cases.

2.3 Positioning of the paper

As previously highlighted, several methods can be applied to forecast the product’s demand. However, firstly, it is not completely clear how these methods perform and compare to each other in the context of a pandemic, both for the forecasting performance and for the resulting inventory
management performance. In addition, while the compartmental model has been used extensively
to predict pandemic behavior, the information provided by this model is not generally used to
predict demand. We believe that relying on such information in the context of a pandemic could
potentially improve forecasts and lead to better performances.

Therefore, the first contribution of our paper is to analyze and compare different forecasting
methods using various types of data and assess their performance. In particular, we compare
two classic forecasting models (i.e., the naïve and Holt methods) and a forecasting model based
on an epidemiological model. In addition to the traditional statistical performance (e.g., root
mean square error), we also compare these methods with respect to their performance within an
inventory management system since a good forecasting performance may not necessarily result in
a good inventory management system. This comparison provides a better understanding of how
the forecasting process impacts the decision-making process and, consequently, the performance of
the system.

We also previously highlighted that data visibility is a challenge that has been studied for quite
some time. While it is known that data visibility influences performance, it is not clear how visibility
affects performance in the specific context of a pandemic. The HSC has suffered tremendously from
the lack of visibility in recent years, particularly during the pandemic. Therefore, there exists an
urgent need to increase our knowledge in this domain.

The second contribution of this paper is the study of different DVIs’ impact on inventory
management performance during a pandemic. We analyze the information delay in two distinct
formats: fixed and random lags. This approach enables us to analyze the impacts of lag elongation
as well as real-world situations where lag lengths are random. Similar to data delay, we investigate
the performance of the system under the influence of aggregated data in two formats: fixed and
random. Finally, we test the impact of both under- and over-reporting of demand (i.e., erroneous
data). To the best of our knowledge, this is the first study that investigates the impact of DVIs (i.e.,
data lag, aggregated data, and erroneous data) on the performance of an inventory management
system in the context of a pandemic.

3 Problem description

The inventory management problem under study is an inventory management problem of medical
supplies in healthcare facilities. In this problem, a manager controls the replenishment of a single
product (e.g., N95 respirators) for a specific region (e.g., country, province, city). Furthermore,
since we assume that facilities within this region can redistribute supplies among themselves as
needed, we only consider the aggregated demand for the region. The entire time horizon (i.e., the
duration of the pandemic wave) is divided into a series of decision epochs with a constant interval
of \( R \) days (i.e., \( R \) is the review period of the system). At each decision epoch, after observing the
current and previous states of the system (e.g., inventory), the manager decides the quantity of the
product that needs to be ordered (if any) while minimizing the cumulative costs of the system.

The cost definition depends on the objective(s) to achieve. The costs can be defined as the monetary value of the ordering and holding processes as well as the associated costs related to the shortages. In practice, however, the focus was on the minimization of the shortages while avoiding too much left-over inventory at the end of the wave. We now further describe the different components of this dynamic system in the rest of this section. It is important to note that the demand in this study is perishable; thus, the unfulfilled demand is considered to be lost.

3.1 State

At the beginning of each decision epoch \( k \in K = \{1, 2, \ldots, K + 1\} \) with fixed intervals (i.e., one week), the system is in the state \( s_k \):

\[
s_k = (d_{k-1}, q_k, p_k, u_k)
\]

where \( d_{k-1} \) denotes the total demand during epoch \( k - 1 \), \( q_k \) denotes the state of the inventory, \( p_k \) denotes the state of the pandemic, and \( u_k \) denotes the state of the supplier.

The state of the inventory is given by \( q_k = (q^a_k, q^t_k) \) where \( q^a_k \) is the available inventory level of the region and \( q^t_k \) is the in-transit inventory vector. Note that the sign of \( q^a_k \) indicates a shortage or surplus of inventory, with a negative value indicating the former. The in-transit inventory consists in a vector tracking the remaining units to be delivered according to how many epochs ago they were ordered. This vector has a length of \( L_{max} \), which is the maximum lead time for the ordered items rounded up to the nearest multiple of the fixed interval.

The state of the pandemic \( p_k \) at the beginning of epoch \( k \) includes information on the daily number of infections and hospitalizations since the previous decision epoch. It also contains information about the government protocol that outlines the consumption of PPE per hospitalized patient, \( CC_k \), at healthcare facilities.

Finally, the state of the supplier \( u_k \) provides information regarding the lead time, the lot size as well as the supplier’s upper and lower limits regarding the quantity of products in each order.

3.2 Action

At each epoch \( k \in K \setminus \{K + 1\} \), the manager takes an action \( a_k \in A(s_k) \), which is a feasible placement of an order to a supplier; note that, at the epoch \( K + 1 \), the manager observes the state, but takes no action. If there is no need for an order at epoch \( k \), the action is \( a_k = 0 \). This action is mainly restricted by the state of the supplier \( u_k \), e.g., the supplier’s upper and lower limits regarding the quantity of products in each order. Constraints such as the budget, storage space, or political aspects are not considered in this study.
The quantity of the order at epoch \( k \), \( a_k \), is generally defined by a policy \( \pi \) which requires the full history of states, i.e., \( a_k = \pi(s_{1:k}) \) where \( s_{1:k} \) denotes the history of states up to epoch \( k \), i.e., \( s_{1:k} = (s_1, s_2, \ldots, s_k) \). Note that the policy is assumed to be stationary. Moreover, the policy is history-dependent since the manager needs to have some knowledge of the historical data in order to make a decision (e.g., to know whether we are in an increasing or decreasing trend in terms of the number of infections). While it is possible to increase the state dimension to capture previous states and recover a Markovian policy, this leads to the curse of dimensionality.

Through these actions, the manager tries to minimize the costs, which are described next.

### 3.3 Cost function

Generally, for a particular state \( s_k \) and action \( a_k \), the manager incurs a cost \( C(s_k, a_k) \) at the end of the epoch \( k \), which can be a combination of the ordering, holding, and shortage costs. In particular, it can be defined as

\[
C(s_k, a_k) = c_f \mathbb{1}_{a_k > 0} + c_u a_k + c_h q_k^{a,+} + c_s q_k^{a,-}
\]  

(2)

where \( c_f \) denotes the fixed ordering cost, \( \mathbb{1} \) denotes the indicator function, \( c_u \) denotes the variable (or unit) ordering cost, \( c_h \) denotes the unit holding cost, \( c_s \) denotes the unit shortage cost, and \( q_k^{a,+} \) and \( q_k^{a,-} \) denote respectively the positive (i.e., inventory) and negative (i.e., shortage) parts of \( q_k^a \).

### 3.4 Transition function

Once the manager takes an action \( a_k \), the system transitions into the next state \( s_{k+1} = (d_{k}, q_{k+1}, p_{k+1}, u_{k+1}) \). The state components can be categorized as being independent or dependent of the agent’s action. On the one hand, it is assumed that the components \( d_k, p_{k+1} \) and \( u_{k+1} \) do not depend on the agent’s action and are updated solely based on the evolution of the pandemic and the characteristics of the supplier; the manager receives these data from external sources. Thus, we assume that the agent’s action (i.e., the replenishment decision) does not influence the pandemic’s evolution or the suppliers’ available inventory.

On the other hand, the inventory \( q_{k+1} = (q_{k+1}^a, q_{k+1}^t) \) is directly impacted by the action \( a_k \). Let \( y_{k+1,j} \) denote a quantity that was ordered \( j \) epochs ago where \( j \in \{1, 2, \ldots, L_{max}\} \) and delivered at the beginning of the epoch \( k + 1 \). We assume \( y_{k+1,j} \) is 0 when \( k + 1 - j < 1 \); in other words, we assume no orders are passed before epoch 1. Then, the available inventory is updated as

\[
q_{k+1}^a = q_k^{a,+} + \sum_{j=1}^{L_{max}} y_{k+1,j} - d_k.
\]  

(3)

Yet, it should be noted that in this paper, the demand of medical supplies is assumed to be
perishable and cannot be back-ordered. Finally, each element $j$ of the in-transit inventory vector is updated as

$$q_{k+1,j} = \begin{cases} a_k - y_{k+1,k} & \text{if } j = 1, \\ q_{k,j-1} - y_{k+1,k+1-j} & \text{if } j = 2, \ldots, L_{\max}. \end{cases}$$  \hspace{1cm} (4)$$

where $q_{k+1,j}$ is the quantity that was ordered $j$ epochs prior to epoch $k + 1$.

### 3.5 Objective function

The objective of this problem is to determine an optimal policy $\pi^*$ that minimizes the total expected cost over the (finite) time horizon, i.e.,

$$\pi^* = \arg \min_{\pi \in \Pi} E \left[ \sum_{k=1}^{K} C(s_k, \pi(s_1:k)) \bigg| s_1 \right]$$  \hspace{1cm} (5)$$

where $\Pi$ is the set of all feasible policies and $s_1$ is the initial state of the system.

Note, however, that even a single shortage may result in deaths within the context of medical equipment. Hence, associating a specific cost to an equipment shortage (i.e., $c_s$) is extremely difficult. Therefore, in this study, our primary measure to compare the different solution methods is the total number of shortages (i.e., the service level) over the time horizon. As additional measures, we consider two types of inventory costs: the left-over inventory at the end of the time horizon (hereon, LOI) and the average inventory cost (i.e., holding costs). Due to the high purchase cost of PPE during the pandemic, we believe that the LOI has a more profound impact on the overall cost of the system than the holding costs. For this reason, we selected the LOI as a secondary measure in this study. We do provide, however, an analysis of the average inventory cost as well.

### 4 Solution methods

In this section, we provide methods that aim to approximate the optimal policy $\pi^*$ of Section 3.5. In contrast to more advanced methods, these methods seek to mimic approaches that can be easily used in practice, which can be greatly beneficial during fast-evolving situations such as pandemics where the required data is scarce at best. In addition, since one objective of this work is to evaluate the impact of data visibility on the performance of inventory management, these methods differ in the type of data they use. In the rest of this section, we describe forecasting methods and the inventory control method used in the decision-making process.
4.1 Forecasting methods

The forecast of the demand is an essential part of the policy \( \pi \), directly affecting the decision-making process. Without such methods, the managers are forced to use their gut feelings to place an order, which can be improved upon. In this section, we present three demand forecasting methods, which can be grouped into two categories based on the types of data they require to make a forecast. The first category consists in forecasting methods that employ demand data to develop a forecast. Most of the classical statistical forecasting methods fall into this category. In the second category, the forecasting method employs epidemiological data of the pandemic as well as government protocols concerning the consumption of PPE. These two categories of methods have distinct methodologies in the forecasting process, which is of importance for the results section. We employ the following forecasting methods to estimate the demand during the first wave of the pandemic.

4.1.1 Methods using demand data

Numerous forecasting methods employ demand data as the primary source of information in their forecasting process. However, a simple model such as the naïve method often performs reasonably well in the absence of reliable historical data (i.e., the context of a pandemic) [31], while being more practical than more advanced methods. We now describe two simple methods in more detail.

4.1.1.1 Modified naïve forecasting method

Based on interviews with managers, a simple forecasting method consists of identifying the maximum daily demand of the previous two epochs (here, two weeks) in order to use it as the average daily demand over the forecasting period. This is an adaptation of the naïve method [86] that takes the current epoch consumption as the consumption in the next epoch. In particular, in this modified naïve method (hereon, the naïve method), the forecast at epoch \( k \) of the total demand is given by

\[
\hat{d}_k^{Naïve} = f_k \max \{ d_{k-1}, \overline{d}_{k-2} \}
\]

where \( f_k \) is the forecast horizon (i.e., the length of forecast) in days at epoch \( k \), and \( \overline{d}_{k-1} \) and \( \overline{d}_{k-2} \) are respectively the maximum daily demand during epoch \( k-1 \) and \( k-2 \). Note that the forecast \( \hat{d}_k^{Naïve} \) can go beyond the epoch \( k \) if \( f_k \) is longer than one epoch. This method is the benchmark for the computational study.

4.1.1.2 Holt forecasting method

The second forecasting method using demand data is the well-established Holt method [86]. It is widely used in the industry due to its relative ease of use and ability to capture the demand’s trend. In the context of a pandemic, capturing the demand’s trend is essential. However, it is not necessarily useful to model seasonality; we only model one
wave of a pandemic in this work. The forecast at epoch $k$ is given by

$$\hat{d}_k^{Holt} = \sum_{h=1}^{f_k} (l_{k \times R} + hb_{k \times R})$$  \hspace{1cm} (7)

where $l_{k \times R}$ and $b_{k \times R}$ denote respectively the estimates for the daily level and trend of the series on day $k \times R$, and $R$ is the review period in days. They are obtained with

$$l_t = \alpha dd_t + (1 - \alpha) (l_{t-1} + b_{t-1})$$  \hspace{1cm} (8)

$$b_t = \beta (l_t - l_{t-1}) + (1 - \beta) b_{t-1}$$  \hspace{1cm} (9)

where $dd_t$ is the daily demand on day $t$, and $0 < \alpha < 1$ and $0 < \beta < 1$ are smoothing parameters for, respectively, the level and trend.

### 4.1.2 Method using epidemiological data – the SEIRHD model

To be able to use epidemiological data for inventory management, we adapt the susceptible-exposed-infected-removed-hospitalized-discharged (SEIRHD) model (see Figure 1). In addition to the typical setup in the SEIR model \[87\], the SEIRHD model includes a path where subjects may be hospitalized and then discharged. There are two types of subjects visiting healthcare facilities during a pandemic, i.e., the infected and non-infected subjects. For the sake of this work, it is assumed that the majority of PPE consumption within healthcare facilities occurs during the handling, treatment, and discharge of infected subjects. Note that this paper analyzes specific types of PPE, such as N95 respirators, which are recommended for utilization only during exposure to infected subjects \[88\]. Hence, the SEIRHD model only tracks the number of hospitalizations of the infected population in the *hospitalized* compartment, which is later used for forecasting purposes.

![Figure 1: The SEIRHD model](image)

Furthermore, since the primary focus of this work is the demand for PPE within healthcare facilities, infected subjects that do not visit these facilities are removed from the system and placed into the *removed* compartment. Using the same analogy, the infected subjects discharged from the healthcare facilities are moved into the *discharged* compartment. For the purpose of this work, we do not distinguish between the recovered and dead population for both of these compartments. A
description of the typical assumptions associated with such a compartment model is provided in Appendix A.

4.1.2.1 Specification of the SEIRHD model  Kermack, McKendrick, and Walker [41] formulated the initial SIR model as a series of differential equations. The adaptation of these equations to our compartment model is as follows

\[
\frac{dS}{dt} = -\frac{\beta SI}{N},
\]

\[
\frac{dE}{dt} = \frac{\beta SI}{N} - \sigma E,
\]

\[
\frac{dI}{dt} = \sigma E - p_H \gamma_H I - (1 - p_H) \gamma_R I,
\]

\[
\frac{dR}{dt} = (1 - p_H) \gamma_R I,
\]

\[
\frac{dH}{dt} = p_H \gamma_H I - \gamma_D H,
\]

\[
\frac{dD}{dt} = \gamma_D H,
\]

where \( S, E, I, R, H, D \) denote the population in each respective compartment (see Figure 1). The other parameters are described in Table 1.

Table 1: Parameters of the SEIRHD model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>Total population</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Number of contacts per unit time, multiplied by the probability of transmission in a contact between a susceptible and an infected subject</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Per-capita incubation rate, i.e., transition rate of exposed subjects to the infected class</td>
</tr>
<tr>
<td>( p_H )</td>
<td>Probability of hospitalization of infected subjects</td>
</tr>
<tr>
<td>( \gamma_R )</td>
<td>Per-capita rate of recovery and death of non-hospitalized subjects</td>
</tr>
<tr>
<td>( \gamma_H )</td>
<td>Per-capita rate of hospitalization</td>
</tr>
<tr>
<td>( \gamma_D )</td>
<td>Discharge rate of hospitalized subjects (dead and recovered)</td>
</tr>
<tr>
<td>( R_{0,1} )</td>
<td>Initial ( R_0 )</td>
</tr>
<tr>
<td>( R_{0,2} )</td>
<td>Final ( R_0 )</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>Midpoint of the logistic function</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Growth rate of the logistic function</td>
</tr>
</tbody>
</table>

With the addition of the hospitalized compartment, the proposed model can predict the disease’s behavior during an outbreak and, more importantly, the number of hospitalizations at healthcare facilities. However, to do so, the model requires correct parameter values. First, several parameters can be assumed as fixed through time and can be determined \textit{a priori} by using data from various sources. In particular, the total population \( N \) of the region can be retrieved from governmental
data. In addition, since the inverse of the parameter $\sigma$ corresponds to the incubation period, it is possible to compute this parameter value as $\sigma = 1/t_{incubation}$, where $t_{incubation}$ corresponds to a commonly agreed incubation period (in days) for COVID-19; as explained in Appendix A, we don’t stratify this incubation period by, for example, age groups since this additional complexity would not lead to additional insights in the case of this study. Furthermore, the hospital’s discharge rate can be estimated as $\gamma_D = 1/t_{ALOS}$, where $t_{ALOS}$ is the average length of stay in days. The probability of hospitalization of infected subjects $p_H$ can be computed as the ratio between the total number of hospital admissions of infected subjects and the total number of infected subjects. Then, the parameter $\beta$ is linked to the basic reproduction number $R_0$, an important parameter in epidemiological studies which was first introduced by Macdonald [89]. $R_0$ describes the intensity of disease transmission, which may change over the course of a pandemic depending on the evolution of disease characteristics (e.g., variants), as well as public and government preventive actions. Hence, similarly to other studies [90, 91] that use the logistic function within epidemiological models, we model a varying $R_0$ that transitions between two values, i.e., from an initial value $R_{0,1}$ to a final value $R_{0,2}$, according to a logistic function, and we estimate the varying $\beta$ from these $R_{0,1}$ and $R_{0,2}$ values using Lemma 1. The proof of Lemma 1 is provided in Appendix B.

**Lemma 1.** For the SEIRHD model, the parameter $\beta$ can be computed as

$$\beta = \left[ \frac{R_{0,1} - R_{0,2}}{1 + e^{-\kappa(t_0 - t)}} + R_{0,2} \right] (p_H \gamma_H + (1 - p_H) \gamma_R)$$

where $t_0$ denotes the time at which $\beta$ is estimated, $t_0$ and $\kappa$ denote the logistic function’s midpoint and growth rate, respectively. The other parameters are defined in Table 1.

Finally, the remaining parameter values, $\gamma_R, \gamma_H, R_{0,1}, R_{0,2}$ and $t_0$, are obtained by fitting the curves of the SEIRHD model to longitudinal data, which includes the daily number of infections, of hospital admissions, of hospitalizations, and of hospital discharges. Further details on the fitting process are provided in Section 5.

**4.1.2.2 Demand forecasting with the SEIRHD model** Once a forecast of the hospitalizations is established, it is possible to forecast the consumption of PPE with the government protocol observable in the element $p_k$ of the state $s_k$. Each Canadian province mandates a specific set of recommendations in dealing with COVID-19-related patients [92]. These government protocols detail the consumption of PPE for all healthcare workers, even those not in close proximity to COVID-19 patients. Assuming that there is always an active protocol and that it is closely respected, it is possible to estimate the consumption of PPE based on the daily number of hospitalizations of the SEIRHD model as

$$d^{SEIRHD}_k = H(f_k)CC_k$$

(17)
where $H(f_k)$ denotes the total number of hospitalization days over the forecast horizon of epoch $k$, and $CC_k$ denotes the coefficient of consumption observed at the beginning of epoch $k$, i.e., the daily number of PPE per hospitalized COVID-19 patient as outlined in the government protocol. Note again that the forecast horizon $f_k$ can be longer than one epoch.

It is important to highlight that this method only predicts the consumption of PPE associated with infected patients in healthcare facilities. Yet, during the height of the pandemic, many healthcare facilities shut down most of their daily routine operations to attend to COVID-19 patients. Therefore, the majority of patients at any given time in these centers were COVID-19-related. In addition, note that some PPE, such as N95 respirators, may be used exclusively with infected patients.

### 4.2 Inventory control – periodic review system

Once a forecast is made, the next step within the policy $\pi$ is the computation of the quantity to order. The manager must minimize the cost function while respecting the system’s constraints. The inventory control method also impacts the system’s performance; hence advanced methods such as robust optimization may be envisioned. However, to be representative of actual methods used in practice, we use the popular periodic review system [93] that is well-known for its efficiency and ease of use.

By using the forecasting methods described in Section 4.1, the manager obtains the predicted demand over the forecast horizon $f_k$ at each epoch $k$. It is then possible to compute the reorder point (ROP) in the context of uncertain demand as

$$ROP = \hat{d}_{R+L} + z \cdot RMSE \cdot \sqrt{R+L}$$ (18)

where $R$ denotes the review period in days, $L$ denotes the lead time in days, $\hat{d}_{R+L}$ denotes the total predicted demand over the $R+L$ period, $z$ denotes the factor associated with the $(1-\alpha)$ service level, and $RMSE$ denotes the root mean square error of the forecast in the last review period. We refer the reader to Section 2.10 of Axsäter [94] for details on how to use the forecast errors to determine the safety stocks. It is important to note here that the forecast horizon $f_k$ may be longer than $R+L$, since the forecast may take into account days before the current epoch in the case of the lagged data scenario (see Section 5.2.2). Thus, we omit the quantity before the current epoch when computing $\hat{d}_{R+L}$. With this method, the ROP’s value is dynamic and re-calculated at each epoch.

Finally, the quantity ordered $a_k$ is given by $ROP - q^{a,+}_k$ subject to the supplier’s minimum and maximum order quantities and rounded to the upper lot size. In particular, if $ROP - q^{a,+}_k$ is above the supplier’s maximum order quantity or below the minimum order quantity, then $a_k$ equals this supplier’s maximum or minimum order quantity, respectively.
5 Computational study

This section presents different scenarios designed to address a specific visibility issue within the healthcare supply chain. In Section 5.1, the simulation process of the demand, as well as the required parameters, are described. The detailed description of each scenario and their relevant results are then presented in Section 5.2. Finally, in Section 5.3, we provide a discussion.

5.1 Data and parameters

Tracking PPE inventory is a difficult task, if at all possible. These types of equipment are often located at multiple (official and unofficial) locations within a healthcare facility, which prevents a physical inventory count. As a consequence, daily consumption data of PPE is generally not available. Furthermore, even in the rare cases where this daily consumption data may be available (e.g., due to strict control measures for the allocation of PPE), it generally does not necessarily correspond to the daily demand data. In the particular case of the COVID-19 pandemic, healthcare workers often had to reuse their PPE due to major PPE shortages. The daily demand data is, thus, severely censored.

For these reasons, this study relies on simulated data, which is based on the pandemic data. Similarly to Lum, Johndrow, Cardone, et al. [53], we assume that the major driving force behind the high demand for PPE is the pandemic, i.e., we assume there exists a strong positive association between the demand and the pandemic-related variables such as the number of infections and hospitalizations because of the government protocols that enforced the number of PPE consumption per patients. While we acknowledge the existence of other factors that influence demand data, these are omitted in this study since the objective is to understand the effect of data visibility and not to reproduce exactly demand data during a pandemic.

This paper employs the data for the Canadian province of British Columbia (BC). In particular, the population of the region consists of 5,147,712 inhabitants [95]. The number of infections is obtained through the daily government updates [96], see Figure 5a in Appendix C. Furthermore, data from the Canada Institute for Health Information (CIHI) [97] provides the COVID-19-related daily hospital admissions, discharges, deaths, and the average length of stay (e.g., see Figure 5b in Appendix C for trend of the number of hospitalized patients in BC). The daily number of hospitalizations is acquired by subtracting the daily discharges and deaths from the daily hospital admissions. In addition, the incubation period, $t_{\text{incubation}} = 5.1$ and the average length of stay $t_{\text{ALOS}} = 12.2$ in Section 4.1.2.1 are derived from this CIHI data.

We then multiply the number of hospitalizations by the coefficient of consumption (i.e., a factor associated with the government protocol prescribing the number of PPE to use per hospitalization) to simulate the number of PPE that are used. As previously discussed, this study assumes these protocols are followed closely. We believe this is a proper method to simulate the demand associated
with a pandemic since, during the first wave of the pandemic, due to shortage concerns, some specific types of PPE, such as the N95 respirators, were prescribed to be used only for the handling of COVID-19 patients. Therefore, employing hospitalization as a trigger for the demand seems reasonable. To create a more realistic setting where some divergences from the government protocol are to be expected, we assume that the consumption coefficient $CC_{\text{sim}}$ is a normally distributed random variable, resampled daily, with mean $\mu_{CC}$ and standard deviation $\sigma_{CC}$, i.e.,

$$CC_{\text{sim}} \sim N(\mu_{CC}, \sigma_{CC}^2).$$

Furthermore, we control the signal-to-noise ratio (SNR) (i.e., the inverse of the coefficient of variation) of this distribution throughout the different iterations to control the mean relative to the spread. The SNR is defined as

$$SNR = \frac{\mu_{CC}}{\sigma_{CC}}.$$  
(20)

Note that $CC_{\text{sim}}$ is the consumption coefficient used to generate the demand data and that it can change from one day to the next. It differs from the previously discussed $CC_k$, which is the government protocol value. In particular, $CC_k$ is fixed to $\mu_{CC}$ in our first three scenarios, while it differs from that value for the last scenario on erroneous data.

The decision epochs are seven days apart, as a weekly review of the system is a common practice. The additional parameters used in all scenarios of this study are the number of units per case, the supplier’s order limits per case, the service level, and the lead time. Each parameter is uniformly sampled in each iteration from a continuous or discrete interval defined in Appendix C and is then kept fixed throughout the iteration; these intervals are chosen to be as realistic as possible and to yield as many insights as possible. Overall, 1,000 iterations are executed for the base scenario using Python 3.7, and the obtained data is reused in the other scenarios to improve comparability. For each individual iteration, at every decision epoch, we observe the demand, make a forecast, take an action, record the performance of the system, and then move to the next epoch. Also note that each iteration is addressed by the three forecasting methods previously described.

Finally, the fitting process of the proposed epidemiological model (i.e., the SEIRHD model) is done with the lmfit package in Python by performing a grid search and minimizing the least square error while searching within pre-specified ranges for these parameters. These ranges consist of realistic values for these parameters and are provided in Table II of Appendix C with the other parameter values. Note that we use this curve fitting process to estimate the least number of parameters possible since this curve fitting is complex and subject to multiple local optima, especially when trying to fit multiple parameters. This is why several parameters are estimated a priori from various data sources.
5.2 Results

We study the following four settings: (1) a scenario without DVI as previously described (i.e., the base scenario), (2) a scenario with lagged data, (3) a scenario with temporally aggregated data, and (4) a scenario with erroneous data. The base scenario is assumed to be the benchmark for the other scenarios since there is no modification to the simulated data. For each scenario, the forecasting methods are evaluated on the percentage bias (PBIAS), root mean square error (RMSE), and mean absolute percentage error (MAPE). For each iteration $i$, these measures are computed as

$$\text{PBIAS}^i = \frac{100}{K} \sum_{k=1}^{K} \sum_{h=1}^{f_k} \frac{\hat{d}_{k,h} - dd_{k,h}}{f_k dd_{k,h}},$$

$$\text{RMSE}^i = \frac{1}{K} \sum_{k=1}^{K} \sqrt{\frac{\sum_{h=1}^{f_k} (\hat{d}_{k,h} - dd_{k,h})^2}{f_k}},$$

$$\text{MAPE}^i = \frac{100}{K} \sum_{k=1}^{K} \sum_{h=1}^{f_k} \left| \frac{\hat{d}_{k,h} - dd_{k,h}}{f_k dd_{k,h}} \right|,$$

where, with a slight abuse of notation, $\hat{d}_{k,h}$ is the $h$-step ahead forecast in epoch $k$ of iteration $i$, and $dd_{k,h}$ is the daily demand $h$ days after the beginning of epoch $k$ in iteration $i$. Note that we observe the states of the epochs $k = 1, 2, \ldots, K + 1$, which contain the demand of the epochs $k = 0, 1, \ldots, K$, but only forecast and take an action in the epochs $k = 1, 2, \ldots, K$. This explains the range of the summations of the previous and following equations. We report the average of these measures over all iterations.

Furthermore, since the end goal is to analyze the performance of these forecasts with respect to inventory management, we also evaluate the periodic review system performance when using these forecasts. As discussed in Section 3.5, this is done by evaluating the shortages and left-over inventory at the end of the time horizon (LOI). In particular, to improve the comparability of the results across the different iterations, we evaluate these methods on the relative shortage (RS) and relative left-over inventory (RLOI) measures, i.e.,

$$\text{RS}^i = \frac{\sum_{k=1}^{K+1} q_{k,-}}{\sum_{k=0}^{K} d_k} \times 100,$$

$$\text{RLOI}^i = \frac{q_{K+1}^{a,+}}{\sum_{k=0}^{K} d_k} \times 100.$$

Unless specified otherwise, we assume that the forecast horizon $f_k$ is constant throughout the decision epochs $k$ and that, for each iteration $i$, $f_k$ corresponds to a period that includes the following epoch and the lead time used in iteration $i$, i.e., $f_k = R + L$. Finally, note that the naïve method is assumed to be the benchmark in each scenario since it is the simplest forecasting method and is used commonly in practice.
5.2.1 Scenario 1: Base scenario

As a benchmark for our study, we first create a scenario where the manager receives the required data in an ideal setting. The data is updated daily and passes through the forecasting process and inventory control at each epoch. The results are provided in Table 2.

Table 2: Mean base scenario results over the 1,000 iterations

<table>
<thead>
<tr>
<th>Method</th>
<th>PBIAS</th>
<th>RMSE</th>
<th>MAPE</th>
<th>RS</th>
<th>RLOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>372.39</td>
<td>717.45</td>
<td>388.02</td>
<td>5.65</td>
<td>35.80</td>
</tr>
<tr>
<td>Holt</td>
<td>73.3</td>
<td>222.6</td>
<td>156.49</td>
<td>11.08</td>
<td>36.13</td>
</tr>
<tr>
<td>Naïve</td>
<td>210.6</td>
<td>264.05</td>
<td>248.53</td>
<td>11.95</td>
<td>58.4</td>
</tr>
</tbody>
</table>

In this scenario, even though the Holt method has the best performance across the forecasting measures (i.e., PBIAS, RMSE, and MAPE), the SEIRHD method outperforms the other methods on the RS and RLOI measures. To explain this counter-intuitive outcome, we analyzed PBIAS before and after the maximum demand (i.e., the peak) in each iteration, only for epochs in which the system placed an order. We refer to these periods as the pre-peak and post-peak periods in Table 3. Note that the demand follows a similar trend to the daily number of hospitalized patients (i.e., Figure 5b in Appendix C).

Table 3: Mean base scenario percentage bias (PBIAS), before and after peak demand

<table>
<thead>
<tr>
<th>Method</th>
<th>Pre-peak</th>
<th>Post-peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>538.19</td>
<td>105.99</td>
</tr>
<tr>
<td>Holt</td>
<td>55.95</td>
<td>178.40</td>
</tr>
<tr>
<td>Naïve</td>
<td>13.49</td>
<td>512.43</td>
</tr>
</tbody>
</table>

During the pre-peak period, the SEIRHD method does not have access to enough data to make accurate forecasts. As a result, the forecasts are over-estimated during this period, which forces the system to place orders with higher quantities. This additional inventory later helps the system when the supplier’s capacity is insufficient for the demand and, hence, explains the better performance with respect to the relative shortage. This conclusion holds regardless of the supplier’s capacity, where we observe similar trends when the supplier’s capacity is not limited, albeit at lower RS levels for all methods. Furthermore, once the demand has plateaued, the SEIRHD method can capture the trend and obtains the best forecasts on average in terms of the percentage bias. This performance in the post-peak period leads the SEIRHD method to generate the best RLOI.

In contrast, the Holt method exhibits a much lower percentage bias in the pre-peak period than the SEIRHD method, affecting its RS. Furthermore, the performance of the Holt method declines in the post-peak period, which affects its RLOI. These explanations also apply to the naïve method with an even greater effect on the RS and RLOI. Overall, the SEIRHD and Holt methods perform better than the naïve method.
5.2.2 Scenario 2: Lagged data

An important aspect of data visibility is the delay in the data flow. In a supply chain, managers are frequently deprived of the latest version of the data. In the particular case of COVID-19, it often took several days to collect the data from the different hospitals. In this scenario, we investigate this common phenomenon by analyzing two distinct lag formats (i.e., fixed and dynamic lag) within the simulated data. The lag is defined as the time in days between the date data is captured and when data is available to the managers. In this scenario’s first version, the lag applied to the data is fixed for all iterations. We then gradually increase the fixed lag to understand its impact on the system. It is important to note that, since a forecast begins at the last known date of the data, the lag period is also included in the forecast horizon \( f_k \); formally, \( f_k = R + L + \text{lag} \). However, the forecasted demand during the lag period is removed prior to applying the periodic review system since this perishable demand has already been realized. The lag is applied to the data from the base scenario. Figure 2 presents the results for the fixed lagged data.

The negative impact of the data delay on the system performance can clearly be established in Figure 2 for the Holt and naïve methods. The gradual increase of the lag length results in the continuous augmentation of the shortage level. In the extreme case of a fixed lag of 14 days, the Holt method experiences 48% more RS than the case with no lag, i.e., the base scenario. A similar pattern is also observed for the naïve method. Furthermore, the SEIRHD method also follows this behavior, where the relative shortage measure has a general upward trend. However, increasing the lag can also be beneficial for this method, as shown with the various local minima of the SEIRHD method in Figure 2a. By increasing the lag length, the forecast horizon \( f_k \) has effectively been increased. Note that the SEIRHD method tends to over-forecast during the early epochs and that these forecasts are shaped as an exponential function. Hence, by increasing the lag, the magnitude of these over-forecasts is increased, and if the supplier’s capacity is not binding, the relative shortages are reduced accordingly. Therefore, even though the SEIRHD method generally follows a similar diminishing performance for the RS and RLOI measures, the behaviour fluctuates with greater volatility toward the larger lag values. The rise in the RLOI of both the Holt and naïve methods can also be observed, albeit to varying degrees, caused by continuous over-forecasting, especially during the post-peak demand period.

In order to emulate real-world circumstances where the delay in the data might not be fixed, dynamic lags are also generated. In particular, 1,000 \((K + 1)\)-dimensional vectors of lag values are sampled from the discrete uniform distribution \( W = \{0, 1, \ldots , 14\} \); we assume that the maximum delay within the data does not exceed 14 days (i.e., two weeks). We then apply each of the 1,000 lag vectors to the 1,000 data sets from the base scenario for a total of 1,000,000 iterations. Table 4 provides the results of the dynamic lag.

Despite producing the worst forecasts, the SEIRHD method provides the best performance on both the RS and RLOI measures. The delay in the delivery of the data interferes with the forecast process, resulting in over-forecasts as shown in Table 5. As a result, the system orders
Figure 2: Relative shortage and LOI for the fixed lagged data. The shaded region represents the 95% confidence interval.

more products before the peak, which lowers the relative shortage compared to the base scenario. In contrast, since the Holt and naïve methods considerably under-estimate the demand in this scenario versus the base scenario, they obtain worse relative shortages than in the base scenario.

An additional analysis of the PBIAS results reveals that the accuracy of the system is better with the SEIRHD method than the other methods after epoch 10 (which is after the peak demand).
Table 4: Mean dynamic lag results over the 1,000,000 iterations

<table>
<thead>
<tr>
<th>Method</th>
<th>PBIAS</th>
<th>RMSE</th>
<th>MAPE</th>
<th>RS</th>
<th>RLOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>5664.34</td>
<td>14901.77</td>
<td>5690.39</td>
<td>4.56</td>
<td>37.97</td>
</tr>
<tr>
<td>Holt</td>
<td>93.37</td>
<td>299.71</td>
<td>222.6</td>
<td>13.36</td>
<td>43.44</td>
</tr>
<tr>
<td>Naïve</td>
<td>221.48</td>
<td>301.14</td>
<td>285.49</td>
<td>14.96</td>
<td>62.07</td>
</tr>
</tbody>
</table>

Table 5: Mean dynamic lag percentage bias (PBIAS), before and after peak demand

<table>
<thead>
<tr>
<th>Method</th>
<th>Pre-peak</th>
<th>Post-peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>8201.85</td>
<td>3971.36</td>
</tr>
<tr>
<td>Holt</td>
<td>25.04</td>
<td>331.09</td>
</tr>
<tr>
<td>Naïve</td>
<td>-27.14</td>
<td>592.28</td>
</tr>
</tbody>
</table>

In particular, the PBIAS results of the SEIRHD, Holt and naïve methods are respectively 97.99%, 291.22% and 689.83%. This is caused by the random lag delaying the realization of the maximum demand. Therefore, having enough time to adjust the inventory level after epoch 10, the SEIRHD method produces the lowest RLOI, comparable to that of the base scenario. In contrast, the over-forecasts in the Holt and naïve methods appear to take place primarily after epoch 10, resulting in higher RLOI than the SEIRHD method.

5.2.3 Scenario 3: Temporally aggregated data

Another potential problem with regard to the supply chain’s visibility is the granularity of the data. In this scenario, the data is not reported on a daily basis, and only the total sum of a specific variable (e.g., the demand) since the last report is available. Hence, the evolution of the daily demand is unknown to the manager, which could hinder the performance of the system.

For this scenario, we investigate aggregated data received at different frequencies, hereon the period info. This aggregated data represents the total demand of PPE over the period info. The daily behaviour of the demand is assumed to be unknown during the forecasting process. However, the products are consumed based on the actual daily demand. Moreover, we apply the same granularity to the pandemic data that is used by the SEIRHD model.

Similar to Section 5.2.2, we analyze the impact of two distinct formats of temporally aggregated data. We first apply a fixed period info on the data for all iterations during the entire simulation and then vary its value based on the set $P_{\mathcal{I}} = \{1, 2, \ldots, 30\}$. This setting analyzes the period info’s influence on the system’s performance. It is important to mention that since the data is aggregated and reported based on specific period info, there exists the possibility of a lag within the system if the period info is not a multiple of seven days (i.e., the duration of an epoch). The lag is defined here as the number of days between the date of the last reported aggregated data to the date of the epoch that is being analyzed. Figure 3 presents the results for this part.
Figure 3: Relative shortage and LOI for the fixed period info. The shaded region represents the 95% confidence interval.

Figure 3a illustrates the negative impact of the temporally aggregated data on the RS measure. The number of shortages increases as the data becomes coarser. The Holt and naïve methods follow the upward pattern in their shortages, with the Holt method almost matching the naïve method’s results for large period info due to the lack of proper data for its fitting process. Additionally, even though the RS measure of the SEIRHD method generally increases as the period info is
increased, the SEIRHD method performs considerably better than the other methods. However, the behaviour of the SEIRHD method becomes unpredictable and erratic once the period info goes above the 20 days mark. The greater temporal aggregation of data pushes the SEIRHD method to display substantial over- or under-forecasts, resulting in fluctuating behaviour. The same behaviour is observed for the RLOI results of the SEIRHD method, albeit with a slightly smoother upward pattern. The results of the RLOI measure in Figure [3b] provide an interesting finding regarding the performance of the naïve method, which holds a relatively steady level of RLOI, between 55% to 60%. An important observation is that the naïve method outperforms both the Holt and SEIRHD methods on the RLOI measure when the period info is large enough (i.e., around 30 days), indicating the reliability of the more advanced methods is challenged as the data becomes coarser.

For the second part of this scenario, we investigate the effect of dynamic period info on the system. This setting reflects real-world situations where the medical centers send their total consumption data at random frequencies. To do so, we generate 1,000 vectors of period info values, uniformly sampled from \( \mathcal{P} \), and apply them to the 1,000 iterations of the base scenario, leading to a total of 1,000,000 iterations. Table 6 presents the results of the dynamic period info.

<table>
<thead>
<tr>
<th>Method</th>
<th>PBIAS</th>
<th>RMSE</th>
<th>MAPE</th>
<th>RS</th>
<th>RLOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>11604.61</td>
<td>15878.67</td>
<td>11699.44</td>
<td>12.34</td>
<td>33.49</td>
</tr>
<tr>
<td>Holt</td>
<td>159.35</td>
<td>293.56</td>
<td>250.48</td>
<td>17.69</td>
<td>59.93</td>
</tr>
<tr>
<td>Naïve</td>
<td>117.68</td>
<td>267.25</td>
<td>215.09</td>
<td>18.53</td>
<td>58.08</td>
</tr>
</tbody>
</table>

As with the fixed period info, the SEIRHD method outperforms the other two methods on the RS measure due to the over-forecasts in this method. The analysis of the bias distribution in Table 7 provides additional explanations on the performance of the system.

<table>
<thead>
<tr>
<th>Method</th>
<th>Pre-peak</th>
<th>Post-peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>19593.08</td>
<td>9518.87</td>
</tr>
<tr>
<td>Holt</td>
<td>-56.50</td>
<td>485.89</td>
</tr>
<tr>
<td>Naïve</td>
<td>-65.31</td>
<td>375.69</td>
</tr>
</tbody>
</table>

As a consequence of dynamic period info, the Holt and naïve methods experience significant under-forecasts before the peak demand, resulting in large RS values. After the peak demand, the Holt method exhibits over-forecasting, which forces the system to place more orders for epochs with much lower demand, and as a result, its RLOI measure is considerably higher than in Scenario 1. However, the results of the SEIRHD method in Table 7 require further analysis since the over-forecasts, both before and after the peak, still translate into the best RLOI value across the different scenarios. A detailed explanation of this ambiguity is provided in Section 5.3.
5.2.4 Scenario 4: Erroneous data

In this scenario, we explore the impact of erroneous data on the performance of the system. There exists a possibility of under- or over-reporting by healthcare facilities within the input data, which influences the forecasting process and, consequently, the performance of each method. To achieve the setting of this scenario, we multiply $CC_{sim}$ by a deviation parameter, $\delta_{CC}$, before generating the demand data for the naïve and Holt methods. For the SEIRHD method, the consumption coefficient $CC_k$ is instead adjusted to $CC_k = \delta_{CC}\mu_{CC}$.

In this scenario, we vary the deviation parameter, $\delta_{CC}$, in the range $[0, 7]$ by increments of 0.25. Thus, the data is simulated to be under-reported when the deviation parameter is in the range $[0, 1)$. In particular, note that no data is reported with $\delta_{CC} = 0$ and, hence, the forecasts are null; yet, the system still orders due to the high resulting RMSE in Equation 18. In contrast, it is simulated to be over-reported when $\delta_{CC} \in (1, 7]$. Note that $\delta_{CC} = 1$ corresponds to the base scenario. The modified demand data is fed directly into the forecasting model for the methods that use the demand data (i.e., the Holt and naïve methods). For the SEIRHD method, it is assumed that the coefficient of consumption employed in the forecasting process (see Equation 17) is being reported by the healthcare facilities; thus, it contains the same erroneous $\delta_{CC}$ as in the demand data. Moreover, the data is assumed to be collected daily and provided to the system at each epoch, similar to the base scenario. Figure 4 presents the results of this scenario.

It is clear from the results that $\delta_{CC}$ has an inverse effect on the RS measure. On the one hand, increasing the level of under-reporting results in significant exponential growth of the RS measure for all forecasting methods. On the other hand, even though over-reporting the data improves the RS measure considerably at the beginning, the impact becomes less significant as we continue increasing the over-reporting level. Note, however, that the shortages are never entirely eliminated. Furthermore, the SEIRHD method consistently outperforms the other methods in the RS measure, primarily due to the over-forecasts before the peak demand. Finally, the RLOI measure is also affected by $\delta_{CC}$ with which it has a strong positive association; Figure 4b reveals that as $\delta_{CC}$ becomes larger, the system experiences higher RLOI.

5.3 Discussion

The forecasting methods that are employed in this paper provide valuable insights into the forecasting process of PPE demand during a pandemic. We conclude that in the absence of any historical data (e.g., in the first few epochs), the naïve method is the only model that can produce reasonable forecasting results, which do not necessarily translate into enhanced inventory management performance due to external factors such as the supplier’s lead time and capacity. As more data is provided to the system, the epidemiological model produces more accurate results, as shown in Table 3. We also analyzed the impact of DVIs on the performance of an inventory management system within the context of the COVID-19 pandemic. We present a unique scenario for each DVI.
that first quantifies the direct impacts of the issue when its magnitude is gradually increased. In general, increasing the DVI magnitude diminishes the system’s performance albeit to varying degrees, which is evident from Fig 2 and Fig 3. Moreover, we randomize the DVI of the delayed and temporally aggregated data scenarios to mimic real-world situations. For a system that experiences random temporal aggregation of data, a performance deterioration in the RS measure is observed in comparison to the base scenario (see Table 6). In contrast, the presence of a random lag causes
the system to produce a lower RS than the base scenario for the SEIRHD method (see Table 4). In Scenario 4 where the system is dealing with erroneous data, it is observed that artificial augmentation of the demand leads to improved performances, but at higher costs due to higher RLOI levels.

Additional analyses are, however, required. In particular, the scenario with the applied randomized lags requires additional analyses, since it generates inconsistent results when the system employs the SEIRHD method. The presence of DVIs in the first three scenarios generally deteriorates the performance of the system for both the RS and RLOI measures. However, the RS measure of the SERIHD method improves when the system is exposed to randomized lags. These contradictory results can be explained through Table 8 that characterizes the forecasts of the SEIRHD method for the different scenarios considered in this study. When a system experiences a randomized DVI (i.e., dynamic lagged data and dynamic temporally aggregated data), the SEIRHD method produces larger and more frequent under-forecasts than in the base scenario. At the same time, the magnitude of the over-forecasts is amplified exponentially. In the case of the dynamic lag, despite the fact that the occurrence of the under-forecast portion is larger than the base scenario, both measures of the over-forecast portion (i.e., the occurrence and the mean percentage) force the system to place additional orders during the pre-peak epochs, which in turn assist the system to have a lower RS value than the base scenario. In comparison, even though the mean of the over-forecast portion for the temporally aggregated data is quite large, it is not frequent enough to affect the RS measure.

Table 8: Forecasting behaviour for SEIRHD method across the scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Estimation</th>
<th>Occurrence</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>Base</td>
<td>Under-forecast</td>
<td>16.7</td>
<td>-9.4</td>
</tr>
<tr>
<td></td>
<td>No bias</td>
<td>1.4</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Over-forecast</td>
<td>81.9</td>
<td>380.9</td>
</tr>
<tr>
<td>Dynamic lagged data</td>
<td>Under-forecast</td>
<td>22.6</td>
<td>-31.4</td>
</tr>
<tr>
<td></td>
<td>No bias</td>
<td>1.3</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Over-forecast</td>
<td>76.1</td>
<td>6309.8</td>
</tr>
<tr>
<td>Dynamic temporally aggregated data</td>
<td>Under-forecast</td>
<td>66.7</td>
<td>-67.1</td>
</tr>
<tr>
<td></td>
<td>No bias</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Over-forecast</td>
<td>32.8</td>
<td>31818.2</td>
</tr>
</tbody>
</table>

We also perform a linear regression analysis on the RS results of both the lagged and temporally aggregated data when the applied distortion is fixed throughout each iteration. Table 9 presents the slope of the fitted lines for each method. We observe that the SEIRHD method is the least affected by the gradual increase in the lag among all forecasting methods since it has the smallest slope. On the other hand, if the system is expecting an increase in the temporal aggregation of data, the naïve method produces more stable results than the other methods. Finally, the Holt
method exhibits mid-range performances compared to other methods regardless of the source that causes the distortion. However, it is not possible to directly compare these slopes across the two scenarios since increasing the lag by one day is not necessarily equivalent to increasing the data aggregation by the same amount. Therefore, we cannot firmly state that one type of DVI is worse than the other. The analysis of each issue should be performed independently.

Table 9: Slopes of the fitted linear regressions on the relative shortages

<table>
<thead>
<tr>
<th>Method</th>
<th>Lagged data</th>
<th>Temporally aggregated data</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEIRHD</td>
<td>0.08</td>
<td>0.44</td>
</tr>
<tr>
<td>Holt</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.40</td>
<td>0.28</td>
</tr>
</tbody>
</table>

In Section 3.5, we provided our justification for the use of the LOI instead of the average inventory. Nonetheless, we also analyze the average inventory for the base scenario, and the randomized lagged data and temporally aggregated data scenarios. Similarly to Equation 25, the relative inventory (RI) is used to improve the comparability of the results, i.e., \( RI^i = 100 \times \frac{\sum_{k=1}^{K+1} q_k}{\sum_{k=0}^{K} d_k} \). Table 10 presents the results of our analysis.

Table 10: Relative inventory

<table>
<thead>
<tr>
<th>Scenario</th>
<th>SEIRHD</th>
<th>Holt</th>
<th>Naïve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>416.65</td>
<td>244.54</td>
<td>309.78</td>
</tr>
<tr>
<td>Dynamic lagged data</td>
<td>421.47</td>
<td>265.69</td>
<td>309.7</td>
</tr>
<tr>
<td>Dynamic temporally aggregated data</td>
<td>306.51</td>
<td>293.14</td>
<td>282.08</td>
</tr>
</tbody>
</table>

The RI measure provides additional insights into the overall cost of the system in each scenario. While the SEIRHD method generates the best results with respect to RS and RLOI in scenarios 1 to 3, its RI is considerably inferior to the other methods particularly for the base and dynamic lagged data scenarios. The RI results of the SEIRHD method are linked to its significant overforecasting behaviour during the pre-peak epochs resulting in higher inventory and consequently higher holding costs than the other methods. Furthermore, the performance of the Holt and naïve methods with respect to the RI appears to be associated with their RLOI and follows a similar pattern.

In the case of Scenario 4, it is shown in Figure 4a that the RS measure never gets to zero as \( \delta_{CC} \) is increased, while the RLOI measure is increasing at a steady pace. The supplier’s capacity is the main obstacle to the complete elimination of shortages. The limitations imposed by the supplier’s capacity become more prominent in this scenario since at high \( \delta_{CC} \) values, the remaining shortages occur during the epochs where the supplier’s capacity has already been reached. Hence, artificially increasing the demand does not have an impact in these epochs. It should again be noted that while erroneous data has a significant impact on the performance of the system, it can not be directly
compared with the other DVIs.

Even though none of the previous literature studied the impact of DVIs on inventory management performance in the context of a pandemic, our results are aligned with several previous studies. In particular, the analysis of the RS results for the Holt and naïve methods are comparable to those acquired by Hoberg and Thonemann [75], where the system’s performance deteriorates as the data lag increases. Yet, similar to the results presented by Hosoda and Disney [76], we also conclude that not all systems benefit from shorter lags, as evident by the results of the SEIRHD method where, due to the over-forecasting, the RS measure is improved with a dynamic lag over the base scenario that has no lag. Regarding the data aggregation, there exist contrasting views among scholars where both positive [82, 83] and negative [85] impacts on the system have been observed depending on the settings of the studies. Our results from Scenario 3 point toward the fact that the temporal aggregation of data worsens the system’s performance. Finally, as presented by Kwak and Gavirneni [69], erroneous data can potentially hinder the performance of a system, which is similar to our findings in Scenario 4 with under-reported data. Overall, our study confirms that several previous findings still hold in the case of inventory management during a pandemic.

Finally, our study brings practical insights for managers and their inventory planning activities. For example, it is possible for the managers to observe the quantitative impact of the lag on the system’s performance with the fixed lag results of Scenario 2. With these results, the managers can perform a cost-benefit analysis to determine if reducing the delay in the data is appropriate. Furthermore, as mentioned previously, the temporal aggregation of data is a major DVI in the Canadian healthcare system, unlikely to be solved in a timely manner. In that regard, our findings of Scenario 3 can assist policymakers in estimating the potential level of reduction in shortages should future enhancements in the healthcare system improve the granularity of the data. Moreover, Scenario 4 provides two valuable practical insights to managers in different echelons of the decision-making process. First, there exists a point for all forecasting methods from which a further increase in over-reporting only results in higher inventory costs in the form of left-over inventory and minimal to no improvements to the shortages. Hence, the results of Scenario 4 become particularly useful to a cost-benefit analysis of the issue of over-reporting. Second, we have observed the significant consequences of under-reporting, and even though it seems unlikely that such situations might occur, the managers should identify the potential sources of under-reporting in the system and act accordingly to prevent this behaviour. Under-reporting could result from numerous sources within the data structures of HSCs. For a system that employs a demand-based forecasting model, the lack of an adequate inventory tracking system could be a potential source of under-reporting; whereas due to limited testing capacities, the system with an epidemiological-based forecasting model might receive under-reported data [70, 71]. These are but a few examples of areas where a manager should investigate. Once the under-reporting is detected in the system, the manager can employ our analysis to indicate the additional shortages imposed by this DVI, assuming the percentage of under-reporting can be estimated. Lastly, it should be pointed out that the managers may not be in the exact same context on which the results of this paper are based. However, by
re-generating the proposed simulation according to their region’s specific settings and parameters, the exact impact of the DVIs could be examined.

6 Conclusions

Visibility is a major contributing factor to the performance of healthcare supply chains. The COVID-19 pandemic further amplified the system’s shortcomings in this regard, both in the upstream and downstream segments of the supply chain system. In this paper, we analyzed the impacts of data visibility on the performance of an inventory management system during a pandemic. We considered four scenarios where the first one contains no DVI and is the base scenario. In the second scenario, the system experienced delays in the flow of information. Then, the temporal aggregation of data was addressed in the third scenario. Finally, the final scenario examined the under- and over-reported demand. From these scenarios, we concluded that while the SEIRHD method is not producing the best forecasts, its RS and RLOI results are often superior to those of the other methods. The benchmark method, the naïve method, which is widely used in healthcare facilities, has consistently performed worst in all categories. There are, therefore, areas for further enhancements. We also observed that, in most cases, the existence of DVIs diminished the performance of the system.

In our study, it was assumed that the demand pattern follows the hospitalization curve as was also theorized in other studies (e.g., [53]). We believe this assumption is realistic since government protocols enforce the number of PPE consumed per patient. However, other factors, such as panic purchasing behaviours, may cause some degrees of deviation. We tried to alleviate this limitation by implementing random noise over the data, but additional studies could analyze if our results still hold with these other factors. Furthermore, our findings only hold for products that are required primarily in the context of COVID-19-positive patients and may not be applicable to other types of PPE, such as surgical masks, that are frequently distributed in other organizations and among the general public. Then, the epidemiological data in this study is based on only one Canadian province, which follows a pattern similar to those observed worldwide. However, the timing of the peak hospitalization and the length of the wave, to name a few, differ not only across different countries but also across different waves. Thus, our results may not necessarily hold in all contexts. Future research could investigate our results in different epidemiological settings, such as the occurrence of multiple peaks of demand or elongated waves. For example, Perakis, Singhvi, Skali Lami, and Thayaparan [98] propose a forecasting method for multiple waves. We also encourage future studies to investigate and include the baseline demand as well as the demand that stems outside of the hospitals. The baseline demand is defined as the need for a product that is not caused by the presence of COVID-19 patients in hospitals. Furthermore, combinations of DVIs could also be part of future research where multiple DVIs are applied simultaneously to the system. A final interesting research avenue is to go beyond the periodic review system and
assess the impacts of data visibility when using optimization methods for inventory management.

7 Acknowledgements

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A Assumptions of the SEIRHD model

The following five assumptions are implied by the SEIRHD model.

**Assumption 1.** The duration of the disease outbreak is short enough to exclude the natural births and deaths in the population of the studied region. This implies that the total population, \( N \), is constant during this study. Therefore, \( N \) can be formulated as

\[
N = S + E + I + R + H + D. \tag{26}
\]

**Assumption 2.** The rate of transmission is proportional to the contact between the susceptible and infectious populations. In the SEIRHD model, this rate is assumed to be constant.

**Assumption 3.** The demographics of the population are homogeneous enough so that the rate of removal, either recovery or death, is constant. Even though the immune system of different age groups varies significantly in regard to a specific type of disease, the average transmission rate for the entire population is assumed to be relatively constant.

**Assumption 4.** In the SEIRHD model, the immunity achieved by the subjects who survived the outbreak is long enough that there will be no re-infection for the duration of the study.

**Assumption 5.** The beginning of a possible outbreak is defined as the time when the first infection is introduced to the model, denoted by \( t = 0 \).

B Proof of Lemma 1

*Proof.* Following the next generation method \cite{99, 101}, we define the vector \( x \) where each element \( x_i \) denotes the number of subjects in the \( i \)th compartment. Let \( F_i(x) \) be the rate of appearance of new infections in compartment \( i \) and let \( V_i(x) = V_i^-(x) - V_i^+(x) \), where \( V_i^+(x) \) and \( V_i^-(x) \) are respectively the rate of transfer of subjects into and out of the \( i \)th compartment, by other means than infection.

Then, we can form the next generation matrix \( FV^{-1} \) where the matrices \( F \) and \( V \) are constructed from the partial derivatives of \( F_i \) and \( V_i \) over the three compartments that contain infected subjects (i.e., the exposed, infected and hospitalized compartments). Specifically,

\[
F = \left[ \frac{\partial F_i(x_0)}{\partial x_j} \right] \quad \text{and} \quad V = \left[ \frac{\partial V_i(x_0)}{\partial x_j} \right],
\]

where \( x_0 \) is the disease-free equilibrium (i.e., \( S = N \) and the other compartments are empty) and \( i, j \) refer alternatively to the exposed, infected and hospitalized compartments.
With Equations 11, 12 and 14 this translates to

\[
F = \begin{bmatrix}
0 & \beta & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}, \quad V = \begin{bmatrix}
\sigma & 0 & 0 \\
-\sigma & p_H \gamma_H + (1 - p_H) \gamma_R & 0 \\
0 & -p_H \gamma_H & \gamma_D \\
\end{bmatrix},
\]

and

\[
FV^{-1} = \begin{bmatrix}
\frac{\beta}{p_H \gamma_H + (1 - p_H) \gamma_R} & \frac{\beta}{p_H \gamma_H + (1 - p_H) \gamma_R} & 0 \\
0 & \frac{\beta}{p_H \gamma_H + (1 - p_H) \gamma_R} & 0 \\
0 & 0 & 0 \\
\end{bmatrix}.
\tag{27}
\]

The basic reproduction number, \(R_0\), is then given by the spectral radius (i.e., the dominant eigenvalue) of the matrix \(FV^{-1}\), i.e.,

\[
R_0 = \frac{\beta}{p_H \gamma_H + (1 - p_H) \gamma_R}.
\tag{28}
\]

Finally, to obtain Lemma 1 we replace \(R_0\) in Equation 28 with a varying basic reproduction number, i.e.,

\[
R_0(t) = \frac{R_{0,1} - R_{0,2}}{1 + e^{-k(t_0 - t)}} + R_{0,2},
\tag{29}
\]

where the various parameters are defined in Table 1. Solving for \(\beta\) results in

\[
\beta = \left[ \frac{R_{0,1} - R_{0,2}}{1 + e^{-k(t_0 - t)}} + R_{0,2} \right] (p_H \gamma_H + (1 - p_H) \gamma_R).
\tag{30}
\]
C SEIRHD and simulation parameters

Table 11 presents the value or range for the parameters of the SEIRHD model.

Table 11: Parameter values of the SEIRHD model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>5,147,712</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Computed with Lemma 1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$1/5.1$</td>
</tr>
<tr>
<td>$p_H$</td>
<td>18.88%</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>$[0.1, 0.7]$</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>$[0.1, 0.7]$</td>
</tr>
<tr>
<td>$\gamma_D$</td>
<td>$1/12.2$</td>
</tr>
<tr>
<td>$R_{0,1}$</td>
<td>$[1.5, 10]$</td>
</tr>
<tr>
<td>$R_{0,2}$</td>
<td>$[0, 10]$</td>
</tr>
<tr>
<td>$t_0$</td>
<td>$[50, 120]$</td>
</tr>
</tbody>
</table>

Table 12 outlines the value, interval or set employed for the different parameters of the simulation.

Table 12: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean consumption coefficient ($\mu_{CC}$), units per patient</td>
<td>${3, 3.5, \ldots, 7}$</td>
</tr>
<tr>
<td>Signal-to-noise ratio (SNR)</td>
<td>[2, 10]</td>
</tr>
<tr>
<td>Number of units per case</td>
<td>12</td>
</tr>
<tr>
<td>Supplier’s minimum order quantity, cases</td>
<td>${1, 2, \ldots, 12}$</td>
</tr>
<tr>
<td>Supplier’s maximum order quantity, cases</td>
<td>${200, 201, \ldots, 400}$</td>
</tr>
<tr>
<td>Service level ($1 - \alpha$), %</td>
<td>${95, 95.1, \ldots, 99.9}$</td>
</tr>
<tr>
<td>Lead time ($L$), days</td>
<td>${5, 6, \ldots, 30}$</td>
</tr>
</tbody>
</table>

Figure 5 presents some of the data from British Columbia that were used in the simulation.
Figure 5: British Columbia data

(a) Daily number of new infections

(b) Number of hospitalized patients
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