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Bureau de Montréal

Université de Montréal
C.P. 6128, succ. Centre-Ville
Montréal (Québec) H3C 3J7
Tél : 1-514-343-7575
Télécopie : 1-514-343-7121

Bureau de Québec

Université Laval,
2325, rue de la Terrasse
Pavillon Palasis-Prince, local 2415
Québec (Québec) G1V 0A6
Tél : 1-418-656-2073
Télécopie : 1-418-656-2624

Multi-layer Network Design for Consolidation-based Transportation Planning

Teodor Gabriel Crainic*

Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT), and School of Management, Université du Québec à Montréal

Abstract. The work on multi-layer network design, and this report which follows from it, is dedicated to the loving memory of Bernard - Professor Bernard Gendron. We miss him so much.

Multilayered networks are used to model systems with multiple interacting components, each being represented by a network layer. The Operations Research *multi-layer network* term is associated to complex multilayered problem settings, encountered in planning transportation and telecommunication systems in particular, in which an arc in a given layer is defined with respect to a set of arcs in another layer. More than two layers may be interconnected within a multi-layer network, through relationships between the design, flow, and attribute variables of their arcs. *Multi-layer network design* aims to simultaneously design all layer-specific networks to satisfy at minimum overall cost a given set of origin-destination demands. Extending the already-difficult network design problem class, multi-layer network design presents additional modelling and algorithmic challenges arising from the connectivity relations and requirements making up the problem settings. We recall and enhance the basic definitions and formulations, and introduce new ones for richer multi-layer networks, with more than two layers and connectivity relations involving several layers simultaneously. We illustrate these concepts through multi-layer service network design models for tactical planning of consolidation-based freight transportation carriers.

Keywords: Network Design, multilayered networks, consolidation-based transportation planning.

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* Corresponding author: teodorgabriel.crainic@cirrelt.net

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1 Introduction

Multilayered networks are used to represent complex natural, social, biological, and technological systems, where each layer stands for a particular component of the system, the components interacting in multiple types of relationships (Kivelä et al., 2014; Areta and Moreno, 2019). One finds a number of network types in the Operations Research literature corresponding to this broad definition, e.g., the *multi-echelon* (Cordeau et al., 2006), *multi-tier* (Crainic et al., 2004, 2009, 2021c), *multilevel* (Balakrishnan et al., 1994; Costa et al., 2011), and *hierarchical* (Obreque et al., 2010; Lin, 2010) networks. The network components within any one of these problem contexts and definitions correspond to particular networks making up a transportation, logistics, or telecommunication system, and interact through transfer links or nodes, providing the means to move flows over multi-component paths. Thus, for example, each component may correspond to the routes of a particular container-shipping navigation company in a regional or global intermodal study, transfers taking place in ports, or to the network of a particular public-transport mode in an urban transportation study, passengers transferring from one mode to another at common stations. It is noteworthy however that, in all these cases, the definitions of the links of a component do not depend on the definitions of the links of another component. A tramway link is not defined in terms of a path of bus or bicycle links, for example. Similarly, the routes between regional warehouses and city fulfillment terminals (first echelon) are not defined in terms of the route segments of delivery tours from the latter to the stores (second echelon).

The Operations Research *multi-layer network* term is generally associated to more complex problem settings, encountered in planning transportation and telecommunication systems, in which an arc in a given layer is defined with respect to a set of arcs in another layer, the arcs in the set often making up a path or a cycle. In freight railway tactical planning, for example, a block (group of cars handled together as a unit) is defined, for possible selection in a block layer, in terms of the path of train-service arcs which will transport it, if selected in the service layer (Chouman and Crainic, 2021; Zhu et al., 2014). Such interwoven definitions imply several connectivity relations and requirements in terms of both design (arc or node selection) and flow-distribution decisions, raising challenging network-design modelling and algorithmic issues.

Crainic et al. (2022) reviewed and synthesized the *Multi-layer Network Design* (*MLND*) literature, presenting a first version of a formal framework and taxonomy of the field. The authors focused on what may be described as the "basic" aspects of the multi-layer problem class, i.e., two layers and one-to-one connectivity requirements between two layers (even when more than two layers are considered), these requirements involving either the design or the flow decisions, but not both.

One continually witnesses, however, the emergence of studies addressing problems with more than two layers interacting in both design and flow decisions. The field of medium to long-time planning of consolidation-based freight transportation is particularly active in this respect, following the increasing interest in integrating the various aspects of the planning process within a comprehensive decision-support methodology. The simultaneous optimization of the service network, the demand-flow itineraries using this network, and the high-level management of resources supporting those services is a case in point.

We focus on these issues in this chapter. The goals are first, to recall and enhance the basic multi-layer definitions and formulations, second, to introduce the definitions and formulations for richer multi-layer networks, with more than two layers and connectivity relations involving several layers simultaneously, and, third, to illustrate these concepts in the context of multi-layer service network design models of consolidation-based freight transportation planning. Following a brief summary of the basic notation, definitions, and formulation of multi-layer network design making up Section 2, Sections 3, 4, and 5 address these three goals, respectively. We conclude in Section 6.

2 Multi-layer Network Design

We recall the basic notation, definitions, and formulation of multi-layer network design problems as formalized in Crainic et al. (2022).

A multi-layer network consists of several networks, one on each layer, coupled through *arc-definition* relations and *connectivity* settings and requirements. Inter-layer arcs complete the network, providing the means for the circulation of flows among the appropriate layers.

Inter-layer connectivity may involve two layers only or several. Most of the literature concerns the former case, as reviewed in Crainic et al. (2022). Hence, while the definitions in this section are general, we illustrate using two layers only.

Let $\mathcal{L}$ be the set of layers of multi-layer network $\mathcal{G} = (\mathcal{N}_l, \mathcal{A}_l) = \bigcup_{l \in \mathcal{L}} \{\mathcal{G}_l = (\mathcal{N}_l, \mathcal{A}_l)\}$, where $\mathcal{G}_l$ is the network on layer $l \in \mathcal{L}$, with $\mathcal{N}_l$ and $\mathcal{A}_l$ the corresponding sets of nodes and arcs. Some of the nodes in $\mathcal{N}_l, l \in \mathcal{L}$, may belong exclusively to layer l, while others may be shared with the layers involved in the connectivity requirements. For simplicity of presentation, but without loss of generality, let's assume that all the arcs in a set $\mathcal{A}_l, l \in \mathcal{L}$, are design arcs, i.e., that they must be selected in order to be included in the final network.

Let $l, l' \in \mathcal{L}$ be two layers of $\mathcal{G}$ coupled by an arc-definition specifying how an arc in layer l' is related to a subset of arcs in layer l. According to Crainic et al. (2022), we then say that l' is *supported* by l and that l is *supporting* l'. To illustrate, consider the 2-layer network in Figure 1, where arcs a, b, and c of the supported layer l' are defined by the sets of arcs (paths, actually) in the supporting layer l $(\beta, \delta, \epsilon)$, (α, ϵ), and $(\gamma, \delta, \epsilon)$, respectively.

The multi-layer network $\mathcal{G}$ is to be designed to satisfy the multicommodity, origin-

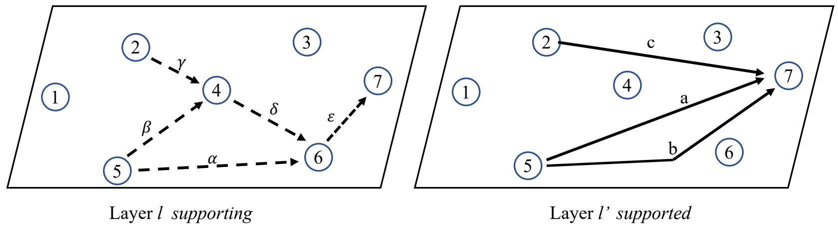

Figure 1: Illustration of arc-definition connectivity

destination (OD), demand $\mathcal{K} = \bigcup_{l \in \mathcal{L}} \mathcal{K}_l$. Notice that, most cases of interest either involve a single set of OD demands only, or one may easily transform the problem to a single-demand-set setting (e.g., when several demands of different product classes are defined between the same pair of origin-destination nodes, one duplicates these nodes and thus creates "new" OD commodities on the same layer Crainic et al., 2021a). Consequently, for simplicity of presentation but without loss of generality, we assume in the following that only one set of OD demands $\mathcal{K}$ is defined on a given layer $\varkappa \in \mathcal{L}$, each $k \in \mathcal{K}$ requiring the transportation of d^k units from its origin $O(k) \in \mathcal{N}_\varkappa$ to its destination $D(k) \in \mathcal{N}_\varkappa$ primarily through the arcs of $\mathcal{A}_\varkappa$. Notice that, the flows on layer $\varkappa$ may be projected on the layers associated with it through arc definitions. In the railway planning case invoked above, for example, flows defined and circulating on the block layer may be summed up to compute the total railcar flow on the train service layer. Sections 3 and 4 detail such connectivity settings and the associated constraints.

The *arc-definition relations* specify that an arc in a given layer is tightly related to (defined by) a set of arcs in at least another layer. The arcs on a defining layer are often, but not always, making up a path or a cycle. Turning again to the railway tactical planning example, a block arc is defined by a path of train-service arcs making up its route from origin to destination.

The *connectivity requirements* specify the degree and type of relations between the arc decision variables and attribute values of coupled (*supporting*, *supported*) layers, yielding constraints in the related MLND formulations (Sections 3 and 4).

The connectivity *degree* indicates whether two or more layers are involved and the direction of involvement:

***One-to-one*:** Two layers, one supporting and the other being supported (Figure 1); A pair of such (supporting, supported) layers in $\mathcal{G}$ is denoted (l, l'), $l, l' \in \mathcal{L}$;

***Many-to-one*:** Several, $m + 1$, layers involved; Represented by the couple $(\mathcal{L}(l'), l')$, it refers to the relations between $m > 1$ supporting layers, gathered in set $\mathcal{L}(l') \subset \mathcal{L}$, and a supported layer $l' \in \mathcal{L}$; The design of a train service (on a supported layer) which may be operated by engines of three different types, the activity of each type being represented on a different supporting layer, illustrates the case;

***One-to-many*:** Several, $m + 1$, layers involved; Represented by the couple $(l, \mathcal{L}(l))$, it refers to the relations between a supporting layer $l \in \mathcal{L}$ and $m > 1$ supported layers, gathered in set $\mathcal{L}(l) \subset \mathcal{L}$; The simultaneous design, on the same infrastructure, of a passenger-train service network and a freight-train service network (on two supported layers), which are to be operated by engines of a unique fleet, managed on a supporting layer, illustrates the case.

Let $C = \{(l, l') \cup (\mathcal{L}(l'), l') \cup (l, \mathcal{L}(l)),\ l, l' \in \mathcal{L}\}$ be the *connectivity-requirement* set of groups of *supporting* and *supported* layers in $\mathcal{G}$. We define three types of layer connectivity relations among the layer groups in C (Crainic et al., 2022, introduced the first two):

Design The selection of an arc in a layer requires the selection of all or some of its supporting arcs in another layer (at least one, for many-to-one case); Thus, to open arc c in the supported layer l' of Figure 1, one must open all the arcs defining it, i.e., γ, δ, ϵ, in the supporting layer l. Design connectivity is fundamental in the definition of multi-layer networks.

Flow The flow on an arc in a layer is a function of the flows on a set of arcs of at least another layer; Thus, for example, the flow on arc ϵ in layer l (Figure 1), supporting several arcs, a, b, and c, in the supported layer l', could be the sum of the flows on these three arcs.

Attribute Generalizing the flow type of connectivity, it concerns the relations between the attributes, e.g., cost, time, and capacity, of the links of the layers involved; Section 4.1 further investigates this connectivity type.

Let arcs $a \in \mathcal{A}_l, l \in \mathcal{L}$, be characterized by a fixed cost f_{al}, commodity-specific unit flow costs $c^k_{al}, k \in \mathcal{K}$, and flow capacity u_{al} (as usual in network design, commodity-specific capacities $u^k_{al}, k \in \mathcal{K}$, and associated constraints, may be defined; for simplicity of presentation but without loss of generality, we do not include those in the following).

A generic multicommodity, fixed-cost, capacitated multi-layer network design formulation may then be introduced (Crainic et al., 2022) with the following decision variable vectors:

Design $\mathbf{y} = [y_{al}] \in \mathbb{Y}$, where $y_{al} = 1$ if arc $a \in \mathcal{A}_l$ of layer l is selected, 0, otherwise; Alternatively, $y_{al} \in \mathbb{N}$ when the arc may be selected more than once (e.g., the departure frequency of a selected transportation service);

Flow $\mathbf{x} = [x^k_{al}] \in \mathbb{X}$ indicating the quantity of demand $k \in \mathcal{K}$ assigned to arc a of layer l; Depending upon the application, the flow variable may be continuous or integer, but always non negative.

Let $\mathcal{A}^+_l(i)$ and $\mathcal{A}^-_l(i)$ represent the sets of outgoing and incoming arcs of node $i \in \mathcal{N}_l$, and w^k_i equal 1 if $i = O(k)$, -1 if $i = D(k)$, and 0, otherwise, for all $i \in \mathcal{N}_l, k \in \mathcal{K}, l \in \mathcal{L}$. The formulation may then be written as

$$\min \sum_{l \in \mathcal{L}} \left\{ \sum_{a \in \mathcal{A}_l} f_{al} y_{al} + \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}_l} c^k_{al} x^k_{al} \right\} \tag{1}$$

$$\text{subject to} \sum_{a \in \mathcal{A}^+_l(i)} x^k_{al} - \sum_{a \in \mathcal{A}^-_l(i)} x^k_{al} = w^k_i \qquad \forall i \in \mathcal{N}_l,\ \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{2}$$

$$\sum_{k \in \mathcal{K}} x^k_{al} \leq u_{al} y_{al} \qquad \forall a \in \mathcal{A}_l, \tag{3}$$

$$(\mathbf{y}, \mathbf{x}) \in (\mathcal{Y}, \mathcal{X})_{ll'} \qquad \forall (l, l') \in C, \forall l \in \mathcal{L}, \tag{4}$$

$$\mathbf{y} \in \mathbb{Y}, \mathbf{x} \in \mathbb{X}. \tag{5}$$

The objective function (1) minimizes the total cost of selecting and using arcs on all the layers of the network. Constraints (2) are the classical *flow-conservation* equations ensuring that the demand flows are routed from their origins to their destinations in each layer, without gains or losses at intermediary nodes. Notice that, flow conservation has to be enforced on a layer only, usually the layer of definition, when the projections on the other layers are simple aggregation (Section 3). For example, when demand is defined on the supported layer (blocks in the railway planning case), flows on the supporting layer arcs (train services) equal the sum of the flows on the corresponding supported arcs (the blocks making up the train).

The aggregated *linking capacity* constraints (3) ensure that only selected, and paid for, arcs are used, enforcing the associated capacity restriction with respect to the total flow assigned to each one of them. Relations (5) define the domains of the decision variables. (Particular so-called network design side constraints, e.g., limiting the total budget or imposing topological design conditions on particular layers, such as the equality between the number of entering and exiting design arcs at nodes, are not discussed in this chapter as they are not related to the multi-layer nature of the problem; we refer the interested reader to Crainic et al., 2021a).

Relations (4) stand for the sets of constraints corresponding to the design, flow, or attribute connectivity requirements proper to the multi-layer network design application at hand. We discuss general constraint classes in Sections 3 and 4.

3 MLND Basic Connectivity Requirements and Constraints

The so-called basic connectivity definitions and requirements are found in 2-layer network design problem settings, which were the object of most of the early contributions. We focus in this section on design and flow-connectivity constraints (see Crainic et al., 2022, for the initial definitions and particular applications).

The set C then reduces to $\{(l, l'),\ l, l' \in \mathcal{L}\}$, that is, the set of (supporting, supported) pairs of layers in $\mathcal{G}$ involved in one-to-one connectivity requirements (the only type possible in this context). We use the notation α, β, etc. for arcs on a supporting layer $l \in C$ and a, b, etc. for arcs on a supported layer $l' \in C$. We then define the following sets:

$\mathcal{A}_{l'}(\alpha l)$ Set of *supported arcs* in layer l' by the supporting arc α in layer l, i.e., $\mathcal{A}_{l'}(\alpha l) = \{a \in \mathcal{A}_{l'} \mid a \text{ is supported by } \alpha\}$, $\mathcal{A}_{l'}(\alpha l) \subseteq \mathcal{A}_{l'}$;

$\mathcal{A}_l(al')$ Set of *supporting arcs* in layer l of the supported arc a in layer l', i.e., $\mathcal{A}_l(al') = \{\alpha \in \mathcal{A}_l \mid \alpha \text{ supports } a\}$, $\mathcal{A}_l(al') \subset \mathcal{A}_l$; This corresponds to the arc definition (without its topology) of a.

Design-connectivity constraints enforce existence relations between supported and supporting arcs.

Also called *all-design* linking constraints for supported arcs, the relation (6) states that all the supporting arcs (e.g., α and ϵ of layer l of Figure 1) must be selected in order for a supported arc (b of layer l') to be eligible for selection,

$$y_{al'} \leq y_{\alpha l}, \quad \forall \alpha \in \mathcal{A}_l(al'), \forall a \in \mathcal{A}_{l'}, \forall (l, l') \in C, \tag{6}$$

while *min-design* linking constraints (7) are introduced when a single supporting arc has to be selected only, in order for the supported arc to be eligible for selection,

$$y_{al'} \leq \sum_{\alpha \in \mathcal{A}_l(al')} y_{\alpha l}, \quad \forall a \in \mathcal{A}_{l'}, \forall (l, l') \in C. \tag{7}$$

Figure 2 illustrates fundamental occurrences of design-connectivity relations and the utilization of constraints (6) and (7). The lower-left side of the figure illustrates the case where arc a of supported layer l' is defined by a link, in other words, a single-link path $\{\alpha\}$, and a path, $\{\delta, \beta\}$, in the supporting layer l. Assuming link a may exist if either one of the two paths is selected, the design-connectivity is enforced through a type (7) constraint for the alternative definitions

$$y_{al'} \leq y_{\alpha l} + y_{\delta l} + y_{\beta l}, \tag{8}$$

and two type-(6) constraints to enforce the two-link path definition

$$\begin{aligned} y_{al'} &\leq y_{\delta l} \\ y_{al'} &\leq y_{\beta l}. \end{aligned} \tag{9}$$

Obviously, the last two constraints may be compressed into a single one when design variables are defined for path selection in the supporting layer.

The lower-right side of Figure 2 generalizes the previous case and illustrates the situation where a supporting arc or path defining an arc in a supported layer is part of one or several

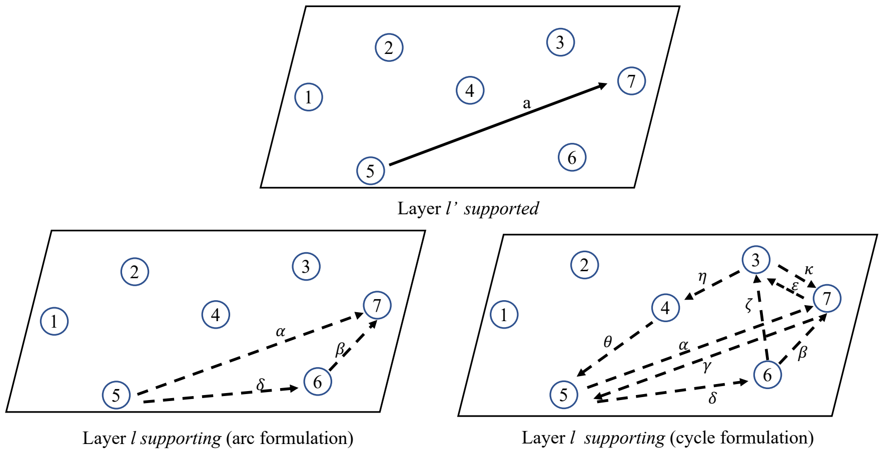

Figure 2: Illustration min-design linking constraints

larger network structures, e.g., cycles. Thus, we see arc α belonging to cycle $\{\alpha, \varepsilon, \eta, \varphi\}$, arc β being part of $\{\delta, \beta, \gamma\}$, while arcs δ and γ belong not only to the latter cycle but also to $\{\delta, \zeta, \kappa, \gamma\}$. Such a case is often encountered in service network design applications when any of a number of possible resources (generally operating cycling sequences of services) may be used to operate a service (Crainic and Hewitt, 2021). Notice that the constraints (8) and (9) are sufficient in this case as well, the cycle structures being ensured in the supporting layer, usually through cycle-selection (design) decision variables and constraints imposing that each arc belongs at most to a selected cycle only.

When a supported arc requires the support of a given number $n_{al'}$ of supporting arcs, the min-design linking constraints take the form

$$n_{al'} y_{al'} \leq \sum_{\alpha \in \mathcal{A}_l(al')} y_{\alpha l}, \quad \forall a \in \mathcal{A}_{l'}, \forall (l, l') \in C. \tag{10}$$

Design-connectivity issues from the point of view of the supporting layers impose particular constraints as well. Thus, the *min-design* linking constraints (11), or (12), enforce the requirement that selecting an arc supporting one or several arcs in a different layer implies that at least one, or $n_{\alpha l} \leq |\mathcal{A}_{l'}(\alpha(l))|$, respectively, of those must be selected as well:

$$y_{\alpha l} \leq \sum_{a \in \mathcal{A}_{l'}(\alpha l)} y_{al'}, \quad \forall \alpha \in \mathcal{A}_l, \forall (l, l') \in C, \tag{11}$$

$$n_{\alpha l} y_{\alpha l} \leq \sum_{a \in \mathcal{A}_{l'}(\alpha l)} y_{al'}, \quad \forall \alpha \in \mathcal{A}_l, \forall (l, l') \in C. \tag{12}$$

Note that, min-design linking constraints are not always present, at least, not in transportation-planning applications. Thus, for example, a link on a resource layer could not be supporting any link in a supported service network design layer when it corresponds to an empty move to reposition the resource as needed.

Finally, it is noteworthy that design requirements between two layers may imply not only the existence of one given the other, but also the amplitude of the relation. This characteristic, proper to multi-layer networks, is captured through the *design capacity* $v_{\alpha l}$ of a supporting arc $\alpha \in \mathcal{A}_l$, limiting the design-related measures of its supported arcs. In its simpler expression, it limits the number of supported arcs, which may be selected simultaneously, by imposing the *design-capacity* constraints

$$\sum_{a \in \mathcal{A}_{l'}(\alpha l)} y_{al'} \leq v_{\alpha l} y_{\alpha l}, \quad \forall \alpha \in \mathcal{A}_l, \forall (l, l') \in C. \tag{13}$$

Thus, setting $v_{\alpha l} = 2$ to the arcs in layer l of Figure 1 and imposing constraints (13) means that at maximum two of the three arcs a, b, and c on layer l' may be selected. Such a design-capacity constraint on the attributes of a given train service (supporting layer) in a freight railway tactical planning application could thus limit the total number of different blocs (supported layers) assigned to it.

Flow-connectivity refers to relations between the flows on the arcs of a (supporting, supported) layer pair.

Flow-accumulation constraints address the case when the demand $\mathcal{K}$ is defined on the supported layer, i.e., $\varkappa = l'$, and state that the commodity flow on a supporting arc equals the sum of that commodity flows on all its supported arcs:

$$x^k_{\alpha l} = \sum_{a \in \mathcal{A}_{l'}(\alpha l)} x^k_{al'}, \quad \forall \alpha \in \mathcal{A}_l, \ \forall k \in \mathcal{K}, \forall (l, l') \in C. \tag{14}$$

Recall that, according to the particular application, demand flows may or may not be split ("bifurcated" in the telecommunication literature), that is, each commodity may either flow on several paths between its origin and destination, or must move through a single path, respectively. When flows cannot be split on the supported layer, the supporting layer inherits the property through the flow-accumulation constraints.

Notice that, when demand is measured in the same units as the capacity of the supporting arc, e.g., number of vehicles or tons, combining the flow-accumulation relations (14) and the linking-capacity constraints (3) enforces the requirement that the sum of the flows on all supported arcs does not violate the supporting-arc capacity.

When this is not the case, that is, when the demand unit of measure, e.g., the container or railcar, is different from the capacity measure of the supporting arc, e.g., length (very frequent in railway planning), explicit *flow-attribute linking-capacity* constraints must be added (complementary to the linking-capacity constraints (3)). Let φ_k be the attribute of commodity

$k \in \mathcal{K}$ corresponding to the measure of capacity $u_{\alpha l}$ of arcs in a supporting layer $l \in \mathcal{L}$. The flow-attribute linking-capacity constraints may then be written as:

$$\sum_{a \in \mathcal{A}_{l'}(\alpha l)} \sum_{k \in \mathcal{K}} \varphi_k x^k_{al'} \leq u_{\alpha l} y_{\alpha l}, \quad \forall \alpha \in \mathcal{A}_l, \forall (l, l') \in C. \tag{15}$$

4 Designing Richer Multilayer Networks

Rich MLND problems involve different, more complex arc-definition relations and connectivity requirements compared to the basic problem setting discussed in the previous section. Connectivity requirements among the attributes of the arcs involved in a (supporting, supported) layer relation are discussed in Section 4.1. Arc-definition relations involving more than two layers, and the associated connectivity requirements, are the topic of Section 4.2. It is noteworthy that rich MLND applications often include several arc definitions and connectivity requirements in the same formulation as discussed in Section 5.

4.1 Attribute Connectivity

Network links may display a wide variety of attributes. Commodity flows together with fixed and unit commodity transportation costs are among the most usual arc attributes, where cost may indeed be a monetary value, but may also be distance or time. It is not unusual to have these three cost attributes simultaneously within an application, while other "cost" measures may also be found, somewhat less frequently (e.g., the touristic interest of the urban zone represented by the arc), however. *Capacity* is another widely encountered link attribute limiting the flow of commodities, total, commodity-specific, or both, one may assign to the arc.

Section 3 addressed inter-layer relations in terms of flows. What about relations among the other attributes, including costs and the "physical" attributes mentioned above? We aim to start answering this question in this section. The (supporting, supported) arcs of Figure 3 serve as illustration. The supporting layer l has four arcs, which support four arcs in layer l', such that $\mathcal{A}_l(a_1) = \{\alpha_1, \alpha_2\}$, $\mathcal{A}_l(a_2) = \{\alpha_2, \alpha_3, \alpha_4\}$, $\mathcal{A}_l(a_3) = \{\alpha_3\}$, and $\mathcal{A}_l(a_4) = \{\alpha_4\}$.

We first notice that not all attributes of interacting-layer arcs are always related. Thus, for example, the fixed costs of supported arcs are often defined independently of those of the supporting ones. Second, many such relations are of a definitional and additive nature. *Distance* generally belongs to this class of attributes, as do unit commodity costs, *time*-related measures, e.g., duration, transport, delay, and waiting times, and most node measures.

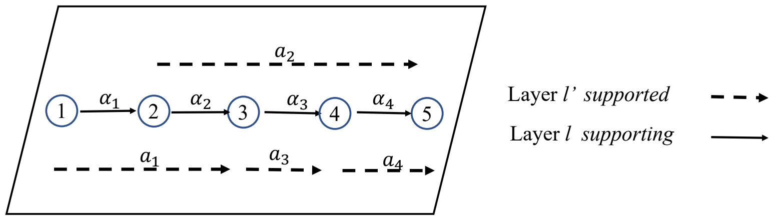

Figure 3: Attribute connectivity illustration

We thus introduce the *supported arc additive-attribute* definition stating that the value $F_{\varrho}(\cdot)$ of any additive attribute ϱ of a supported-layer arc a is given by the sum of the values of the corresponding attributes of the supporting arcs, i.e.,

$$F_{\varrho}(al') = \sum_{\alpha \in \mathcal{A}_l(al')} F_{\varrho}(\alpha l), \;\; \forall \varrho, \; \forall a \in \mathcal{A}_{l'}. \tag{16}$$

Definition (16) must be verified for the relevant attributes when all the potential arcs on the supporting and supported layers are given as input in the problem setting, e.g., when a potential block in a railway planning application is defined *a priori* as moving on a given sequence of potential train services. The situation is different when the arcs of the supported layer are to be dynamically generated during problem solving, in which case, constraints of the (16) type have to be included in the variable-generation model or procedure.

A different case is observed when addressing arc capacity, as feasibility issues have to be addressed. Similar to the discussion above, when the potential supported arcs are pre-defined, verifying that their capacities are not higher than the lowest capacity among the respective supporting arcs, i.e.,

$$u_{al'} \leq \min_{\alpha \in \mathcal{A}_l(al')} \{u_{\alpha l}\}, \; \forall a \in \mathcal{A}_{l'}, \tag{17}$$

guarantees feasibility, together with intra-layer linking (3), the (supporting, supported) all-design design-connectivity (6), and the flow-connectivity, e.g., (14), constraints of the MLND model.

The situation is less straightforward when the arc capacities in the supported layer are not known *a priori*, rather belonging to the set of decisions characterizing the problem setting. In such cases, the capacity of a supported arc a in layer l' becomes a decision variable, its "optimal" value to be determined by the interplay among the design decisions in both layers, the capacity of each supporting arc $\alpha \in \mathcal{A}_l(al'), (l, l') \in C$, and the allocation of the latter to all the arcs it supports, that is, $a \in \mathcal{A}_{l'}(\alpha l)$. The *multi-layer network design problem with capacity decisions* then aims to determine simultaneously the selection of the design arcs on all layers, the arc capacities on the supported layer, and the distribution of demand flows over the resulting multi-layer network, to minimize the total generalized cost of the system.

To illustrate, consider the case of railway tactical planning, where a train service of known capacity (length or number of cars, usually based on demand as well as physical and operational policies) may haul one or several of a given set of blocks. While each of the latter has a maximum capacity (based, e.g., on the physical characteristics of the terminal where it is built), its actual capacity cannot be higher than the residual capacity of the service once the capacities of the other blocks selected to be moved by the service have been determined. The tactical plan should therefore provide not only the selected service network and the composition of each service in terms of the blocks it hauls, but also indications regarding the maximum volume of demand which may be assigned to each block. The latter information is important during operations when the actual blocks and trains are built, particularly when the blocks are not built within the railway's own terminals (e.g., in the port terminal by port crews working with guidelines provided by the railway). One has, therefore, to allocate the capacity of the service, the supporting arc, to blocks it hauls, the supported arcs, and this has to be done for all supporting and supported arcs simultaneously with their own selection.

Define the decision variables $\vartheta_{al'}, \forall a \in \mathcal{A}'$, the capacity of arc a in the supported layer l', and let $c_{al'}$ be the unit cost of assigning capacity to arc a. The range of $\vartheta_{al'}$:

$$0 \leq \vartheta_{al'} \leq u_{al'} = \min_{\alpha \in \mathcal{A}_l(al')} \{u_{\alpha l}\}. \tag{18}$$

When $|\mathcal{A}_{l'}(\alpha l)| = 1$, for $\alpha \in \mathcal{A}_l(al')$, i.e., when there are no other arcs in layer l' competing for the capacity of the supporting arc α, constraints (6) and the feasibility range (18) are sufficient.

Shared-capacity requirements and constraints are involved when, on the contrary, the supporting-arc capacity must be allocated among several supported arcs, i.e., when $|\mathcal{A}_{l'}(\alpha l)| > 1$, each allocation being bounded from above by the *residual* capacity of the supporting arc given the allocations to the other supported arcs.

From the point of view of the supported arcs, shared-capacity requirements may be represented by:

$$\vartheta_{al'} \leq \min_{\alpha \in \mathcal{A}_l(al')} \left\{ u_{\alpha l} y_{\alpha l} - \sum\nolimits_{b \in \mathcal{A}_{al'}(\alpha l) \setminus \{a\}} \vartheta_{bl'} y_{bl'} \right\}, \forall a \in \mathcal{A}_{l'}, \forall (l, l') \in C, \tag{19}$$

where $\mathcal{A}_{al'}(\alpha l) \subseteq \mathcal{A}_{l'}(\alpha l)$ stands for the set of arcs supported by α which includes a. Recalling that, the (supporting, supported) all-design design-connectivity (6) constraints also apply in this case, the previous expression becomes

$$\vartheta_{al'} \leq \min_{\alpha \in \mathcal{A}_l(al')} \left\{ u_{\alpha l} - \sum\nolimits_{b \in \mathcal{A}_{al'}(\alpha l) \setminus \{a\}} \vartheta_{bl'} y_{bl'} \right\}, \ \forall a \in \mathcal{A}_{l'}, \ \forall (l, l') \in C. \tag{20}$$

Indeed, when the supporting arc is not selected, i.e., $y_{\alpha l} = 0$, all the supported arcs are not selected either, i.e., $y_{al'} = y_{bl'} = 0$, and the corresponding arc flows are also zero. Any capacity distribution satisfying relations (18) would do in this case, in particular that of an initial solution setting $\vartheta_{al'} = 0$. Moreover, when $c_{al'} > 0$, any solution with a positive capacity on such an arc will have a higher cost, hence not optimal, compared to a solution with zero capacity. Hence, to illustrate, the capacities of the four supported arcs in Figure 3 would have to satisfy the system of non-linear relations:

$$
\begin{aligned}
\vartheta_{a_1} &\leq u_{\alpha_1} \\
\vartheta_{a_1} &\leq u_{\alpha_2} \quad -\vartheta_{a_2} y_{a_2} \\
\vartheta_{a_2} &\leq u_{\alpha_2} \quad -\vartheta_{a_1} y_{a_1} \\
\vartheta_{a_2} &\leq u_{\alpha_3} \quad -\vartheta_{a_3} y_{a_3} \\
\vartheta_{a_2} &\leq u_{\alpha_4} \quad -\vartheta_{a_4} y_{a_4} \\
\vartheta_{a_3} &\leq u_{\alpha_3} \quad -\vartheta_{a_2} y_{a_2} \\
\vartheta_{a_4} &\leq u_{\alpha_4} \quad -\vartheta_{a_2} y_{a_2}.
\end{aligned}
$$

One can also examine the shared-capacity requirements from the point of view of the supporting-layer arcs, which yields relations:

$$\sum\nolimits_{a \in \mathcal{A}_{l'}(\alpha l)} \vartheta_{al'} y_{al'} \leq u_{\alpha l}, \ \forall \alpha \in \mathcal{A}_l, \ \forall (l, l') \in C, \tag{21}$$

where the $y_{\alpha l}$ design variables on the supporting layer are dropped given the all-design design-connectivity relations. A two-layer network design with capacity decisions formulation may then be written (recall that, $\mathcal{L} = \{l, l'\}$ with $C = \{(l, l')\}$ the couple of (supporting, supported) layers):

$$\min \sum_{\alpha \in \mathcal{A}_l} f_{\alpha l} y_{\alpha l} + \sum_{a \in \mathcal{A}_{l'}} (f_{al'} y_{al'} + c_{al'} \vartheta_{al'}) + \sum_{k \in \mathcal{K}} \sum_{\varkappa \in \{l, l'\}} \sum_{\alpha \in \mathcal{A}_\varkappa} c^k_{\alpha\varkappa} x^k_{\alpha\varkappa} \tag{22}$$

$$\text{subject to} \sum_{a \in \mathcal{A}^+_{l'}(i)} x^k_{al'} - \sum_{a \in \mathcal{A}^-_{l'}(i)} x^k_{al'} = w^k_i, \ \forall i \in \mathcal{N}_{l'}, \qquad \forall k \in \mathcal{K} \tag{23}$$

$$\sum_{k \in \mathcal{K}} x^k_{al'} \leq \vartheta_{al'} y_{al'}, \qquad \forall a \in \mathcal{A}_{l'} \tag{24}$$

$$\sum_{k \in \mathcal{K}} x^k_{\alpha l} = \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}_{l'}(\alpha l)} x^k_{al'} \leq u_{\alpha l} y_{\alpha l}, \ \ \forall \alpha \in \mathcal{A}_l, \tag{25}$$

$$\sum_{a \in \mathcal{A}_{l'}(\alpha l)} \vartheta_{al'} y_{al'} \leq u_{\alpha l} y_{\alpha l}, \ \ \forall \alpha \in \mathcal{A}_l, \tag{26}$$

$$y_{al'} \leq y_{\alpha l}, \ \ \forall a \in \mathcal{A}_{l'}, \ \forall \alpha \in \mathcal{A}_l(al'), \tag{27}$$

$$0 \leq \vartheta_{al'} \leq u_{al'}, \ \ \forall a \in \mathcal{A}_{l'}, \tag{28}$$

$$\mathbf{y} \in \mathbb{Y}, \mathbf{x} \in \mathbb{X}. \tag{29}$$

A linear version of this formulation is obtained by, first, using (24) to replace (26) with

$$\sum_{a \in \mathcal{A}_{l'}(\alpha l)} \sum_{k \in \mathcal{K}} x^k_{al'} \leq \sum_{a \in \mathcal{A}_{l'}(\alpha l)} \vartheta_{al'} y_{al'} \leq u_{\alpha l} y_{\alpha l}, \forall \alpha \in \mathcal{A}_l, \tag{30}$$

and, second, by replacing constraints (24) and (28) with the sets of constraints

$$\sum_{k \in \mathcal{K}} x^k_{al'} \leq \vartheta_{al'}, \quad \forall a \in \mathcal{A}_{l'}, \tag{31}$$

$$\vartheta_{al'} \leq u_{al'} y_{al'}, \quad \forall a \in \mathcal{A}_{l'}, \tag{32}$$

$$\vartheta_{al'} \geq 0, \qquad \forall a \in \mathcal{A}_{l'}. \tag{33}$$

Rouhani et al. (2023) further detail these issues and present Benders decomposition solution methods.

4.2 Multi-layer Connectivity

As briefly mentioned in Crainic et al. (2022), multi-layer network-design models with more than two design layers are increasingly proposed when addressing the planning and management of complex systems in as a comprehensive way as possible. This trend is particularly observed in consolidation-based freight transportation planning (Section 5).

The concepts, notation, and definitions presented earlier in this section certainly also apply to L-layer networks with $L > 2$, in particular the one-to-one connectivity discussed in Sections 3 and 4.1. On the other hand, inter-layer relations may involve more than two layers as, for example, when a transportation service (supported layer) may be operated by one of two (or more) different resource types (the supporting layers) or even a combination of those, as discussed in more detail in Section 5. We therefore generalize the previous definitions and formulations to many-to-one and one-to-many connectivity relations in L-layer networks with $|\mathcal{L}| > 2$.

Three general *many-to-one design-connectivity* classes may be encountered when the arcs of a supported layer are defined in terms of several supporting layers: exclusive, required, and complementary.

The arc-definitions of an arc on a supported layer, $l' \in \mathcal{L}$, in terms of the arcs of the supporting layers $l \in \mathcal{L}(l')$, together with the design-connectivity requirements, are said to be *exclusive* when at most one of these definitions may be selected. Hence, for example, only one of a given number of ship types (supporting layers) may be assigned to a particular navigation line/service (supported layer).

Additional decision variables must be added to the MLND formulations to capture such exclusive design-assignment decisions. Let $y_{al'l} = 1$ if the $\mathcal{A}_l(al'), l \in \mathcal{L}(l')$, definition of arc $a \in \mathcal{A}_{l'}$ is selected, 0, otherwise. The *design-exclusive-assignment* arc-definition equation (34), together with constraints (6), (7), or (10) written for the new $y_{al'l}$ variables, enforce the exclusivity of the connectivity relations,

$$y_{al'} = \sum_{l \in \mathcal{L}(l')} y_{al'l}, \quad \forall a \in \mathcal{A}_{l'}, \forall (\mathcal{L}(l'), l') \in \mathcal{C}. \tag{34}$$

The supporting-supported layer relation is *required* when at least an arc must be selected on each of the supporting layers in $\mathcal{L}(l')$ for the supported arc l' to be selected. Thus, for example, both traction-power units, e.g., locomotives or tractors, and crews are required to operate rail or road freight transportation services, respectively. This translates into

$$n_{al'l}y_{al'} = y_{\alpha l}, \;\; \forall \alpha \in \mathcal{A}_l(al'), \forall(\mathcal{L}(l'), l') \in C, \tag{35}$$

where, $n_{al'l}$ generalizes the definition of Section 3, to indicate the required number of arcs in the supporting layer l, assuming of course that $|\mathcal{A}_l(al')| > 1$.

Notice that, this formulation assumes binary arc-selection decision variables in layer l, which, when $n_{al'l} > 1$, implies parallel supporting arcs in layer l, as illustrated in Figure 2. Alternatively, non-negative integer values may be allowed for the arc-design variables in the supporting layer l, modelling, for example, the number of resource units moving on the link. The required design-connectivity relations may then be modelled as constraints (36), together with the *design-utilization* constraints (37) indicating how the intensity on the supporting arc is shared among the supported ones (design-capacity constraints (13) may complete the model):

$$n_{al'l}y_{al'} \leq y_{\alpha l}, \;\; \forall \alpha \in \mathcal{A}_l(al'), \forall(\mathcal{L}(l'), l') \in C, \tag{36}$$

$$y_{\alpha l} = \sum_{l' \in \mathcal{L}(l)} \sum_{a \in \mathcal{A}_{l'}(\alpha l)} n_{al'l}y_{al'}, \;\; \forall \alpha \in \mathcal{A}_l, \forall l \in C. \tag{37}$$

Complementary, or additive (Crainic et al., 2022), arc-definitions and design-connectivity requirements generally stand for the possibility to select more than one definition for an arc on a supported layer $l' \in \mathcal{L}$, among its supporting layers $l \in \mathcal{L}(l')$. Hence, for example, a freight train service may be defined in terms of power units of, say, three types of locomotives, yielding three layers similar to the one illustrated in Figure 2. This translates into *design-complement-assignment* arc-definition equation (38):

$$y_{al'} \leq \sum_{l \in \mathcal{L}(l')} y_{al'l}, \;\; \forall a \in \mathcal{A}_{l'}, \forall(\mathcal{L}(l'), l') \in C. \tag{38}$$

It seems clear, however, that these constraints would not capture the actual setting in transportation applications, as the same service cannot be operated simultaneously with different definitions. In such cases, complementary design connectivity also models the requirements in terms of how to combine the selected definitions to achieve a certain level of particular attributes of the supported arc, e.g., a service requiring a certain minimal power, which can be obtained through different combinations of resource types.

Let $\varphi_{al'}$ be the intensity of some specific attribute (power in the previous example) the arc a on layer $l' \in \mathcal{L}$ must achieve through a combination of arcs on a subset of its supporting layers $l \in \mathcal{L}(l')$, and let $\varphi_{ll'}$ be the value of the corresponding attribute on an arc $\alpha \in \mathcal{A}_l(al')$

(these values may be different for different (l, l') pairs). The *design-complement* constraints then become

$$\varphi_{al'} y_{al'} \leq \sum_{l \in \mathcal{L}(l')} \varphi_{ll'} y_{al'l}, \;\; \forall a \in \mathcal{A}_{l'}, \forall(\mathcal{L}(l'), l') \in C. \tag{39}$$

The $y_{al'l}$ assignment decision variables are defined as non-negative integers when several "units" of supporting arcs have to be assigned to a supported arc in order to achieve the required attribute value.

The supporting-layer-specific design-connectivity requirements and constraints (11) - (13), defined for the one-to-one connectivity degree class (Section 3), apply straightforwardly to both many-to-one and one-to-many classes. Moreover, in the latter case, one can generalize them to 1) force that at least a certain number of supported links be selected in order to select the supporting link, constraints (40) or, 2), limit the number of supported links one may select, constraints (41):

$$n_{\alpha l} y_{\alpha l} \leq \sum_{l' \in \mathcal{L}(l)} \sum_{a \in \mathcal{A}_{l'}(\alpha l)} y_{al'}, \;\; \forall \alpha \in \mathcal{A}_l, \forall(l, \mathcal{L}(l)) \in C, \tag{40}$$

$$\sum_{l' \in \mathcal{L}(l)} \sum_{a \in \mathcal{A}_{l'}(\alpha l)} y_{al'} \leq v_{\alpha l} y_{\alpha l}, \;\; \forall \alpha \in \mathcal{A}_l, \forall(l, \mathcal{L}(l)) \in C. \tag{41}$$

Similar generalizations apply to the *flow-connectivity* relations and constraints (14), yielding the *total-flow-accumulation* constraints (42). One may also define, when relevant, layer-specific flow-accumulation decision variables, $x^{kl'}_{\alpha l} \geq 0, \alpha \in \mathcal{A}_l, k \in \mathcal{K}, (l, \mathcal{L}(l)) \in C$, with $x^k_{\alpha l} = \sum_{l' \in \mathcal{L}(l)} x^{kl'}_{\alpha l}$. The *layer-specific flow-accumulation* constraints then become equations (43). Finally, flow-attribute capacity connectivity may also be generalized, yielding constraints (44).

$$x^k_{\alpha l} = \sum_{l' \in \mathcal{L}(l)} \sum_{a \in \mathcal{A}_{l'}(\alpha l)} x^k_{al'}, \;\; \forall \alpha \in \mathcal{A}_l, \; \forall k \in \mathcal{K}, \forall(l, \mathcal{L}(l)) \in C, \tag{42}$$

$$x^{kl'}_{\alpha l} = \sum_{a \in \mathcal{A}_{l'}(\alpha l)} x^k_{al'}, \;\; \forall \alpha \in \mathcal{A}_l, \; \forall k \in \mathcal{K}, \forall(l, \mathcal{L}(l)) \in C, \tag{43}$$

$$\sum_{l' \in \mathcal{L}(l)} \sum_{a \in \mathcal{A}_{l'}(\alpha l)} \sum_{k \in \mathcal{K}} \varphi_k x^k_{al'} \leq u_{\alpha l} y_{\alpha l}, \;\; \forall \alpha \in \mathcal{A}_l, \forall(l, \mathcal{L}(l)) \in C. \tag{44}$$

5 Multi-layer Network Design for Consolidation-based Freight Transportation Planning

From its first appearance in Western vocabulary in the XVth century, *consolidation*, in the broadest sense of the term, means to join together several diverse things into one whole and, thus, to unite and strengthen that whole (Merriam-Webster, 2023). The concept and process of consolidation is used extensively in a large gamut of human activity fields, particularly in transportation where it refers to grouping together freight loads, vehicles, or people for economically and operationally efficient transport for the entire, or part of, the journey between their respective origins and destinations. Consolidation is performed by passenger or freight transport companies and systems, commonly identified as *carriers* (Crainic and Laporte, 1997). Public-transport carriers, operating urban (bus, trolleybus, light rail, collective taxi, etc.) or inter-urban (coach, rail, airplane, ship, etc.) networks, consolidate passengers who do not desire or cannot move by a dedicated vehicle (Mauttone et al., 2021).

Less-than-truckload (LTL) motor carriers, railroads, ocean/maritime liner navigation companies, land- and water (coastal, river, etc)-based intermodal carriers, postal and small-package transportation companies perform consolidation-based services for freight (Crainic, 2003; Crainic and Kim, 2007; Crainic et al., 2021b; Crainic and Hewitt, 2021). Consolidation is also at the heart of innovative transportation-system business and operation models introduced for urban, e.g., City Logistics (Crainic et al., 2023a,b) and interurban, e.g., Physical Internet (Ballot et al., 2014), Synchromodality (Ambra et al., 2021; Giusti et al., 2019), and multi-stakeholder (Taherkhani et al., 2022) settings.

Consolidation offers the means to lower unit-transportation costs and raise service quality, through economies-of-scale of loading large groups of loads on large and efficient vehicles (or convoys) travelling frequently. For shippers with relatively small loads (or value), it means avoiding paying high fees for small vehicles or excess waiting for a sufficiently large volume of freight to warrant a larger vehicle. The downside of these benefits is that carrier consolidated services cannot be tailored-made for each individual customer request for transportation. Consolidation-based services must rather be planned to address the potential demand of as many customers as possible, in a manner to satisfy the closest possible their requests, preferences, and expectations. The goal for consolidation-based freight carriers is to be profitable while achieving these demand-service objectives and, thus, to best use their material and human resources, and operate at the lowest possible cost.

Setting up such a service network is generally part of tactical, medium-term, planning, addressed in most cases through *Service Network Design*, *SND*, methodology. Crainic et al. (2021a) presents state-of-the-art literature and methodology syntheses and reviews for the general SND problem (Crainic and Hewitt, 2021) and applications, including rail (Chouman and Crainic, 2021), motor-carrier interurban road transport (Bakir et al., 2021), liner-shipping navigation (Christiansen et al., 2021), and City Logistics (Crainic et al., 2021c).

Closely related to network design (Magnanti and Wong, 1984; Crainic et al., 2021a), SND presents particular characteristics, stemming from the various applications and the increasingly major trend of integrating into the same model and decision process several system aspects and related decisions. The simultaneous optimization of the service network, with the corresponding demand itineraries, and the high-level management of resources supporting those services is a case in point. Such trends yield multi-layer network design problem settings and formulations, which inspired quite a number of the generic developments of the previous sections.

We discuss the main classes of SND problem settings (following, for the most part, the vocabulary and notation of Crainic and Hewitt, 2021) and multi-layer network design formulations. We identify common characteristics, review briefly the literature, and discuss challenges and possible future developments. The fundamental concepts and definitions are briefly recalled in Section 5.1. Two and L-layer SND problem settings and formulations are addressed in Sections 5.2 and 5.3, respectively.

5.1 Single-layer Service Network Design

Consolidation-based carriers operate on physical networks made up of terminals (nodes) connected by physical, e.g., roads and rail tracks, or conceptual, e.g., sea and air links, arcs. Terminals are facilities where most of the demand originating in nearby regions is brought in to be consolidated (sorted, grouped, loaded into vehicles) before transport, and where the demand flows terminating their journeys are processed (unloaded, deconsolidated) before distribution to the final destinations. Rail stations and yards, LTL terminals, deep-sea and river/canal ports, in-land intermodal platforms, and airports are examples of terminals in consolidation-based networks. Some of these terminals, the so-called *hubs*, play a particularly important role in such systems, by structuring the flows for long-haul transport to take full advantage of the economies of scale of consolidation. Most terminals in a given region are connected to such a large terminal. The loads originating herein, most of which cannot be economically shipped directly, are then sent to that hub to be classified (sorted) and consolidated into larger flows, which are routed to other hubs by high-frequency, high-capacity services. Loads may thus go through more than one intermediary hub before reaching their regional destination terminal, being either transferred from one service to another or undergoing re-classification and re-consolidation.

For network-planning purposes, the multicommodity *demand* for transportation is defined between pairs of terminals in the physical network. Besides the origin and destination terminals, each commodity is also characterized by its amplitude (e.g., quantity, weight, or volume), unit revenue or transportation cost (or both), time-related requirements, i.e., availability date (and time) at the origin terminal and due date or time interval of delivery at the destination, as well as, possibly, various service requirements, e.g., vehicle type, handling conditions, etc.

Carriers respond to demand by offering a network of more or less scheduled services between their terminals. A *service* follows a route through the physical network moving either directly between its origin and destination, or stopping at intermediary terminals to drop and pick up loads and, eventually, vehicles when convoys are being operated (e.g., car and blocks for railroads and trailers for LTL motor carriers operating multi-trailer road trains). The set of services the carrier selects to operate makes up the *service network*. *Itineraries* are then defined for each individual origin-destination (OD) demand, specifying how it is moved through the service network, i.e., the sequence of services and the operations at inter-service terminals, transfer or classification and consolidation.

Tactical planning addresses the issue of building an operationally and economically efficient transportation plan, that is, the service network and schedule to satisfy the contemplated demand. The plan is generally built for a medium-term planning horizon, the so-called season. The carrier determines the season's duration, from one to six months, more rarely a year, depending on the transportation mode(s) involved. The goal is a "stable" operation context under the tactical plan, in terms of environment (regulatory, economic, seasonal, etc.), available resources, and *regular* demand, that is, demand requiring service at regular intervals (e.g., daily or weekly), at some expected (or contracted) time, with a certain predicted (or contracted) amplitude. This regularity is inherited by the transportation plan, which includes regular services for a given *schedule length*, e.g., a day or a week, defined relative to the regularity of demand. The plan is then to be repetitively applied. schedule-length period after schedule-length period, for the duration of the season.

The basic, *static*, single-layer Service Network Design problem setting involves selecting services, out of a set of potential services, to satisfy a set of OD demands, assuming the regularity and stability mentioned previously. Each individual demand is defined by its origin and destination nodes in the physical network, as well as the quantity of freight to move between these. The nodes of the SND network correspond to the nodes of the physical network, while the arcs correspond to the considered services among which the selection has to be made. When services include intermediary stops, each service leg, that is, the path in the physical network connecting two consecutive terminals on the route of the service, yields a particular arc. The selection, and the cost it incurs, are still per service, however, not per leg. Arcs (services) are characterized by the fixed selection (activation) cost, a unit flow transportation cost, as well as, potentially, commodity-specific ones, and a capacity, possibly leg specific. The SND network may include parallel arcs representing different services or service legs which could run between the same pair of terminals.

Two sets of decision variables are defined, design and flow. *Design*, or *service-selection*, variables are binary in most papers in the literature, but may also take non-negative integer values to decide the service frequency, that is, the number of times it will be repeated within the schedule length. The continuous or integer-valued *flow* variables determine the quantity of each demand moved on each arc of the network. They become binary, selecting whether the arc belongs to the itinerary of the demand or not, when demand cannot be split. The objective function of the SND formulation minimizes the total system cost, computed as the sum of the total design cost for the services selected and the total flow-distribution cost. The formulation includes flow conservation constraints for each demand, design-flow linking and capacity constraints for each arc and demand, plus possible constraints modelling particular system requirements (e.g., terminal capacity).

The notation and formulation of this single-layer problem are generally very similar to that of the classic network design problem and are the same, without the layer-specific notation (indexing) and constraints, to that of Section 2 and formulation (1) - (3), plus constraints (5) standing for system requirements and the decision-variable types.

Single-layer *time-dependent* problem settings and formulations arise when time-requirements, often availability at origin and due-date at destination, characterize demand. Services are then also defined by time characteristics, generally taking the form of a *schedule* specifying the departure and arrival times at each terminal on their routes (the degree of precision of the schedule may vary with the particular application). A so-called *time-space network* is the widely-used modelling device in such cases, being obtained by, implicitly or explicitly, extending the physical network along the time dimension with a given granularity partitioning the schedule length into periods. Demands and potential services are then represented on the time-space network, their respective origins, destinations, and intermediate service stops being defined as the copy of the respective physical node at the appropriate time moment. The moving arcs of the time-space network are the service legs operating in space and time, while holding arcs connect two consecutive copies of the physical nodes.

The single-layer SND defined on such a time-space network is generally named *Scheduled Service Network Design*, *SSND*, problem setting and formulation. It is noticeable that, once the corresponding definitions of the system parameters and decision variables are adjusted for the time dimension, the model is the same, mutatis mutandis, as that mentioned above for the SND (with a much higher computational complexity and difficulty). See Crainic and Hewitt (2021) for richer SSND settings, which we also discuss in the following.

5.2 Two-layer service network design

The previous SND formulations are single-layer. Two-layer formulations are found when the service involves convoys, that is, more than a single vehicle and particular vehicle organization structures. They are also found when resource management concerns are explicitly integrated into the tactical planning process.

The first case is typical of railway freight transportation where railcars are consolidated into *blocks*, which are then assembled into and moved between their respective origins and destinations by *train services* (Chouman and Crainic, 2021). This *classification* process aims to reduce the cost of handling individual railcars at intermediate terminals, and is particularly important when trains are long and travel long distances. In such a setting, railcars with possibly different origins and destinations are classified (sorted) at some terminal (origin or intermediary) and consolidated into a block, that is, into a group of cars which travel together for part of their journeys (until the destination of the block, where it is disbanded and the cars not at their destination terminal undergo classification again), being handled as a single unit when transferred from one service to another. Similar issues may be observed for other modes when convoys are involved, such as road-trains and platoons for road transport, and multi-barge assemblies for river/canal navigation.

The tactical planning problem then involves two related sets of design decisions, first, the services and, second, the “blocks”, the latter being defined as sequences of the former. As also discussed in Crainic et al. (2022), SSND models in such cases are defined on a two or three-layer time-space network. The first design layer targets the train-service selection and supports the second design layer targeting the block selection, each supported potential block being defined as a path of arcs in the train-service layer. The OD demand and the flow-decision variables are defined on the supported block layer in a two-layer formulation. A separate layer may be defined for the handling of demand at terminals, availability at origin, classification, arrival at destination for delivery, etc. Such a three-layer formulation simplifies the modelling of complex terminal operations, classification in particular, as shown in Zhu et al. (2014), where the time-space network on each layer is built on two time-nodes for each physical one. This representation provides the means to capture the incoming and outgoing flow at the node at each particular time period, the link connecting the two nodes modelling the terminal activities, classification in particular.

Such two-layer formulations include one-to-one design and flow connectivity requirements and constraints, and thus belong to the Design-Flow Connect class of the MLND taxonomy proposed by Crainic et al. (2022). The former are of the all-design connectivity type, imposing constraints (6), while flow-accumulation constraints (14) belong to the latter group.

A different class of two-layer MLND problems emerges when integrating SND and the issue of providing the resources required to support the selected services. The initial contributions (e.g. Kim et al., 1999; Andersen et al., 2009b; Pedersen et al., 2009) focused on the basic case of a single resource type, a single resource unit required to operate a service, and given levels of available resources at each terminal. The resource movements in space and time are not explicitly addressed, however. The formulations require only the service network to be *balanced*, i.e., the same number of services, hence, resources, to flow into and out of every node (topological constraints on the node in and out degrees). Consequently, design-balanced SND models, without explicit resource management, do not require multi-layer formulations.

Most human (crews) and material (power units, loading units, vehicles, etc.) mobile resources in transportation are operated according to cycles in space and time, generally starting and ending at the same terminal. At the operational planning level, the management of mobile resources yields rather complex problems due to the various requirements of work agreements, periodic inspections, and maintenance, notably. The integration of resource-management concerns into tactical service-network requires a more macro view of the resource cycles to capture the essence of operations without unnecessary detail. This yields a two-layer MLND problem, with services being selected on the supported layer and resource cycles being selected on the supporting one. The demand is then moved on the service layer as in Andersen et al. (2009a), which shows the superiority of cycle-based formulations compared to the design-balanced ones, and Crainic et al. (2014), where explicit service and resource-cycle selection decisions are introduced together with a number of resource management rules (e.g., duration limit and number of returns to the origin terminal within a cycle). All-design connectivity constraints (6) link the two design layers only (no flow connectivity is considered), placing this MLND problem class in the Design Connect category of the Crainic et al. (2022) taxonomy.

5.3 Multi-layer Service Network Design

Richer L-layer SSND models are increasingly proposed, mainly due to the growing interest in adequately addressing problem settings with several service classes or providers and several resource types connected through various design and flow connectivity requirements. They are generally known as SSND with resource management, *SSND-RM*, and SSND with revenue and resource management *SSND-RRM*, and belong to the Design-Flow Connect class of the MLND taxonomy (Crainic et al., 2022). They involve several, but not all, connectivity relations discussed previously.

A first observation is the worth of explicitly including a *commodity layer* to manage the entry and exit of OD demand into and out of the system, as well as the various activities involving demand flows in terminals, e.g., railcar consolidation and waiting for departure (Zhu et al., 2014) or container waiting for assignment and loading on selected intermodal railcars (Kienzle et al., 2023).

The commodity layer is then part of multi-layer network design formulations with several service and resource layers. Crainic et al. (2018) extends the model and solution method proposed in Crainic et al. (2014) to include service-outsourcing possibilities in some parts of the network, and several resource layers representing different resources with particular management rules. The outsourcing possibilities are modelled as a particular resource type, each arc supporting a particular service that may be outsourced. Thus, while own resources move on cycles, outsourcing ones do not. The authors also explicitly include the possibility to acquire or rent new resources and the decisions to (re-)position resources to terminals before the tactical plan starts to be applied. Services require one unit of one of the resource types available, yielding min-design connectivity constraints (7). Hewitt et al. (2019) extends these concepts and propose a L-layer SSND-RM model explicitly integrating demand uncertainty. This is one of the very few contributions addressing uncertainty issues in multi-layer network design.

Intermodal rail transportation planning raises additional challenges compared to the general rail case. Indeed, containers come in different types (dimensions and particular requirements, e.g., power needs to for refrigeration), as do railcars, which also come in different settings in terms of the type and number of containers they may carry. The problem becomes even more complex when, as in North America, containers may be stacked, one on top of another, with particular feasibility and security stacking rules. The proposed L-layer SSND-RM (Kienzle et al., 2023) thus includes train-service and block design layers, a commodity (several types of containers) layer, and a railcar layer corresponding to as many resource layers as the different railcar fleets considered in the problem. The movement of resources in this model is represented on arcs, the management of the various railcar fleets involving the assignment and loading of containers, the management of inventories at terminals at appropriate time moments, the possible empty movements to fulfill the needs and ensure inventory conservation in space and time, and the initial allocation to terminals. The model introduces constraints describing the container-to-railcar assignment and loading activity. It also includes one-to-one all-design connectivity constraints (6) between the service and block layers, flow-accumulation connectivity constraints similar to (42) between the railcar and block layers, and flow-attribute capacity constraints (26) relative to the total length of train services.

Bilegan et al. (2022) proposes what appears to be the first SSND-RRM contribution, integrating revenue and resource management considerations and the SSND modelling framework to address the tactical planning problem of an intermodal barge transportation system. The revenue management context is represented by the segmentation of demand and service types. Three types of customers are considered, regular with established contracts or understandings for the planning period considered, which must be satisfied; partial-spot, which may be satisfied, if selected, for part of their forecast demand; and full-spot, which must be fully satisfied, if selected. Two service levels are also defined, standard and express, with corresponding fare classes being defined for each demand type. The OD demand is associated to the service layer. The problem setting considers two resource types, that is, two vessel types with particular characteristics in terms of capacity and speed. Vessels operate cyclic routes made up of service legs, the corresponding inter-service leg, holding arcs at intermediary terminals, as well as waiting-at-terminal arcs when idly waiting for the next service to start. The goal is to maximize profit by selecting the appropriate services, and the vessels to support them, to satisfy the regular and selected spot demand. The authors wrote a “flat” model, projecting the two resource layers onto the service layer, by *a priori* associating a vessel to each potential service (which implies that parallel service arcs are possible), and enforcing vessel-type specific fleet size and design-balancing constraints at each port (node) at all time periods.

6 Conclusions and Perspectives

Multi-layer network design emerges as an important problem class within the network-design combinatorial optimization field. On the one hand, it raises interesting methodological challenges stemming from the problem structure and the relations among the design and flow variables and attributes defined on the different layers. On the other hand, it provides the means to adequately represent and address complex planning problems in major application domains, transportation, logistics, and telecommunications, in particular

Following the structuring of the field proposed in Crainic et al. (2022), this chapter updated the basic concepts and properties of MLND, and developed the connectivity-requirement definitions and modelling to address richer problem settings. Attribute-based connectivity was thus defined, in particular related to non-additive attributes, such as arc capacities. The chapter also significantly extended the study of connectivity relations involving the design and flow-distribution decisions on more than two related layers. These relations and the corresponding constraint representation within MLND formulations were then discussed from the point of view of tactical planning in consolidation-based freight transportation and the service network design methodology used to address it.

The field is still emerging, however, particularly when more than two layers are involved. Several relations described in this chapter are only defined in mathematical terms, applications still waiting to be proposed, as the brief SSND literature analysis illustrates. Moreover, other problem characteristics, important methodologically and in practice alike, e.g., uncertainty, robustness, and resilience, have still to be studied. Continuing to address the MLND challenges appears thus important, particularly given the strong mutually beneficial relations between methodological development and applications proper to Operations Research.

Solution-method developments are particularly necessary. As already observed in Crainic et al. (2022), this development may follow two related directions. First, extend the methods proposed for the general network design problems (see, e.g., Crainic and Gendron, 2021; Crainic and Gendreau, 2021, for recent surveys on exact and heuristic methods, respectively). Second, take advantage of the MLND structure to strengthen formulations and develop exact, meta-heuristic, and matheuristic solution methods. Decomposition appears particularly relevant given the multi-layer structure of the problem class, Lagrangian, Benders, and Integrative Cooperative Search offering rich research perspectives.

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Appendix - Flow Accumulation and Conservation Conformity

Crainic et al. (2022) discuss the issue of so-called reachable arcs and the apparent contradiction between flow-connectivity and flow-conservation constraints. Two arcs (x_1, y_1) and (x_2, y_2) are defined as reachable when there is a path from x_1 to y_2, or from x_2 to y_1. Thus arcs a and b of the supported layer l' of Figure 4 are reachable, assuming there is a path in l' from node 3 to node 4. Define arcs a and b as the paths $\{\alpha, \beta\}$ and $\{\epsilon, \beta, \zeta\}$ is the supporting layer l. Assume d units of flow are to be moved from node 1 to node 6 on the supporting and supported arcs. The arc labels in figure display the arc identifier and the quantity of flow assigned to it in this chapter.

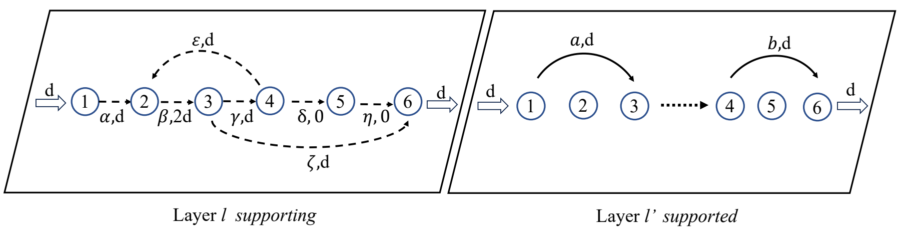

Figure 4: Flow accumulation and conservation illustration

Crainic et al. (2022) claim that, the flow-accumulation constraints force a flow of $2d$ units on arc β, which contradicts the flow-conservation constraints at node 3 putting d units on arc ζ and 0 units on arc γ. Yet, the equality between the flows on the two layers imposed by the flow-accumulation constraints operates in both directions, impacting also the flow distribution on the supporting layer. Hence, a solution sending d units on arcs γ and ζ, and 0 units on arcs δ and ν (see Figure 4,) respects the two types of constraints. No contradiction is observed in this case.

One may ask when such cases may arise. Clearly, the "cost" attributes on the supporting and supported layers would have an impact and should therefore reflect such flow distributions. More generally, the problem setting must require it. We are not aware of general settings with network configurations such at those of Figure 4. The only situation we are aware of concerns a particular setting of the Scheduled Service Network Design (SSND) problem (Crainic et al., 2021b; Kienzle et al., 2023). Recall that SSND models are often formulated on time-space networks covering activities for a given schedule length. Wrap-around arcs are then defined to represent activities, e.g., holding and services, starting during the schedule length but terminating in the next occurrence of the plan (schedule). As resources move according to (explicit or implicit, depending on the problem setting) cycles and all arcs point in the same time direction, the situation discussed above does not happen. The case may be observed, however, when resources are injected into the system at the first occurrence of the plan, and representations similar to that around node 2 in the figure are built. (Notice, however, that Figure 4 is not an illustration of a time-space network with wrap-around arcs as some flows, on arc ζ, do not return.)