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The Two-Echelon Multicommodity Location-Routing Problem with Stochastic and Correlated Demands

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Abstract. This study introduces the stochastic two-echelon multicommodity location routing problem with stochastic and correlated demands. We propose a two-stage stochastic programming formulation, with second-echelon facilities design decisions defining the first stage, while recourse decisions, which are made in the second stage, establish how the observed demands are distributed. The overall objective is to optimize the cost of the first-stage design decisions plus the total expected routing cost incurred in the second stage. To solve this formulation, we propose a progressive hedging metaheuristic with a series of algorithmic enhancements to accelerate the exploration of the solution space. These enhancements include: 1) population structures to obtain alternative and diverse solutions for the scenario subproblems that need to be solved throughout the search process; 2) alternative strategies to define the reference solutions which are used to guide and accelerate the overall search; and 3) a reset procedure that reduces the risk of the method becoming trapped in local optima. We assess the efficiency and effectiveness of all proposed strategies through extensive computational experiments, evaluating their capability to generate high-quality solutions across various problem characteristics and demand correlations.

Keywords: Two-echelon location-routing problem, stochastic demand, multicommodity, origin-destination demands, progressive hedging

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1 Introduction

The two-echelon location-routing problem (2ELRP) is an important class of combinatorial optimization problems with a wide rage of applications in the freight transportation industry. At its core, the concept is to design a two-layer freight transportation system that enables indirect freight transportation between platforms (distribution centers) and customers through a set of intermediate facilities named satellites. The 2ELRP has been defined as the preferred methodology for efficiently capturing the simultaneous decisions concerning the location of one or two levels of facilities (platforms and/or satellites) and creating a limited set of routes at both echelons to effectively serve all customer demands (Cuda et al., 2015). Despite the growing number of scientific contributions and advances in this field, most research on 2ELRP has focused on models and solution methods for "classic" problem variants and deterministic cases, while uncertain factors are often overlooked (Gendreau et al., 2014).

Considering demand uncertainty and its interrelation is of great significance for when planning decisions are involved. In logistics planning, which encompasses strategic and tactical choices in distribution network design, obtaining accurate information about customer demand variations is essential for long-term planning (Lium et al., 2009). Several sources of uncertainty related to demand can be observed, such as variations in volume, inaccuracies in forecasted values, or unexpected demand fluctuations between specific origin-destination pairs (Crainic et al., 2011). While studies in LRPs with stochastic demands often assume statistically independent request fluctuations, variability and correlation are observed in many logistics contexts (Verma and Campbell, 2019). Different demand values often display degrees of positive or negative correlation relative to other customer demands (Bucci et al., 2006). Seasonal demand variations serve as an example, though they may not entail high uncertainty due to their predictability. Correlations and more intricate covariation gain importance during planning, especially when systematic relationships exist among customer demands (e.g., regions, product types, time periods) (Heath and Jackson, 1994; Thapalia et al., 2012; Verma and Campbell, 2019; Mirhedayatian et al., 2019). One can thus assume that demand correlation exhibits a *mixed* nature, rather than being purely positive or uncorrelated. To the best of our knowledge, 2ELRPs considering correlated and uncertain demands, specifically involving non-substitutable demand with a known origin and destination, remains unexplored. This study is aimed at deepening the understanding of the effects of the integrated treatment of uncertain and correlated non-substitutable demands on location and routing decisions. Our goal is to provide a methodology to respond to the modelling and algorithmic challenges and, thus, to contribute toward filling the gaps in the literature.

This paper address a 2ELRP with stochastic and correlated multicommodity, origin-to-destination (OD) demands. We thus introduce the *Two-Echelon Multicommodity Location-Routing Problem with Stochastic and Correlated Demands* (*2E-MLRPSCD*) as a unified view of the attributes considered. The problem centers around design decisions concerning the selection of satellite facilities and the allocation of multicommodity origin-destination (OD) demands to these satellites, while also encompassing the definition of a limited set of routes at both echelons to efficiently fulfill the demand. To address the uncertainty, we propose a stochastic programming approach oriented towards devising a singular system design capable of maintaining cost-effectiveness in the presence of diverse demand realizations. Specifically, we present a two-stage stochastic programming formulation, with satellite facility design decisions defining the first stage, while recourse decisions which are made in the second stage, establish how the observed demands are distributed. We thus represent the demand uncertainty through a finite set of scenarios, which must approximate the uncertainty inherent in the planning context. However, employing scenario-based uncertainty modeling yield large-scale models that may prove impractical to address using standalone exact solution methods King and Wallace (2012).

The proposed work thus introduces a progressive hedging-based metaheuristic for addressing the 2E-MLRPSCD, building on the work of Crainic et al. (2011) for the network design problem. From a methodological standpoint, the classic progressive-hedging (PH) algorithm iteratively solves deterministic subproblems derived from the scenario-based decomposition of the stochastic program. The PH metaheuristic iterates by adjusting the mathematical formulation of scenario subproblems using aggregated solutions until reaching an optimal solution when a general consensus among non-scenario-dependent decisions is observed. However, the classic structure of the PH metaheuristic and the metaheuristic methods derived from it lack alternative aggregation methods to effectively derive key insights from the subproblem solutions. To address this, we present a specialized PH-based metaheuristic with a series of algorithmic enhancements. These enhancements include: population structures to obtain alternative and diverse solutions for the scenario subproblems that need to be solved throughout the search process; 2) alternative strategies to define the reference solutions which are used to guide and accelerate the overall search; and 3) a reset procedure that reduces the risk of the method becoming trapped in local optima. In the computational study, we analyze the cost sensitivity, infrastructure usage, and a comparison between the uncertain and deterministic definition of the demand to derive insights of the effectiveness of the proposed solution method.

The remaining parts of the paper are organized as follows. Section 2 is dedicated to describing the problem definition. An overview of the related scientific literature is provided in Section 3. Section 4 presents the system modelling and the proposed mathematical formulation. The solution method we developed is described in Section 5. Computational results are then presented and analyzed in Section 6.

2 Problem definition

This section introduces the 2E-MLRPSCD, which involves addressing a 2E-LRP with stochastic and correlated multicommodity origin-to-destination (OD) demands. The section is divided into two parts. Section 2.1 presents the physical problem setting of the 2E-MLRPSCD. Section 2.2 outlines representation of stochastic and correlated demands as well as the main lines, objective and requirements, of the problem.

2.1 The 2E-MLRPSCD setting

The two-echelon system consists of three main components: *platforms* (primary facilities serving as demand origins), *satellites* (intermediate facilities), and *customers* (demand destinations).

Formally, the 2E-MLRPSCD is represented as a complete weighted directed graph $N = (V, A)$, with vertices $V = P \cup Z \cup C$, divided into three disjoint sets: platforms P, satellites Z, and customers C. Platforms are large-sized facilities with a known set of commodities to be distributed to customers. Satellites are medium- to small-sized multimodal infrastructures that serve as intermediate facilities, allowing the consolidation and sorting of freight between the two transportation echelons involved in distributing goods to customers. Each satellite location $z \in Z$ is associated with a limited storage capacity Q_z and a fixed opening cost F_z.

Demand is defined between platforms and customers, each individual demand being characterized by an origin, a destination and a requested volume to be delivered. Let K denote the set of origin-destination (OD) demands. For the deterministic version of the 2E-MLRPSCD, each OD demand $k \in K$, is thus characterized by a volume vol_k, an origin $O(k)$ associated with a platform node in P, and a destination $D(k)$ associated with a customer node in C. Additionally, a fixed allocation cost Δ_{pzk} represents the cost of serving OD demand $k \in K$ through platform $p \in P$ and satellite $z \in Z$.

Each arc $(i, j) \in A = A^1 \cup A^2$ is associated with a non-negative cost ζ_{ij} for a vehicle to travel between i and j. Let A^1 denote the set of arcs of the first echelon, corresponding to the connections between platforms P and satellites Z and between satellites. The set A^2 includes the arcs of the second echelon, that is, the connection of the satellites Z with the final customers C and between customers.

Freight delivery is performed by two homogeneous fleets of vehicles $H = H^1 \cup H^2$ with limited load capacities cap_1 and cap_2, which are respectively available for the first and second echelon, and are able to transport any demand. Vehicles are assumed to be available at each existing facility for each echelon, where vehicles start and end their routes.

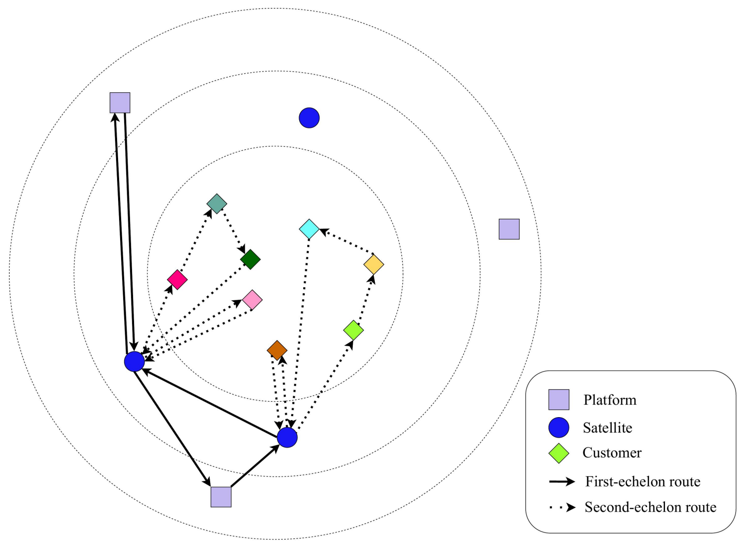

Figure 1: Topology of the 2E-MLRPSCD.

The considered problem involves the selection of satellite facilities, the allocation of OD demands to satellites, as well as the routing of vehicles at each echelon to deliver the freight from platforms to customers, going through satellite facilities. As depicted in Figure 1, each OD demand that is made available at its originating platform has to be moved by a first-echelon vehicle to a given satellite to be then transferred to a second-echelon vehicle. Loads delivered at satellites are then transshipped and consolidated into second-echelon vehicles, which will perform the deliveries to the final destinations.

2.2 The stochastic setting

The 2E-MLRPSCD involves uncertainty in the volume of demand stemming from random changes occurring between correlated OD pairs. We assume that probability distributions exist to describe the variation in the random events affecting the volume of demands. Moreover, the problem setting involves correlation among OD pairs, where each OD pair can be either positively or negatively correlated with other distinct OD pairs. The problem is characterized by two sets of OD pairs used to represent the correlation; OD pairs within each set are positively correlated, while all correlations between OD pairs in different sets are strongly negative (i.e., low demands in one set result in high demands in the other).

The 2E-MLRPSCD problem setting addresses strategic and tactical planning decisions in multiple application fields. In terms of decision-making and information processing, the design and allocation decisions during the planning stage must be defined based on an evaluation/estimation of their impact on operations, including the available recourse actions to adapt the plan to the observed demands. The recourse actions in the present case involve the definition of the optimal routes to fulfill observed ("realized") customer demands including, when necessary, the use of external outsourcing services with high additional operational costs.

The 2E-MLRPSCD then consists in the selection of the locations of the satellite facilities, the allocation of OD demands to satellites, as well as the construction of a limited set of routes for the first and second echelons vehicles in such a way that: (i) the demand of each platform is assigned to an open satellite; (ii) every route of the first echelon starts and ends at the same platform; (iii) every route of the second echelon starts and ends at the same satellite; (iv) all the customers' demands are satisfied either by the system or an outsource service; (v) the load capacity of each vehicle is not exceeded; (vi) each customer served by the system is visited by only one vehicle; (vii) the total demand assigned to a satellite facility must not exceed its capacity; and (viii) the sum of the fixed location and allocation costs and the expected routing costs (the recourse action) is minimized.

3 Literature review

The 2E-MLRPSCD belongs to the *Location-Routing Problem* (*LRP*) category, which constitutes an important problem class that contains a vast number of contributions and ongoing works in the literature. LRPs fundamentally appear in the context of the planning process that seeks to open one or more platforms from a given set of pre-defined locations, define the customer assignments to them and establish a variety of routes required to meet the demands of each customer considered. Studies dedicated to the LRPs and the 2E-LRP are increasingly gaining attention, in particular on realistic multi-attribute problem settings (Escobar-Vargas and Crainic, 2022). This section aims to situate the 2E-MLRPSCD within the relevant literature on both the 2E-LRP and LRP, specially pointing out the gaps in knowledge concerning how to deal with stochastic demands in this setting. A brief discussion on the progressive-hedging strategy is also provided, focusing on the challenges and gaps of the application of this method when tackling integer programming problems. Works on 2E-LRP and LRP dedicated to their deterministic versions or stochastic aspects other than demand uncertainty are out of the scope of this study. Therefore, we refer the interested readers to the recent surveys by Cuda et al. (2015), Schiffer et al. (2019) and Mara et al. (2021).

Because of its practical relevance, the LRP has attracted much attention from the research community resulting in a wide variety of high-quality solution approaches for its deterministic versions since its introduction in Maranzana (1964). While studies on demand uncertainty are still scarce, more attention has been devoted to this variant spurred by the desire to solve more realistic distribution planning problems (Cuda et al., 2015; Escobar-Vargas and Crainic, 2022). Because of the complexity of considering demand uncertainty in LRP, most studies have focused on proposing heuristics methods to solve the problem setting considered. The literature is notably characterized by the extensive use of local-search-based metaheuristic frameworks to address the underlying transportation problems to guide two- or multi-stage heuristics, where location, allocation and routing decisions are treated by different heuristics at different stages (see, Albareda-Sambola et al., 2007; Huang, 2015; Marinakis, 2015; Marinakis et al., 2016; Zhang et al., 2019). A different approach is proposed by Quintero-araujo et al. (2019), where a simheuristic algorithm is proposed to deal with the LRP with stochastic demands. This simheuristic algorithm then hybridizes a Monte Carlo simulation with an iterated local search metaheuristic. In spite of the advances in the field, the literature in LRPs with demand uncertainty is quite limited, especially in the case of non-substituable demands. The case where stochastic demands are statistically independent remains the most predominant setting studied in the literature. Research concerning correlation features and their impact on the decision making process has yet to be addressed. Important contributions are also still required to deepen the understanding of the impact of richer problem settings and their influence on location decisions under uncertainty.

The literature on 2E-LRP with uncertain demands is very limited. To the best of our knowledge only Snoeck et al. (2018) have presented a stochastic mixed-integer linear programming formulation to model a two-echelon capacitated location-routing problem with uncertain demands arising from a practical application. However, particular developments are required in the field, especially in relation to explicitly consider demand correlations, non-substituable demand considerations, and meeting the modelling and algorithmic challenges these considerations imply.

Aside from the modelling aspects, there is a fundamental need for more effective solution procedures for 2E-LRP with uncertainty considerations. Concerning exact and approximate solution frameworks, decomposition-based methods have shown very promising results for solving two- and multi-stage stochastic optimization models Atakan and Sen (2018). The effectiveness of such methods rely on how the stochastic problem can be decomposed. Two general decomposition strategies are usually applied here. The first strategy decomposes the model according to the scenarios used to formulate the uncertain phenomena, while the second strategy separates the model according to the decision stages that define the optimization model. The progressive hedging algorithm is one of the most used dual decomposition frameworks in the field. Rockafellar and Wets (1991) developed progressive hedging to solve convex stochastic programs. The algorithm involves decomposing the stochastic problem by scenario, solving each of the resulting scenario subproblems independently, and then determining the stochastic problem's solution based on the consensus (or averaging) of all scenario subproblems solutions. However, converging to a globally optimal solution for mixed-integer stochastic programs in a computationally-efficient manner is challenging, primarily because of the non-convex nature of the feasible set (Atakan and Sen, 2018). To overcome such computational burden several studies have proposed different heuristic frameworks following the progressive hedging algorithm to allow the application of the method to integer programming formulations (see, Løkketangen and Woodruff, 1996; Haugen et al., 2001; Crainic et al., 2011; Lamghari and Dimitrakopoulos, 2016; Alvarez et al., 2021). To reach a consensus for the complete integer stochastic problem, the standard structure of these PH-based metaheuristics, usually relies on obtaining the best solution possible (not necessarily optimal) for each scenario subproblem. This strategy enables the use of the best decisions to guide the search of the solution space. Nonetheless, considering the single best solution possible for each scenario subproblem can also reduce the overall diversity of solutions of the complete stochastic problem, which is crucial at each iteration of the PH. The current work seeks to close these gaps in the literature by extending and enhancing a progressive-hedging-based metaheuristic to the 2ELRP, by introducing a specialised set of heuristics to allow the consideration of diverse alternative solutions for each scenario subproblem, as well as a set of novel techniques to accelerate consensus for the stochastic problem.

4 Modelling

Section 4.1 introduces the initial outline of the modelling approach, followed by the proposed mathematical formulation in Section 4.2.

4.1 Modelling uncertainty

The 2E-MLRPSCD is formulated as a two-stage stochastic program to account for the strategic planning decisions. The proposed two-stage model consists of a first stage, where the location of satellite facilities and the OD demand to satellite allocation decisions are made, and a second stage, where the vehicle routes for both echelons are determined when customer demands are observed. Additionally, the option of resorting to ad-hoc, outsourced capacity when necessary is also part of the second stage, where an operational cost R is associated with the percentage of the demand volume that is served by an outsourced service.

The demand uncertainty is represented through known distributions, while correlations are given by matrices.

We model the demand uncertainty and correlation in this system through the generation of a set of scenarios, obtained by sampling probability distributions, each scenario representing a possible realization of the random event affecting the demands. Let S denote the set of scenarios, where scenario $s \in S$, represents a possible realization of the random events which sets the demand values of each customer and the correlations. Let ρ_s be the probability of occurrence of scenario s, such that $\sum_{s \in S} \rho_s = 1$. Then for a given $s \in S$, there is a demand volume fixed to $vol_k(s)$ for all $k \in K$, such that $vol_k(s) \geq 0$.

4.2 Two-stage formulation for the 2E-MLRPSCD

This section presents the Mixed-Integer Programming (MIP) formulation for the 2E-MLRPSCD, as a two-stage stochastic programming problem using a three-index vehicle-flow formulation. Two sets of decision variables are defined. First-stage variables address the satellite location and OD demand to satellite allocation decisions. Vehicle-routing decisions at both echelons are made in the second stage. Following the general trend in the literature, we save space and present the formulation directly in terms of the set of scenarios S. This yields second-stage variables indexed by scenario, while first-stage ones are not (they are not supposed to be modified in the second-stage). The following definitions describe the decision variables that constitute the extensive form of the proposed two-stage formulation:

- $y_i \in \{0,1\}, i \in Z$: location variable, 1 if a satellite is opened in location i, 0 otherwise;
- $f_{pzk} \in \{0,1\}, p \in P, z \in Z, k \in K$: allocation variable, 1 if satellite z is assigned to platform p to serve the demand k, 0 otherwise;
- $u^s_{pzkh} \in \{0,1\}, p \in P, z \in Z, k \in K, h \in H^1, s \in S$: vehicle allocation variable, 1 if vehicle h is assigned to serve satellite z from platform p with demand k for scenario s, 0 otherwise;
- $v^s_{zch} \in \{0,1\}, z \in Z, c \in C, h \in H^2, s \in S$: vehicle allocation variable, 1 if vehicle h is assigned to serve the customer c with satellite z for scenario s, 0 otherwise;
- $x^s_{ijh} \in \{0,1\}, (i,j) \in A, h \in H, s \in S$: vehicle flow variable, 1 if arc (i,j) is used by vehicle h for scenario s, and 0 otherwise;
- $w^s_{zkh} \geq 0, z \in Z, k \in K, h \in H^2, s \in S$: percentage of demand k served by a satellite z with a vehicle h for scenario s;
- $o^s_k \geq 0, k \in K, s \in S$: percentage of demand k that is outsourced for scenario s;
- $b^s_{kh} \geq 0, k \in K, h \in H^1, s \in S$: percentage of demand k dispatched with a vehicle h for scenario s;
- $L^s_{zh} \geq 0, z \in Z, h \in H^1, s \in S$: integer variable used to record the position of the satellite z in the route assigned to the first-echelon vehicle h for scenario s;
- $N^s_{ch} \geq 0, c \in C, h \in H^2, s \in S$: integer variable used to record the position of the customer c in the route assigned to the second-echelon vehicle h for scenario s;

The extensive two-stage formulation of the 2E-MLRPSCD then becomes:

$$\min \sum_{s\in S} \rho_s \left(\sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k \right) + \sum_{i\in Z} F_i y_i + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Delta_{ijk} f_{ijk} \tag{1}$$

subject to

$$\sum_{j\in(P\cup Z), i\neq j} x^s_{ijh} \leq 1 \quad \forall i \in (P\cup Z), h\in H^1, s\in S \tag{2}$$

$$\sum_{i\in(P\cup Z), i\neq j} x^s_{ijh} - \sum_{i\in(P\cup Z), i\neq j} x^s_{jih} = 0 \quad \forall j \in (P\cup Z), h\in H^1, s\in S \tag{3}$$

$$L^s_{ih} - L^s_{jh} + |Z|\, x^s_{ijh} \leq |Z| - 1 \quad \forall i,j\in Z, i\neq j, h\in H^1, s\in S \tag{4}$$

$$\sum_{h\in H^2} \sum_{j\in(Z\cup C), i\neq j} x^s_{ijh} = 1 \quad \forall i\in C, s\in S \tag{5}$$

$$\sum_{i\in(Z\cup C), i\neq j} x^s_{ijh} - \sum_{i\in(Z\cup C), i\neq j} x^s_{jih} = 0 \quad \forall j \in (Z\cup C), h\in H^2, s\in S \tag{6}$$

$$\sum_{h\in H^2} \sum_{j\in C} x^s_{ijh} \leq |H^2| y_i \quad \forall i\in Z, s\in S \tag{7}$$

$$N^s_{ih} - N^s_{jh} + |C|\, x^s_{ijh} \leq |C| - 1 \quad \forall i,j\in C, i\neq j, h\in H^2, s\in S \tag{8}$$

$$\sum_{j\in(Z\cup C), i\neq j} x^s_{ijh} + \sum_{j\in(Z\cup C), l\neq j} x^s_{ljh} - v^s_{lih} = 0 \quad \forall i\in C, l\in Z, h\in H^2, s\in S \tag{9}$$

$$\sum_{h\in H^2} \sum_{i\in Z} v^s_{ijh} = 1 \quad \forall j\in C, s\in S \tag{10}$$

$$\sum_{i\in P} \sum_{h\in H^1} u^s_{ijkh} = \sum_{h\in H^2} v^s_{jD(k)h} \quad \forall j\in Z, k\in K, s\in S \tag{11}$$

$$\sum_{h\in H^2} \sum_{i\in Z} w^s_{ijh} + o^s_j = 1 \quad \forall j\in K, s\in S \tag{12}$$

$$w^s_{ijh} \leq v^s_{iD(j)h} \quad \forall i\in Z, j\in K, h\in H^2, s\in S \tag{13}$$

$$b^s_{kh} \geq w^s_{ikl} - (2 - v^s_{iD(k)l} - \sum_{p\in P} u^s_{pikh}) M$$

$$\forall h\in H^1, l\in H^2, i\in Z, k\in K, s\in S \tag{14}$$

$$\sum_{k\in K} vol_k(s) \sum_{h\in H^2} w^s_{ikh} \leq Q_i \quad \forall i\in Z, s\in S \tag{15}$$

$$\sum_{k\in K} vol_k(s) \sum_{i\in Z} w^s_{ikh} \leq cap_2 \quad \forall h\in H^2, s\in S \tag{16}$$

$$\sum_{k\in K} b^s_{kh} \leq cap_1 \quad \forall h\in H^1, s\in S \tag{17}$$

$$\sum_{h \in H^1} u^s_{ijkh} = f_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{18}$$

$$\sum_{h \in H^2} v^s_{zD(k)h} = \sum_{p \in P} f_{pzk} \quad \forall z \in Z, k \in K, s \in S \tag{19}$$

$$y_i \in \{0,1\} \quad \forall i \in Z \tag{20}$$

$$f_{pzk} \in \{0,1\} \quad \forall p \in P, z \in Z, k \in K \tag{21}$$

$$u^s_{pzkh} \in \{0,1\} \quad \forall p \in P, z \in Z, k \in K, h \in H^1, s \in S \tag{22}$$

$$v^s_{zch} \in \{0,1\} \quad \forall z \in Z, c \in C, h \in H^2, s \in S \tag{23}$$

$$x^s_{ijh} \in \{0,1\} \quad \forall (i,j) \in A, h \in H, s \in S \tag{24}$$

$$w^s_{zkh} \geq 0 \quad \forall z \in Z, k \in K, h \in H^2, s \in S \tag{25}$$

$$o^s_k \geq 0 \quad \forall k \in K, s \in S \tag{26}$$

$$b^s_{kh} \geq 0 \quad \forall k \in K, h \in H^1, s \in S \tag{27}$$

$$L^s_{zh} \geq 0 \quad \forall z \in Z, h \in H^1, s \in S \tag{28}$$

$$N^s_{ch} \geq 0 \quad \forall c \in C, h \in H^2, s \in S \tag{29}$$

The objective function (1) seeks to minimize the sum of the expected total routing and outsourced costs and the total fixed cost of opening satellites and allocating them to the platforms. Constraints (2) ensure that each available vehicle is assigned to at most one platform. Constraints (3) are the flow conservation constraints for platforms and satellite facilities. Constraints (4) are the sub-tour elimination constraints for the first-echelon vehicles. Constraints (5) ensure that every customer is served by a single second-echelon vehicle. Constraints (6) are the flow conservation constraints at satellites and at customers. Constraints (7) state that second echelon vehicles can only be used from located satellites. Constraints (8) are sub-tour elimination constraints for the second-echelon vehicles. Constraints (9) link the allocation and routing variables. Constraints (10) impose that each customer has to be assigned to a satellite.

Constraints (11) are the flow conservation constraints for each commodity k at each satellite z. Constraints (12) ensure that the portion of the customer demand served by a satellite and the portion served by an outsourced service meet the complete demand for each customer. Constraints (13) ensure that each satellite can only serve its assigned customers. Constraints (14) ensure that, for each commodity k, the portion of the demand that is serviced via the located satellite corresponds to the inbound portion that originates from the associated platform. Constraints (15) impose that flow leaving an open satellite z is less or equal than its storage capacity. Constraint (16) and (17) guarantee that the commodity flow carried by each vehicle, in the first and second echelon, respectively, is less than or equal to its own capacity. Constraints (18) and (19) link allocation and vehicle allocation variables for the first echelon and second echelon vehicles, respectively. Constraints (20)-(29) impose the integrality and non-negativity of each decision variables in the model.

5 A progressive hedging-based metaheuristic for the 2E-MLRPSCD

This section presents a progressive hedging-based metaheuristic to address the 2E-MLRPSCD, building on the work of Crainic et al. (2011) for the stochastic network design problem.

As its name implies, the methodology derives from the progressive hedging (PH) algorithm introduced by Rockafellar and Wets (1991) for multi-stage stochastic optimization problems. From a methodological perspective, the 'classic' progressive-hedging algorithm, iteratively solves the set of deterministic subproblems, which result from the scenario-based decomposition of the extensive formulation. At each iteration, the PH metaheuristic solves each scenario-specific deterministic subproblem separately, thus producing a series of solutions that may differ from one another. The search then proceeds by computing a reference solution (the expected value of the best scenario-specific solutions is traditionally used), which also serves to assess the overall level of consensus among the scenario-specific solutions. The formulations of the scenario subproblems are then adjusted to incentivize agreement (i.e., to make subproblems move toward the same implementable solution). This general process is repeated until either a consensus solution is found or another stopping criterion is reached (e.g., a computation time limit).

It is well known that the PH algorithm does not necessarily converge to an optimal solution when it is applied in the case of mixed-integer programs, such as the 2E-MLRPSCD. A significant algorithmic challenge also arises from the computational load of solving a series of NP-hard problems (one for each scenario) at each iteration of the PH metaheuristic. There is a clear need of an efficient guiding strategy and procedures to direct the algorithm toward finding a consensus solution more quickly. We thus introduce a PH metaheuristic with a set of algorithmic and methodological enhancements aimed at accelerating the search for an efficient implementable solution. These enhancements encompass: (1) a set of population structures to obtain alternative and diverse solutions for the scenario subproblems, (2) a set of novel scenario-selection strategies that effectively derive key insights from subproblem solutions to identify potential consensus, (3) a specialized heuristic to define a high-quality reference solution in the first PH iteration, and (4) a reset procedure to prevent the PH metaheuristic from getting trapped in local optima. This section presents the structure of our proposed PH metaheuristic and the novel strategies developed to accelerate the consensus.

5.1 General structure

The proposed PH metaheuristic, illustrated in Figure 2, follows the general structure of the method proposed by Crainic et al. (2011). The algorithm starts with the scenario decomposition of the extensive formulation introduced in Section 4.2. This results in a set of subproblems that take the form of a deterministic 2E-MLRPSCD for each scenario $s \in S$. Unlike the standard structure of Crainic et al. (2011), the proposed PH metaheuristic is set up to define a group of alternative solutions for each scenario subproblem, instead of using the single best solution, aiming to broaden the design options, specifically for location and allocation decisions.

We introduce two population structures to handle the group of alternative solutions for each scenario subproblem: a set of *local populations*, one for each scenario subproblem, and a single *global population* for the complete problem.

Each *local population* serves to organize scenario-specific alternative solutions. These *local populations* start empty and are updated at each iteration of the PH with the objective of maintaining the best representative solutions for each scenario subproblem. To assess the value of the solutions obtained for each scenario at each PH iteration, a ranking measure is defined. Each scenario-specific solution is ranked based on its quality and contribution to *diversity*. This ranking is performed with respect to the solutions already present in the *local population* and determines whether a solution should be included in the *local population* at each PH iteration. This ranking prioritizes *diversity* in solutions, favoring those that exhibit the most dissimilarity with respect to the first-stage decision variables compared to other solutions already present in the same *local population*.

The *global population* is constructed at each iteration of the PH, based on the best subset of solutions from each *local population*. A general reference solution is then determined based on a selected subset of solutions from the *global population* defined by one of the proposed scenario-selection strategies. This reference solution is used to guide the search by adjusting the costs in the objective function of each scenario subproblem, aiming to reach a consensus on the first-stage decisions across all scenarios.

Finally, the algorithm ends when a consensus is reached on the first-stage decisions or when external stopping criteria are met, while saving the best feasible solution obtained at each iteration of the PH. In the following sections, we provide a more in-depth description of each step of the proposed PH metaheuristic.

5.2 Scenario decomposition for the 2E-MLRPSCD

The decomposition strategy applied to the extensive formulation, requires the first-stage decisions to be reformulated (detailed reformulation in the supplementary material Appendix A. Specifically, these decisions need to be defined as scenario-dependent and constraints must be added to ensure that first-stage variables are the "same" in all scenario-subproblems. Let y_i^s and f_{ijk}^s be the reformulation of the first-stage variables for each scenario $s \in S$, for the location and allocation decisions, respectively. In doing so, con-

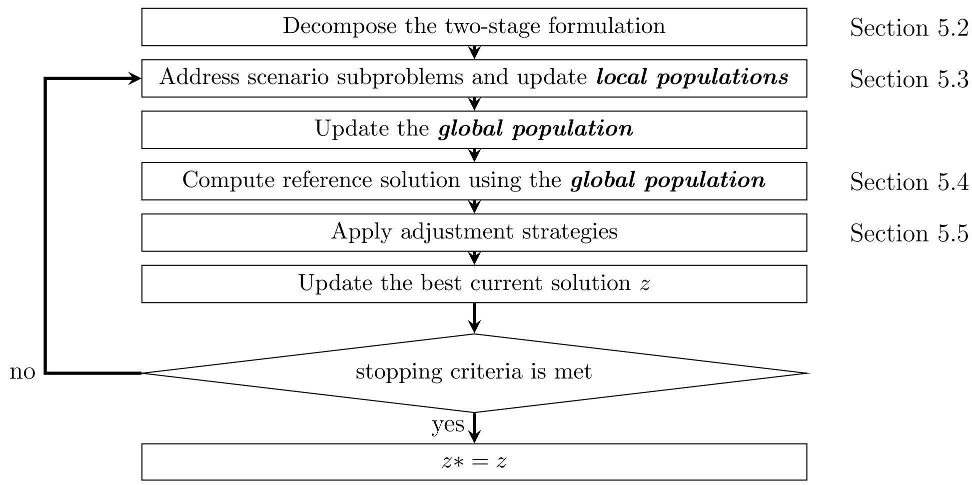

Figure 2: Progressive Hedging-based metaheuristic for the 2E-MLRPSCD

straints (7), (18) and (19) are reexpressed according to the scenario-specific location and allocation first-stage decisions. This reformulation explicitly includes the following non-anticipativity constraints, which prevent the first-stage decision variables to be set to different scenario-specific values:

$$y_i^s = \bar{y}_i \quad \forall i \in Z, s \in S \tag{30}$$

$$f_{ijk}^s = \bar{f}_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{31}$$

$$\bar{y}_i \in \{0,1\} \quad \forall i \in Z \tag{32}$$

$$\bar{f}_{ijk} \in \{0,1\} \quad \forall i \in P, j \in Z, k \in K. \tag{33}$$

The non-anticipativity constraints (30) and (31) ensure that the first-stage solutions will be the same for all the scenarios, with variables $\bar{y}_i$ and $\bar{f}_{ijk}$ serving as the reference variables for the first-stage decisions. This ensure that a single set of facility location and allocation decisions are made for all the scenarios (thus preventing tailored scenario-specific decisions to be made). Then, following the decomposition scheme, originally proposed by Rockafellar and Wets (1991), constraints (30) and (31) are relaxed using an augmented Lagrangean method, which results in the following relaxed reformulation of the extensive model:

$$\begin{aligned} \min \sum_{s\in S} \rho_s \Bigg(\sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x_{ijh}^s + \sum_{k\in K} Ro_k^s + \sum_{i\in Z} \left(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma \bar{y}_i \right) y_i^s \\ + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \left(\Delta_{ijk} + \mu_{ijk}^s + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \right) f_{ijk}^s \Bigg) \end{aligned} \tag{34}$$

subject to

$$(2) - (6)$$
$$(8) - (17)$$
$$\sum_{h \in H^2} \sum_{j \in C} x^s_{ijh} \leq |H^2| y^s_i \quad \forall i \in Z, s \in S \tag{35}$$
$$\sum_{h \in H^1} u^s_{ijkh} = f^s_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{36}$$
$$\sum_{h \in H^2} v^s_{zD(k)h} = \sum_{p \in P} f^s_{pzk} \quad \forall z \in Z, k \in K, s \in S \tag{37}$$
$$y^s_i \in \{0, 1\} \quad \forall i \in Z, s \in S \tag{38}$$
$$f^s_{ijk} \in \{0, 1\} \quad \forall i \in P, j \in Z, k \in K, s \in S. \tag{39}$$

The objective function now involves the Lagrangean multipliers λ^s_i and μ^s_{ijk} for the relaxed non-anticipativity constraints corresponding to the location and allocation decisions, respectively, and a penalty term γ. Constraints (35) state that second echelon vehicles can only be used from located satellites. Constraints (36) and (37) link the facility allocation variables with the vehicle allocation variables. Constraints (38) and (39) impose the integrality and non-negativity of each decision variables in the model.

For a given overall design $\bar{y}_i$ and $\bar{f}_{ijk}$, the relaxed reformulation then undergoes a scenario-based decomposition (the initial values for the overall design are discussed in Section 5.5). This decomposition yields individual scenario subproblems, each adopting the structure of a deterministic scenario-specific problem with modified fixed costs. For a particular scenario subproblem, the Lagrangean multipliers λ^s_i and μ^s_{ijk}, along with the term γ, penalize the discrepancies between the values of the location and allocation decision in the local design and those present in the current overall design. The following sections examine the proposed strategies to extract the overall design and allocation decisions and the approach to adjust the fixed costs of the scenario subproblem to guide the search toward consensus of the first-stage variables.

5.3 Subproblem algorithm

This section presents the algorithm proposed to address the scenario subproblems. The objective of the proposed subproblem algorithm is twofold, (1) generate a set of candidate solutions to represent each scenario subproblem; (2) rank and define the set of candidate solutions in terms of diversity and quality. In what follows, we describe the strategies that are proposed to achieve these two objectives.

First, we solve the MIP defined for each scenario $s \in S$ to identify a sufficient number of high-quality alternative solutions for each subproblem. The MIP for each scenario subproblem consists of the objective function (34), the constraint set: (2)-(6), (8)-(17), (35)-(39), as well as a complete *a priori* enumeration of the subtour elimination constraints.

To handle the set of alternative solutions defined for each scenario subproblem we introduce two types of solution population, a *local population* for each scenario subproblem, and a *global population* for the complete problem. Each *local population* serves to organize the scenario-specific alternative solutions found at each iteration of the PH. It is chraracterized by a total number ψ_T of individual alternative solutions associated with each scenario subproblem, including a reduced number ψ_E of elite solutions.

At each iteration of the PH, our approach involves the individual evaluation of all the feasible solutions found for the MIP defined by each scenario subproblem $s \in S$. This evaluation serves to determine whether an individual solution should be retained in the *local population* of its respective scenario subproblem. To evaluate each scenario-specific solution, we define a ranking measure. This ranking is determined based on the contribution of each individual solution in terms of both quality and diversity relative to the other solutions present in the same *local population*.

The ranking of each individual solution is determined by a *fitness* measure. We define this *fitness* measure by combining both the objective value and the *diversity contribution* of a given solution Sol_i. In this context, the *diversity contribution* or $\Xi(Sol_i)$ refers to the average distance between solution Sol_i and the set of solutions $\mathcal{N}$ present in the respective *local population*, as calculated according to equation (40), where $|\mathcal{N}| \leq \psi_T$. This *diversity contribution* aims to favor solutions that exhibit the greatest dissimilarity with respect to the first-stage decision variables when compared to other solutions already present in the same *local population*.

To measure the dissimilarity between two distinct solutions Sol_i and Sol_j, we propose a normalized Hamming distance $\sigma(Sol_i, Sol_j)$, inspired by the work of Vidal et al. (2012). We define $\sigma(Sol_i, Sol_j)$ as a measure of the dissimilarities between: (1) the satellite allocation decisions $\xi_k(Sol_i)$ and (2) the negative correlation score $\phi_k(Sol_i)$ for each OD demand $k \in K$. For each OD pair $k \in K$, the negative correlation score $\phi_k(Sol_i)$ represents the number of different OD pairs assigned to the same satellite that share a negative correlation with k. It is worth noting that the negative correlation score $\phi_k(Sol_i)$ is introduced to take advantage of negative correlations between OD pairs, as suggested by the opportunities arising from consolidating negatively correlated demands and their impact on system efficiency (King and Wallace, 2012). The proposed Hamming distance is defined according to equation (41), where $\mathbf{1}\,(cond)$ is an indicator function that returns the value 1 if condition *cond* is true, and 0 otherwise.

$$\Xi(Sol_i) = \frac{1}{|\mathcal{N}|} \sum_{Sol_j \in \mathcal{N}} \sigma(Sol_i, Sol_j) \tag{40}$$

$$\sigma(Sol_i, Sol_j) = \frac{1}{2|K|} \sum_{k \in K} \mathbf{1}(\xi_k(Sol_i) \neq \xi_k(Sol_j)) + \mathbf{1}(\phi_k(Sol_i) \neq \phi_k(Sol_j)) \tag{41}$$

We then define a *biased fitness* function $BF(Sol_i)$ computed according to equation (42), where, $RKQ(Sol_i)$ and $RKD(Sol_i)$ define the ranks of the solution Sol_i with respect to the *local population*, in terms of the objective function (34) and the *diversity contribution* $\Xi(Sol_i)$, respectively,

$$BF(Sol_i) = RKQ(Sol_i) + \left(1 + \frac{\psi_E}{\psi_T}\right) RKD(Sol_i). \tag{42}$$

This ranking process is performed while solving each scenario subproblem until all *local populations* have been updated. It is important to mention that the *local populations* are designed to be updated rather than being built from scratch at each iteration of the PH. This allows each *local population* to serve as a 'memory' for the PH, as it can retain well-ranked solutions from previous iterations.

5.4 Defining the reference solution

Once the ranking process presented in Section 5.3 is completed, one can build the *global population* of size ψ_G by including all the subset of elite solutions from all *local populations*. Given that all alternative solutions for each scenario subproblem must be considered in the *global population*, we have that $\psi_G \geq \psi_E|S|$. This *global population*, defines the base that is used to obtain the reference solution for the 2E-MLRPSCD in the subsequent steps of the PH metaheuristic.

This section presents four scenario-selection strategies to determine the reference solution at each iteration of the PH. These selection strategies include a classic strategy, which is an adaptation of the original method traditionally used in PH-based methods (Crainic et al., 2009), and three novel selection strategies. Moreover, a specialized heuristic is also introduced to define the reference solution for the first PH iteration. In the following sections, we provide a comprehensive description of each of these strategies.

5.4.1 Classic strategy

This approach represents the steps of the 'classic' selection strategy proposed by Crainic et al. (2011). Fundamentally, the classic strategy follows the guidelines of the Rockafellar and Wets (1991), by defining an aggregation operator to combine the scenario solution into a single solution, given a weight for each scenario $s \in S$. This classic strategy defines the reference solution using the single best solution obtained for each scenario subproblem. We describe the classic strategy by means of the population structures introduced in this work (see, Section 5.3) to maintain a consistent notation throughout the paper.

To describe the classic strategy, let Λ_s be the set of alternative solutions present in the *global population* for each scenario $s \in S$. For this scenario-selection strategy, we have that $|\Lambda_s| = 1, \ \forall s \in S$, to emulate the use of the single best solution for each scenario subproblem. Let ν be the index of iterations performed by the proposed PH metaheuristic. Let y^{ν}_{ai} and f^{ν}_{aijk} be the value of each alternative first-stage variable $a \in \Lambda_s$ defined for each subproblem associated with each scenario $s \in S$. Similarly, let $\bar{y}^{\nu}_i$ and $\bar{f}^{\nu}_{ijk}$ be the reference solution for iteration ν of the PH. The values of $\bar{y}^{\nu}_i$ and $\bar{f}^{\nu}_{ijk}$ can then be computed by equations (43) and (44) based on the content of the *global population* at iteration ν and the probability of occurrence ρ_s of each scenario $s \in S$,

$$\bar{y}^{\nu}_i = \sum_{s \in S} \sum_{a \in \Lambda_s} \rho_s y^{s\nu}_{ai} \quad \forall i \in Z, \tag{43}$$

$$\bar{f}^{\nu}_{ijk} = \sum_{s \in S} \sum_{a \in \Lambda_s} \rho_s f^{s\nu}_{aijk} \quad \forall i \in P, j \in Z, k \in K. \tag{44}$$

Notice that when $\bar{y}^{\nu}_i \in \{0,1\}, \ \forall i \in Z$ and $\bar{f}^{\nu}_{ijk} \in \{0,1\}, \ \forall i \in P, j \in Z, k \in K$, at a given iteration ν, this means that the method has reached consensus for the first-stage decision values. The PH metaheuristic has found thus an implementable solution for the stochastic problem. In most cases, however, the integrality requirements of the first-stage variables are not enforced, i.e., with $0 < \bar{y}^{\nu}_i < 1$ and $0 < \bar{f}^{\nu}_{ijk} < 1$, implying that the current reference solution is infeasible. Although these values are not feasible for the complete stochastic problem, they can still be used to indicate a trend of facility usage and allocation over the system. Therefore, if $\bar{y}^{\nu}_i \approx 0$, then one can interpret this as a trend towards not opening the facility i, whereas $\bar{y}^{\nu}_i \approx 1$ indicates the reverse (i.e., a trend towards opening the facility). Finally, the same observations can be made regarding the reference solution values associated with the allocation decisions.

5.4.2 Probabilistic strategy

Similar to the classic strategy presented in Section 5.4.1, we define an aggregation operator to combine the scenario solution into a single solution. The reference solution is defined by means of the given weights determined by the probability of occurrence ρ_s and the set of alternative solutions Λ_s of each scenario $s \in S$. Unlike the classic strategy, this strategy uses more than one scenario-specific solutions to define the reference solution, meaning that $|\Lambda_s| \geq 1, \ \forall s \in S$. Let y^{ν}_{ai} and f^{ν}_{aijk} be the value of each alternative first-stage variable $a \in \Lambda_s$ defined for each subproblem associated with each scenario $s \in S$

for iteration ν. The values of the reference solution defined by $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ can then be computed by equations (45) and (46) based on the content of the *global population* for each scenario $s \in S$ at iteration ν,

$$\bar{y}_i^\nu = \sum_{s \in S} \frac{\rho_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} y_{ai}^{s\nu} \quad \forall i \in Z \tag{45}$$

$$\bar{f}_{ijk}^\nu = \sum_{s \in S} \frac{\rho_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} f_{aijk}^{s\nu} \quad \forall i \in P, j \in Z, k \in K. \tag{46}$$

The probabilistic strategy enables the consideration of a broader range of options for the first-stage decision variables available in the *global population* for each scenario subproblem $s \in S$. At the same time, the aggregation operator defined to combine the scenario solution into a single solution is versatile enough to be used with other selection strategies and can be adapted to behave like the classic strategy simply by setting $|\Lambda_s| = 1,\ \forall s \in S$. We, therefore, use the same aggregation operator with the remaining scenario-selection strategies in this section.

5.4.3 Social strategy

The idea behind the social strategy is to define the set of solutions with the best *social score* among the *global population*. We define $\pi(Sol_i, Sol_j)$ as a normalized Hamming distance, which is computed using equation (47). This metric evaluates the similarity between two distinct solutions, Sol_i and Sol_j. We thus define $\chi_z(Sol_i)$ and $\kappa_k(Sol_i)$ as the functions that return the location decision of each $z \in Z$ and allocation of each OD demand $k \in K$, respectively, of a given solution Sol_i,

$$\pi(Sol_i, Sol_j) = \frac{1}{|K|} \sum_{k \in K} \mathbf{1}(\kappa_k(Sol_i) = \kappa_k(Sol_j)) + \frac{1}{|Z|} \sum_{z \in Z} \mathbf{1}(\chi_z(Sol_i) = \chi_z(Sol_j)). \tag{47}$$

The *social score* for a given solution Sol_i is defined by summing the values of equation (47) between the solution Sol_i and all other solutions Sol_j in the *global population*, where $i \neq j$. Using these *social score* values, the solutions in the *global population* can be ranked. It is important to note that this rank favors solutions that share the most similarities with other solutions, meaning that solutions with the most commonalities in both location and allocation first-stage decisions within the *global population* will be ranked higher.

There are two main approaches for determining the reference solution based on the rank of each scenario subproblem. The first approach involves selecting a reduced set of elite solutions from the complete *global population* to define the reference first-stage decisions. This can be achieved by following the steps defined for the probabilistic strategy, as described in Section 5.4.2. The second approach involves identifying a single elite solution, whose first-stage decisions will be used as the reference solution. However, preliminary experiments conducted using these two strategies have shown that using a single elite solution as the reference solution can cause the PH metaheuristic to become trapped in local optima; consequently, we exclusively utilize the first strategy.

5.4.4 Decision-based scenario clustering strategy

The *decision-based scenario clustering strategy* is proposed to identify scenario groups that lead to mutually acceptable solutions (i.e., solutions that remain efficient when considering all the subproblems associated with the scenarios included in the group). Fundamentally, the proposed strategy uses a *dissimilarity function* inspired by the *opportunity cost*, originally proposed by Hewitt et al. (2022). This *opportunity cost* is defined as a measure to quantify the impact of implementing the first-stage decisions associated with a given scenario s_1 when another scenario s_2 occurs. This measure relies on the existence of a single solution for each of the scenarios involved. This characteristic prevents the direct application of the *opportunity cost* defined by Hewitt et al. (2022) for our PH metaheuristic, which uses a set of alternative solutions for each scenario subproblem to determine the *reference solution*. Therefore, this section introduces the proposed *decision-based scenario clustering strategy* to leverage the alternative solutions associated with each scenario subproblem.

The proposed strategy aims to define a specialized *opportunity cost* measure based on the subset of solutions in the *global population* associated with each scenario. Therefore, let Λ_s be the set of indices of the solutions in the *global population* that are associated with scenario $s \in S$. Let $g_i^{\nu}((\hat{y}_n^{\nu*}, \hat{f}_n^{\nu*}); s_j)$ be the updated value of the objective function (34), evaluated with the set of the best first-stage decision variables $\hat{y}_n^{\nu*}$ and $\hat{f}_n^{\nu*}$ at iteration ν, obtained for the solution $n \in \Lambda_{s_i}$ with scenario s_i, when a scenario s_j occurs. The *opportunity cost*, denoted by $\theta^{\nu}(s_i|s_j)$, represents the value of the decision associated with scenario s_i under the assumption that scenario s_j actually occurs. This quantity is calculated using equation (48) as the minimum value obtained by evaluating all the combinations of solutions associated with each pair of distinct scenarios s_i and s_j in S with $i \neq j$.

$$\theta^{\nu}(s_i|s_j) = \min_{n \in \Lambda_{s_i};\, m \in \Lambda_{s_j}} \{g_i^{\nu}((\hat{y}_n^{\nu*}, \hat{f}_n^{\nu*}); s_j) - g_j^{\nu}((\hat{y}_m^{\nu*}, \hat{f}_m^{\nu*}); s_j)\} \tag{48}$$

Based on the *opportunity costs* determined for each scenario within the *global population*, one can then define an *opportunity cost dissimilarity function* by equation (49) for each pair of scenarios s_i, $s_j \in S$ with $i \neq j$, which represents the loss incurred by optimizing under the assumption that scenario s_i happens, when scenario s_j occurs instead, and vice versa.

$$d^{\nu}(s_i|s_j) = \theta^{\nu}(s_i|s_j) + \theta^{\nu}(s_j|s_i) \tag{49}$$

A *Normalized Spectral Clustering* is then used to determine which scenarios are close to each other in terms of *opportunity cost distance function* building upon the approach proposed by Hewitt et al. (2022). This process yields a set of clusters $CL = \{cl_1, cl_2, \ldots, cl_{|CL|}\}$ of the scenarios. Once the clusters are determined, we define a set of representative scenarios Υ, where each representative scenario corresponds to the *medoid* of each cluster (i.e., the scenario with the minimum average opportunity cost dissimilarity function to all other scenarios within the same cluster). Subsequently, we assign the probability η_i to each representative scenario $s \in cl_i$ for all $cl_i \in CL$, computed as the sum of the probabilities ρ_s of all scenarios within the same cluster, as shown in equation (50). Finally, the reference solution for the location and allocation decisions can be determined by computing equations (51) and (52) for each representative scenario $s \in \Upsilon$ and the subset of solutions Λ_s associated with each scenario.

$$\eta_i = \sum_{s \in cl_i} \rho_s \quad \forall cl_i \in CL \tag{50}$$

$$\bar{y}_i^{\nu} = \sum_{s \in \Upsilon} \frac{\eta_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} y_{ai}^{s\nu} \quad \forall i \in Z \tag{51}$$

$$\bar{f}_{ijk}^{\nu} = \sum_{s \in \Upsilon} \frac{\eta_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} f_{aijk}^{s\nu} \quad \forall i \in P, j \in Z, k \in K \tag{52}$$

5.4.5 First iteration reference solution

A major goal of our PH metaheuristic is to efficiently guide the process searching for solution consensus. An initial reference solution $\bar{y}_i$ and $\bar{f}_{ijk}$ must be defined to enable the scenario-based decomposition of the extensive formulation, as described in Section 5.2. These initial values are used to define the solutions for the scenario subproblems obtained in the first iteration of the PH. However, no 'memory' is available in the PH to assess the quality of these solutions since the *local populations* are empty at this stage. Defining a high-quality reference solution at the end of the first iteration is crucial in this case, considering that the consensus search is performed by successively adjusting the objective function costs of the scenario subproblems to gradually encourage agreement. The quality of the decisions defined in the first iteration will greatly influence the subsequent ones.

We propose an heuristic to define the reference solution to be applied in the first iteration of the PH metaheuristic and the initial *global population.* Let us recall that the *global population* is composed of at least one elite solution picked from the *local population* of each scenario subproblem. The 'quality' of the initial reference solution will thus be a function of both the quality of the solutions present in the initial *global population* and the specific selection strategy that is used to obtain the point.

The proposed heuristic generates an initial *global population* by comparing two independent population generation strategies. Let GP_1 and GP_2 be two independent *populations*, each constructed using one of the proposed heuristic strategies. GP_1 is populated with the set of elite alternative solutions from the *local population* of each scenario subproblem, while GP_2 is populated with the single best solution found for each subproblem. Once GP_1 and GP_2 are populated, we let the given selection strategy determine the reference solution of the first-stage decisions for each population. Notice that the resulting reference solutions may contain decision variables with continuous values. To address this, the proposed heuristic *approximates* each reference solution by rounding each continuous value to the nearest discrete value. Each *approximation* now represents an integer solution for the first-stage decisions, which can be evaluated in the extensive formulation. After evaluating each *approximation*, the proposed heuristic selects the reference solution leading to the best objective function to determine which of the two populations should be defined as the first *global population*.

To keep the solutions used to define the best reference solution in subsequent iterations, one must update each *local population* accordingly, as the *global population* is built from scratch at each iteration. Note that *local populations* are only modified when GP_2 is selected as the best initial *global population*. In such case, each *local population* is updated to retain only the best single scenario-specific solution, rather than the complete set of alternative solutions.

It is worth mentioning that preliminary experiments conducted using the proposed heuristic at each iteration of the PH led to the method relying on the *approximation* of the reference solution as the main guiding strategy. This caused the method to become trapped in local optima. Consequently, after the first iteration, the PH continues to work with the set of alternative solutions for each scenario subproblem.

5.5 Consensus procedure

This section describes the heuristics to adjust the costs of the scenario subproblems aiming to guide the PH method towards a consensus for the first-stage solutions over all the scenario subproblems. We build on the work of Crainic et al. (2011) and present two adjustment heuristics to modify the location and allocation costs in the scenario subproblems, specifically, a global adjustment designed for the overall search and a local adjustment to influence the search for each scenario subproblem.

The proposed global adjustment begins with the reference solution defined by $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ at iteration ν to identify trends among the scenario solutions. The costs are defined according to the objective function (34). In this context, we define the costs $\bar{B}_i^{s\nu} = \left(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma\bar{y}_i\right)$ and $\bar{E}_{ijk}^{s\nu} = \left(\Delta_{ijk} + \mu_{ijk}^s + \frac{1}{2}\gamma + \gamma\bar{f}_{ijk}\right)$ as the location and allocation costs of the scenario subproblem, respectively.

As mentioned previously, low values of $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ indicate that most of the scenario solutions share the decision to keep the given facility closed, while high values mean that the facility is open in the majority of the scenario solutions. Therefore, we introduce a parameter $\beta > 1$ as the adjustment rate of the costs, and threshold parameters $0 \leq \epsilon^y \leq 0.5$ and $0 \leq \epsilon^f \leq 0.5$ to determine when the values $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ should be considered either high or low. Specifically, when $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ are lower than ϵ^y and ϵ^f, the fixed costs are increased to incentivize the subproblems to avoid opening the corresponding facility and performing the associated allocation. On the other hand, when $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ are higher than $1 - \epsilon^y$ and $1 - \epsilon^f$, the fixed costs are decreased to encourage the subproblems to include the facility in the network design and perform the allocation. We define this procedure with equations (53) and (54):

$$\bar{B}_i^\nu = \begin{cases} \beta B_i^{\nu-1} & \text{if } \bar{y}_i^{\nu-1} < \epsilon^y, \\ \frac{1}{\beta} B_i^{\nu-1} & \text{if } \bar{y}_i^{\nu-1} > 1 - \epsilon^y, \\ B_i^{\nu-1} & \text{otherwise;} \end{cases} \tag{53}$$

$$\bar{E}_{ijk}^\nu = \begin{cases} \beta \bar{E}_{ijk}^{(\nu-1)} & \text{if } \bar{f}_{ijk}^{(\nu-1)} < \epsilon^f, \\ \frac{1}{\beta} \bar{E}_{ijk}^{(\nu-1)} & \text{if } \bar{f}_{ijk}^{(\nu-1)} > 1 - \epsilon^f, \\ \bar{E}_{ijk}^{(\nu-1)} & \text{otherwise.} \end{cases} \tag{54}$$

The second adjustment strategy is performed at the level of each scenario subproblem $s \in S$, where the costs of variables with large differences between the value of the current reference solution at iteration ν, are further adjusted using the equations (55) and (56). In this context, we define $0.5 < \delta^y < 1$ and $0.5 < \delta^f < 1$ as the thresholds that prescribe when a local adjustment has to be applied for the location and allocation variables, respectively:

$$\bar{B}_i^{s\nu} = \begin{cases} \beta B_i^\nu & \text{if } |y_i^{s(\nu-1)} - \bar{y}_i^{\nu-1}| \geq \delta^y \text{ and } y_i^{s(\nu-1)} = 1, \\ \frac{1}{\beta} B_i^\nu & \text{if } |y_i^{s(\nu-1)} - \bar{y}_i^{\nu-1}| \geq \delta^y \text{ and } y_i^{s(\nu-1)} = 0, \\ B_i^\nu & \text{otherwise;} \end{cases} \tag{55}$$

$$\bar{E}_{ijk}^{s\nu} = \begin{cases} \beta \bar{E}_{ijk}^\nu & \text{if } |f_{ijk}^{s(\nu-1)} - \bar{f}_{ijk}^{(\nu-1)}| \geq \delta^f \text{ and } f_{ijk}^{s(\nu-1)} = 1, \\ \frac{1}{\beta} \bar{E}_{ijk}^\nu & \text{if } |f_{ijk}^{s(\nu-1)} - \bar{f}_{ijk}^{(\nu-1)}| \geq \delta^f \text{ and } f_{ijk}^{s(\nu-1)} = 0, \\ \bar{E}_{ijk}^\nu & \text{otherwise.} \end{cases} \tag{56}$$

Given that there is no reference solution in the original extensive formulation (Section 4.2), we set the values of the initial overall design variables $\bar{y}_i$ and $\bar{f}_{ijk}$ to determine the initial fixed costs for each scenario subproblem. We, therefore, define the initial overall design in terms of the location costs $\bar{B}_i^{s\nu}$ and allocation costs $\bar{E}_{ijk}^{s\nu}$ at iteration $\nu = 0$ (i.e., before the PH starts its first iteration). The location and allocation costs of the scenario subproblems are initialized with their original costs. Therefore, we set

$\bar{B}_i^{s(0)} = F_i, \ \forall i \in Z, s \in S$, and $\bar{E}_{ijk}^{s(0)} = \Delta_{ijk}, \ \forall i \in P, j \in Z, k \in K, s \in S$. Note that the values of the initial overall design $\bar{y}_i$ and $\bar{f}_{ijk}$ will be updated based on the reference solution obtained at the end of the first iteration of the PH to adjust the costs for each scenario subproblem.

The proposed PH is designed to terminate once either a consensus solution is found, or another stopping criterion is reached (e.g., a limit on computation time). A consensus solution is determined when all first-stage decisions $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ have reached a general consensus at a given iteration ν. However, consensus on all first-stage decisions may not be observed at the end of each iteration of the PH metaheuristic. When such a situation occurs, the PH is designed to define a feasible solution for the 2E-MLRPSCD by using the extensive formulation presented in Section 4.2. The approach to defining a feasible solution consists of fixing the location and allocation variables for which consensus is obtained by the PH metaheuristic, and then solving the restricted mixed-integer program defined by the extensive formulation. The results of solving the proposed formulation yield a feasible solution for all the design decisions. One can then update the best solution obtained and continue with the PH metaheuristic.

5.6 Reset procedure

As described previously, the proposed PH algorithm relies on the *global population* to determine the reference solution at each iteration. This *global population* is constituted by the collection of elite solutions from the *local population* of each scenario subproblem. As the search progresses, certain solutions may remain in the *local population* of each scenario subproblem for several iterations of the PH. Consequently, the *global population* may also end up comprising the same set of solutions over consecutive iterations of the PH. If the *global population* remains unchanged over several iterations, the reference solution may become trapped on a series of values that hinder the overall search for a consensus solution. To mitigate such occurrences, we propose a reset procedure that partially reinitializes the overall search process. Specifically, the reset procedure is triggered when the values of the reference solution do not change for ι consecutive iterations. When triggered, the reset procedure clears the contents of all *local populations* and repopulates them with solutions from their corresponding scenario subproblems obtained in the ongoing iteration. It is important to note that while this process defines a new set of alternative solutions for the *global population*, the current costs corresponding to the first-stage decisions in each scenario subproblem, which were updated over the previous PH iterations, remain unchanged.

6 Computational results

This section presents the results of the computational experiments that were conducted to assess: (1) the stability of the scenario generation procedure (Section 6.2), (2) the performance of the proposed PH-based metaheuristic (Section 6.3), (3) the effectiveness of the proposed acceleration procedures for the 2E-MLRPSCD and (4) evaluate the need to explicitly consider stochastic demands and the impact of correlations (Section 6.5 and Section 6.4). We first introduce the instances and the scenario generation procedure in Section 6.1.

The experiments were conducted on a single machine with Intel(R) Core(TM) i7-7800X processor, with 128 GB of RAM running Linux. The mathematical formulation and the proposed solution method are implemented in C++ using IBM ILOG CPLEX concert technology 20.1. The MIPs used within the solution method were solved with an optimality gap tolerance of 1% as the stopping criterion. Finally, the computation times reported are in seconds. The tables display summarized results; more detailed results are provided in the supplementary material in Appendix B.

6.1 Instances

We define our testbed based on the instances introduced by Dellaert et al. (2019) for the 2EVRPTW, since no instances were available in the literature involving the integrated treatment of all of the attributes considered in the 2E-MLRPSCD. The instances introduced by Dellaert et al. (2019) simulate an urban area constituted of platforms, satellites, and customers. The original instances generated do not consider stochastic correlated OD demands, which are explicitly included in the 2E-MLRPSCD. Furthermore, the original instances included delivery time windows, which are not considered in the present setting. Therefore, adjustments were made to the original instances to obtain the testbed for the present study. These adjustments involved the introduction of stochastic and correlated OD demands and the exclusion of the temporal components in the original instances.

Our instance set consists of 60 instances, each with 15 OD demands. We randomly assigned to each platform facility an unique set of OD demands. The same load capacities for vehicles set in Dellaert et al. (2019) were used. The first-level vehicles have thus a capacity of $cap_1 = 200$ and the second-level vehicles have a capacity of $cap_2 = 50$. Travel costs are computed as the ceiling of the Euclidean distances.

Scenarios are generated using the copula-based method proposed by Kaut (2014) to adhere to the statistical properties defined for the stochastic OD demands. This procedure requires the target marginal distribution for each OD demand (which can be specified using a set of marginal distributions available in the method), and the correlation

category	distribution	mean	standard deviation
CA	left-skewed lognormal	2.7	0.4
CB	symmetrical lognormal	2.7	0.1
CC	left-skewed lognormal	3.25	0.4
CD	symmetrical lognormal	3.25	0.1

Table 1: Instance category description

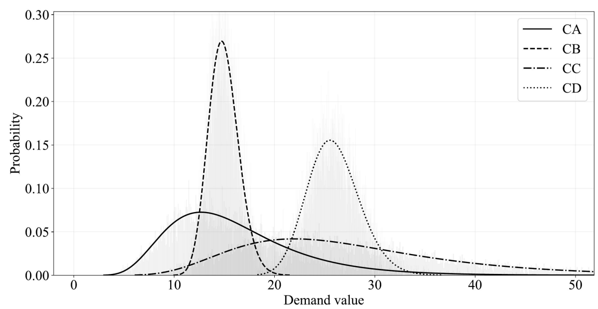

Figure 3: Instance category distribution for scenario generation.

matrix between OD pairs as inputs.

To determine the marginal distribution for all OD demands, we considered the original demand values in the set of instances defined by Dellaert et al. (2019). We identified the distribution that provided the closest fit (among the marginal distributions available in the copula-based method) to the value of the OD demands in the complete instance set. This led us to select a lognormal distribution (with similar mean and standard deviation values) as the best fit for the given demand values. We then defined a set of four lognormal distributions with different mean and standard deviation values to capture the impact on the variation of the marginal distributions representing the demand. Table 1 introduces the proposed instance set, categorized into four groups: CA, CB, CC, and CD. As illustrated in Figure 3, a lognormal distribution with consistent mean and standard deviation values, as defined in Table 1, is utilized for each instance category.

To define demand correlation, two testbed instances are proposed: one considering demand correlation and the other without demand correlation. In the correlated case, correlation matrices are randomly generated. Correlations between OD pairs are determined using a standard normal distribution with a range of [-0.6, 0.6] for each correlation value.

A properly defined correlation matrix must be positive semidefinite (Xu and Evers, 2003). Before applying the scenario generation method, this condition is verified for each correlation matrix obtained (i.e., correlation matrices that do not meet this condition are ignored by the copula-based method). Scenarios are generated, once the positive semidefinite condition is verified. The copula-based heuristic uses the mean and standard deviation of the distributions for each instance category and the correlation matrix defined for each instance to generate a predefined number of scenarios $|S|$ with equal probability. This means that the probability of occurrence ρ_s for scenario $s \in S$ is $\rho_s = 1/|S|$.

6.2 Scenario stability

This section presents the computational experiments conducted to assess the stability of the chosen scenario generation procedure with and without demand correlation. Assessing scenario stability aims to guarante that there is no significant influence by the scenario trees utilized with respect to the results obtained when solving the considered stochastic problem (Kaut and Wallace, 2007). In our case, we used the copula-based method introduced by Kaut (2014) to generate the scenario set. This method, in contrast to other ones (such as sampling methods), has a high probability of producing identical scenario trees when consecutive runs are conducted with the same correlation and distribution inputs. Using 'standard' in-sample and out-of-sample stability tests (see, Kaut and Wallace, 2007) are inappropriate, as these stability tests could overestimate the quality of the scenario generation method (Guo et al., 2019). Therefore, our scenario stability tests build on the work of Zhang et al. (2021) to derive a valid variant of the 'standard' approach for our problem setting.

Based on the guidelines proposed by Zhang et al. (2021), stability tests require creating and evaluating a subset of scenario trees for each problem instance, with a fixed scenario tree size. To test the stability of a scenario tree of size $|S|$, it is necessary to define a set of $2m+1$ scenario trees of sizes $|S|-m, |S|-(m-1), ..., |S|, ..., |S|+m$, where m is a positive integer. Let $Z_{|S|+i}$ denote the optimal (or best-known) solution for each $i \in [-m, m]$ of the $2m+1$ scenario trees obtained for each problem instance. The proposed PH metaheuristic is used to solve the 2E-MLRPSCD resulting from each of the $2m+1$ scenario trees, resulting in $2m+1$ solutions $Z_{|S|+i}$, one for each scenario tree. These solutions are then evaluated by calculating the objective function $F(Z_{|S|+i})$ for each of the $2m+1$ scenario trees, yielding a set of $2m+1$ objective function values for each solution $Z_{|S|+i}$. Finally, for each problem instance, the maximum $(F^+(Z_{|S|+i}))$, minimum $(F^-(Z_{|S|+i}))$, and variance $(\sigma_{|S|+i})$ are defined based on the objective function values of each solution $Z_{|S|+i}$. Stability is then determined by computing the relative difference (RD) between the maximum and minimum values and the variance (VAR) of each scenario tree, as follows:

$$RD = \max_{i \in [-m,m]} \left\{ \frac{F^+(Z_{|S|+i}) - F^-(Z_{|S|+i})}{F^+(Z_{|S|+i})} \times 100\% \right\} \tag{57}$$

$$VAR = \max_{i \in [-m,m]} \{\sigma_{|S|+i}\} \tag{58}$$

$\|S\|$	RD (%)					VAR		
	VSR	AISR	MIN	AVERAGE	MAX	MIN	AVERAGE	MAX
10	38	3.96	0	1.79	8.58	0	1041.89	15222.89
20	47	3.17	0	1.13	5.93	0	335.62	3351.09
30	54	2.65	0	0.82	3.55	0	190.04	1944.3
40	53	2.25	0	0.58	2.25	0	108.05	1142.38
50	53	2.01	0	0.45	2.01	0	74.62	933.34
100	60	n.a	0	0.28	1.44	0	32.52	525.93

Table 2: Stability tests: summarized results of relative difference and variance for different scenario sizes.

Table 2 presents a summary of the relative difference (RD) and variance (VAR) values obtained for the $2m+1$ scenario trees defined for each problem instance. The table shows the number of scenarios for each scenario tree ($|S|$), as well as the minimum (MIN), average (AVR), and maximum (MAX) values for the relative difference and variance. In order to assess stability, a criterion of RD $\leq 2\%$ is defined. Two additional performance measures are used to present the number of instances satisfying the stability criterion (valid stability requirement or **VSR**) and the average RD of the instances failing to meet the criterion (average invalid stability requirement or **AISR**). Experiments are performed using multiple scenario trees with varying numbers of scenarios ($|S|$) and m set to 4, based on the work of Guo et al. (2019). To reduce noise in the results, only the best objective function obtained for each instance by the proposed PH metaheuristic is used for the stability tests.

The results reported in Table 2 show the expected reduction of the relative difference with respect to the increased number of scenarios considered. It is worth mentioning that due to the randomness and heuristic nature of the scenario generation procedure, small fluctuations exist in the relative error for some instances, which do not affect the validity of the proposed stability tests. In general, the use of 30 scenarios represents the best balance between solution stability and scenario size, as it provides a relative error of less than 2% for 53 out of 60 instances, with an average relative error of 2.6% for the remaining instances. Although larger scenario trees are desirable for achieving a smaller relative error, solving the resulting subproblems for the entire instance set using CPLEX at each iteration of the PH metaheuristic becomes exceedingly challenging.

Figure 4 displays the relative difference values of each instance type as a function of the number of scenarios used for the stability testing. One can observe that the dispersion of the demand distributions significantly affects the stability of solutions. Notably, instances of type CA and CC, characterized by more dispersed demand distributions, exhibit more fluctuations in the relative difference values. This behavior can be attributed to the copula-based method generating a diverse set of scenarios, leading to increased volatility in the objective function values and greater fluctuations on recourse actions.

In conclusion, the results presented in Table 2 indicates that using $|S| = 30$ achieves the best balance between solution stability and the size of the scenario tree. Additionally,

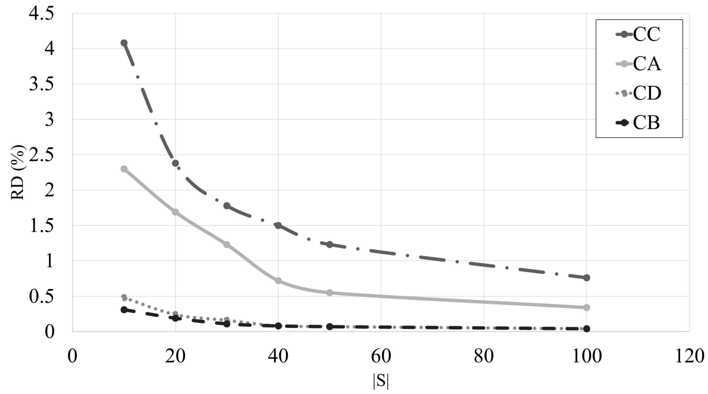

Figure 4: Stability test: Relative difference for each instance type vs scenario size.

Figure 4 shows that increasing the number of scenarios beyond $|S| = 30$ only marginally improves the relative difference percentage in terms of stability. This in turn indicates that using scenario trees with $|S| > 30$ is not favorable due to the significantly increased computational burden required to address this greater number of scenarios at each iteration of the PH metaheuristic.

6.3 Performance of the PH metaheuristic

This section presents a performance analysis of the proposed PH metaheuristic. Computational tests were conducted to compare the performance of the PH metaheuristic to that of CPLEX when solving the complete stochastic model. The results presented in this section focus on the quality of the upper bound obtained and the computational time needed by each solution method. The stopping criteria for all solution methods were set to a maximum running time of 2 hours. Additionally, the PH metaheuristic was limited to a maximum of 60 iterations. CPLEX was used with the default parameter settings, with a thread limit of 6 imposed when solving the overall stochastic model and a thread limit of 1 specified when solving the scenario subproblems within the PH metaheuristic.

To address the stochastic problem, computational tests were conducted by solving the complete two-stage stochastic model with CPLEX or by employing the proposed PH metaheuristic. Additionally, experiments were performed using a *classic strategy* as the baseline for the PH metaheuristic. This approach represents the steps of the 'classic' PH metaheuristic proposed by Crainic et al. (2011) (Section 5.4.1). In all tables, the results obtained using CPLEX and the PH metaheuristic are labeled as 'CPLEX' and 'PH', respectively. Moreover, the PH metaheuristic results are differentiated based on the specific version of the procedure used that is: the classical approach (CL), the probabilistic strategy (PS), the social strategy (SS), and the decision-based clustering strategy (DCS). Each table presents the average optimality gap expressed as a percentage (OG), the average computational time in seconds, and the average upper bound differences between PH and CPLEX (Diff. UB).

Table 3 presents the results of solving instances with no demand correlation. One can clearly observe that solving the overall stochastic problem using CPLEX is challenging. CPLEX achieves an average optimality gap of 21% within the time limit of 2 hours. The PH metaheuristic outperforms CPLEX. The classic approach (CL), achieves an average improvement of 14.5% in solution quality over CPLEX. Moreover, the classic approach is able of reaching consensus and generating high-quality upper bounds for 53 of the 60 instances within the 2-hour limit. One notices that the exclusive use of the best quality solutions of each scenario subproblem to define the reference solution, is not effective enough to reach consensus over the complete set of first-stage decisions. This general adverse effect is particularly evident when analysing the results obtained when solving the CA and CC instances. For these instances, the demand values are sparse, which leads to scenario subproblems which, when solved, tend to produce more diverse first-stage decisions.

Compared to the classic approach, the proposed strategies can achieve consensus for the entire set of instances. The probabilistic strategy, which builds on the classic approach, demonstrates significant improvements in both solution quality and runtime. Specifically, the probabilistic strategy achieves an average improvement of 16.2% compared with CPLEX and a 35.3% decrease of average runtime compared with the classic approach. To reach consensus efficiently, our PH metaheuristic benefits from including alternative solutions for each scenario subproblem. This approach increases the number of complementary first-stage decisions that are used to define the aggregation at each iteration. Notwithstanding the general improvements made by PH utilising the probabilistic strategy, the social and cluster strategies are able of leveraging more efficiently the existing alternative solutions to further improve the overall performance of the algorithm.

The social strategy consistently yields reduced runtimes, with the largest average decrease of 68.3% compared to the classic approach. The consensus-driven approach, which ranks the global population, is also able to help PH metaheuristic reaching consensus faster and cut down on computation time. However, reaching general consensus faster does not necessarily guarantee good-quality solutions. An illustration of this can be seen when comparing the results obtained with the decision-based clustering strategy to those obtained with the social strategy. Although the runs of the PH using the social strategy produces results, on average, 40% faster, the use of the decision-based clustering strategy results in consensus solutions of higher quality. On average, the optimality gap achieved by the PH metaheuristic with the decision-based clustering strategy is 2.6%, compared to 6.8% when using the social strategy.

Computational tests on instances with demand correlation are reported in Table 4. Similar to the results obtained on instances with no demand correlation, solving the complete two-stage formulation with CPLEX leads to the worst optimality gap while

Instance type	CPLEX		PH											
			CL			PS			SS			DCS		
	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)
CA	30.91	7200	-7.02	28.24	4458.16	-13.54	23.71	1323.66	-38.65	8.09	884.99	-44.98	3.76	2295.22
CB	23.66	7200	-32.54	1.54	3568.78	-32.54	1.54	1350.34	-32.62	1.47	1081.83	-32.66	1.44	1239.97
CC	17.42	7200	-9.07	10.07	4651.69	-9.30	9.87	4243.97	-9.04	10.21	2326.02	-19.35	1.95	2442.96
CD	15.79	7200	-9.19	8.11	4520.49	-9.32	8.01	4210.59	-10.11	7.38	1151.61	-14.92	3.47	2200.99
Averages	***21.94***	***7200***	***-14.46***	***11.99***	***4299.78***	***-16.17***	***10.78***	***2782.14***	***-22.61***	***6.79***	***1361.11***	***-27.98***	***2.65***	***2044.79***

Table 3: Summarized results on instances with no demand correlation.

Instance type	CPLEX		PH											
			CL			PS			SS			DCS		
	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)
CA	28.70	7200	1.49	29.77	7200.00	-29.94	10.14	2393.54	-38.05	4.71	2178.57	-42.06	2.35	1663.38
CB	24.35	7200	-33.52	0.86	4071.12	-33.76	0.67	1384.67	-33.76	0.67	1989.33	-33.88	0.58	1920.12
CC	13.03	7200	-11.17	3.57	4622.19	-12.50	2.47	2095.32	-12.71	2.29	2167.33	-13.64	1.48	2091.91
CD	11.77	7200	-7.76	5.10	4809.67	-10.45	2.84	1609.31	-11.35	2.02	2061.57	-11.87	1.56	1781.87
Averages	***19.46***	***7200***	***-12.74***	***9.82***	***5175.74***	***-21.66***	***4.03***	***1870.71***	***-23.97***	***2.42***	***2099.20***	***-25.36***	***1.49***	***1864.32***

Table 4: Summarized results on instances with demand correlation.

reaching the maximum time limit on all instances. On the other hand, the classic approach presents a significant performance improvement, where consensus is reached for 40 out of the 60 instances within the given time limit. The probabilistic and social strategies both show considerably improved performances in solution quality when compared to CPLEX, with an optimality gap of 4% and 2.4%, respectively. That being said, the decision-based clustering strategy outperforms all selection strategies in terms of both time and solution quality. It obtains the best solutions for 50 out of the 60 instances with an average runtime reduction of 64% when compared to the classic approach.

The performance of the PH metaheuristic with each proposed aggregation strategy shows significant variations when tested on instances with and without demand correlation. Scenario trees generated assuming that demands are entirely uncorrelated often result in scenarios with a large number of high demand values. These scenarios have more predominant solution structures, as the first-stage decisions defined under high demand values are more likely to fit scenarios with lower demand values. This effect is less likely to occur for instances where negative demand correlation is considered. The likelihood of producing scenarios with high demand values decreases as the degree of negative correlation between demands increases. As a result, there is increased diversity in demand values for the complete set of scenarios. This diversity leads to a more varied set of first-stage decisions, which, in turn, poses challenges for the PH metaheuristic to reach consensus. This general effect explains the improved performance of the proposed acceleration strategies when demand correlation is considered.

6.4 Correlation impact

Our experiments involve three sets of instances, each considering a different correlation level. Scenarios are generated for instances involving 15 OD demands, with weak, moderate, and strong correlation levels. These levels are defined by a rescaled normal distribution presented in Table 5. Figure 5 illustrates different correlations in the same instance.

Computational studies are conducted using the proposed PH framework with the decision-based clustering strategy, which is selected due to its superior overall performance. We compare the results of weak and strong correlations to those of the moderate case. Table 6 displays the average relative differences in upper bound value (UB dif.), the standard deviation (STD dif.), and run time (Time dif.). The standard deviation is used as a measure to illustrate how dispersed the demand values are within the scenarios generated with different levels of correlation. A negative value in the UB dif. or the Time dif. column indicates a reduction in the measure for instances with either weak or strong levels of correlation. Similarly, a negative value in the STD dif. column indicates that scenarios generated with a moderate level of correlation are more dispersed than the other two correlation levels.

The reported results in Table 6 show changes in different instance categories. For instance types CA and CB, characterized by lower demand values, there is an average 4% increase in the upper bound value. In contrast, instance types CC and CD, characterized by greater demand values, show an average decrease in the upper bound value of 14%. Several factors contribute to this behavior. One is the dispersion of the scenarios. Achieving consensus over a more diverse set of scenarios often results in solutions with larger numbers of routes or more intricate route compositions to accommodate the demands of the complete scenario set without extensive use of recourse actions. Another factor is the ratio of positive and negative demand correlations. A higher ratio of positive correlation leads to scenarios with a higher number of large demand values compared to those with lower levels of correlation. It is worth mentioning that while the probability distribution for each category type remains constant for the three correlation types, some correlation matrices lead to more dispersed scenarios or to scenarios with higher demand values than the other two correlation types, greatly influencing the solution of each instance when determining consensus.

In instances of type CA and CB, moderate correlations display a lower ratio of positive correlations in 19 and 13 out of the 30 instances compared to those with weak and strong correlations, respectively. This, combined with the lower standard deviation observed in scenarios using moderate correlation, leads to an overall decrease in the objective function by at least 3% compared to the other two correlation levels. In contrast, instances of type CC and CD, using moderate correlations, exhibit scenarios with higher dispersion, resulting in a notable increase in the objective function, by at least 13%, compared to scenarios with weak and strong correlations. This high objective function is influenced by the necessity for a greater number of routes or the implementation of more complex route compositions to fulfill all OD demands and reduce outsourced deliveries. In terms of computational time, it is evident that the PH method generally requires more time to address instances with weak and strong correlations, particularly in cases with lower variability, due to the need for a more extensive application of the reset procedure to

Correlation type	Distribution	Range
Weak	standard normal	[-0.3, 0.3]
Moderate	standard normal	[-0.6, 0.6]
Strong	standard normal	[-1.0, 1.0]

Table 5: Parameters for each correlation type.

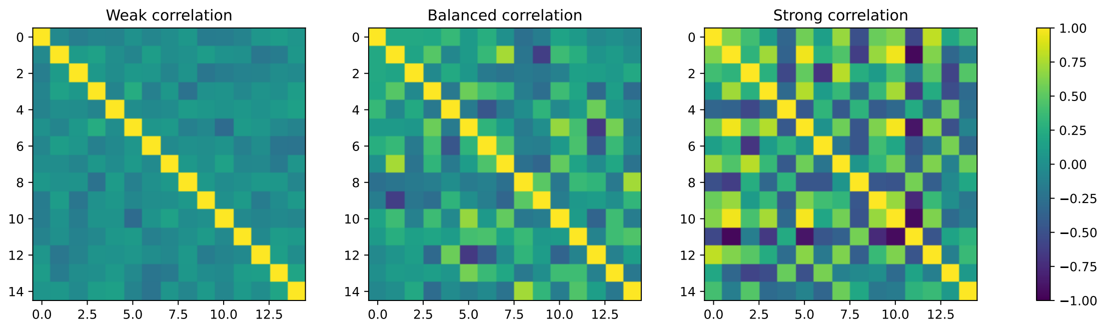

Figure 5: Correlation types for instance Cd5-6,4,15.

avoid local optima.

Overall, it appears that the level of correlation significantly affects solution quality and runtime. Negative correlation provides opportunities to consolidate negatively correlated demands, often leading to a reduced variance in the total demand within the routes in the solution. This facilitates cost-effective routes that serve more OD demands. However, greater variability in demand values often results in more intricate route structures to accommodate the demands of all scenarios, potentially leading to higher operational costs. It can be concluded that this variability is not exclusively attributable to the correlation levels, but rather to the interplay between the demand distribution used by the copula-based method and the correlation. This highlights the importance of accurately capturing any dependence between OD demands.

Instance type	Weak correlation			Strong correlation		
	UB dif.	STD dif.	Time dif.	UB dif.	STD dif.	Time dif.
CA	5.17	1.82	2.81	6.99	2.63	18.50
CB	1.83	-24.12	7.84	2.43	-21.31	5.03
CC	-1.21	5.52	-14.41	-1.74	4.56	-7.50
CD	-26.46	-355.18	5.08	-27.07	-345.28	17.53

Table 6: Summarized results with different type of correlation types.

6.5 Value of the stochastic solution

This section reports on the value of the stochastic solution (VSS), which is a bound to assess the added value of using the stochastic model compared to the deterministic formulation problem for the 2E-MLRPSCD. Experiments are performed using both instances with and without demand correlation. Following the general trend in the literature, we use the deterministic formulation (DF) with the mean approximation of the demand, where the stochastic demands are estimated as their mean values obtained from the scenario sets that are considered. The integrated design and routing decisions are then determined based on the average value of demands. Results for the deterministic formulation of the 2E-MLRPSCD are obtained by solving each instance using CPLEX under a 2-hour time limit. The computational tests conducted using the DF are then compared to the solutions obtained by solving the 2E-MLRPSCD (i.e., the results reported in Section 6.3.

Feasible solutions for all instances can be obtained by using the PH metaheuristic and the DF. Therefore, we conduct experiments by comparing the results obtained from the DF approach against the stochastic approach with respect to the general cost increase percentages. Figures 6 and 7 present the results of these experiments by illustrating the increasing percentage of the objective function (Cost Diff.) and the use of outsourced services (Out Diff.) for the instances with and without demand correlation, respectively. The results are organized to depict the minimum, average, and maximum cost increase associated with the solutions of the deterministic formulation, using the stochastic approach as the baseline. Tables 7 and 8 present the density of location and allocation decisions on satellite facilities for each approach and each instance type for instances with and without demand correlation, respectively. Each table shows the number of satellites ($|Z|$) for each instance type, and the average value of increased percentages of the objective function (Cost Diff.) of the DF against the PH metaheuristic. Moreover, two additional measures are presented at each table to quantify the spatial homogeneity of the distribution of located satellite facilities and customer allocation to them. These measures correspond to: 1) the *satellite location density* (SLD), which represents the average number of open satellites for each instance type, and 2) the maximum and minimum *customer allocation density* (CAD), which represents the average number of customers assigned to each open satellite.

The results presented in Figure 6 and Figure 7 indicate that the stochastic formulation consistently outperforms the deterministic formulation in terms of overall solution quality. The deterministic formulation incurs significantly higher design costs and outsourced services. This discrepancy can be attributed to the fact that the deterministic formulation, fails to capture important demand variations, resulting in higher expected costs. This is particularly evident in design planning decisions, where the deterministic formulation produces facility configurations that are insufficient to conduct the necessary vehicle operations during the second stage for all scenarios. This issue is evident in both

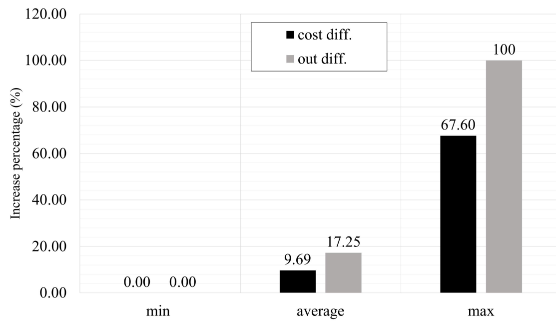

Figure 6: Comparison of the deterministic versus the stochastic formulation of the 2E-MLRPSCD on instance with demand correlation.

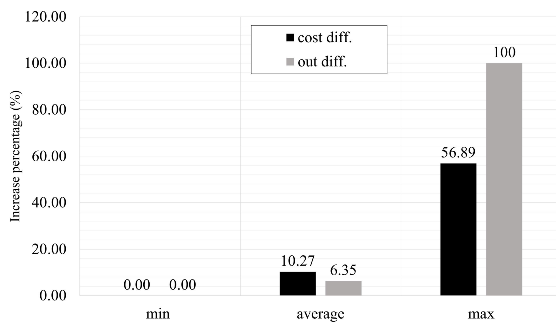

Figure 7: Comparison of the deterministic versus the stochastic formulation of the 2E-MLRPSCD on instance with no demand correlation.

instance sets, where the deterministic formulation used on average 17% and 6% more outsourced services for instances with and without demand correlation, respectively.

Distributions covering a wide range of low demand values, exemplified by instance type CA, demonstrate a clear distinction between instances with and without demand correlation. When demand correlation is considered for instance type CA, there is a tendency to employ more satellite facilities to accommodate the greater diversity of scenarios. In contrast, instances without demand correlation tend to utilize fewer satellite facilities, focusing on addressing scenarios with high demand values within the scenario set, which are typically low when compared to other instance types. For instance types CC and CD, where distributions yield higher demand values, the results usually show a high number of open satellite facilities with a more homogeneous set of customer allocations. Regardless of demand correlation, high demand values force both approaches to lean towards higher satellite usage to handle the high-value demand variations. Interestingly, a very narrow and low-value demand distribution, as in instance type CB, allowed the deterministic (based on the use of the average demands) approach to better approximate the demand distribution, which in turn lead to the deterministic approach to produce results comparable to those of the stochastic approach. These observations hold even when demand correlation is not considered, with a general increase in customer

Instance type	$\|Z\|$	cost diff.	PH			DF		
			SLD	CAD		SLD	CAD	
				max	min		max	min
CA	3	14.43	2.60	10.40	2.00	1.60	13.40	1.60
	5	47.46	2.60	5.60	3.80	1.20	12.00	3.00
	4	39.34	2.60	9.00	0.20	1.00	15.00	0.00
CB	3	0.00	1.20	12.00	3.00	1.20	12.00	3.00
	5	0.28	1.20	12.00	3.00	1.20	12.00	3.00
	4	0.00	1.00	15.00	0.00	1.00	15.00	0.00
CC	3	0.75	3.00	6.40	3.00	3.00	5.00	5.00
	5	2.07	3.60	5.40	0.40	3.20	5.00	0.40
	4	4.93	3.80	6.20	1.60	4.00	5.00	2.20
CD	3	3.53	3.00	6.20	4.00	3.00	5.00	5.00
	5	1.97	3.60	5.60	1.40	3.60	4.80	1.40
	4	4.43	3.40	6.20	1.20	4.00	5.00	2.40
Averages		***9.93***	***2.60***	***8.33***	***1.97***	***2.33***	***9.10***	***2.25***

Table 7: Location/allocation density by instance-types with demand correlation

Instance type	$\|Z\|$	Dif. UB	PH			DF		
			SLD	CAD		SLD	CAD	
				MAX	MIN		MAX	MIN
CA	3	16.64	1.20	12.40	2.00	1.60	13.40	1.60
	5	37.24	1.20	13.20	1.80	1.20	12.00	3.00
	4	35.09	2.60	9.60	0.20	1.00	15.00	0.00
CB	3	0.06	1.20	13.20	3.00	1.20	13.20	3.00
	5	0.00	1.20	13.20	3.00	1.00	13.20	3.00
	4	0.05	1.00	15.00	0.00	1.00	15.00	0.00
CC	3	7.15	3.00	7.80	3.00	3.00	5.00	5.00
	5	5.42	2.8	8.60	2.20	3.20	5.00	0.40
	4	9.32	2.80	6.80	2.60	4.00	5.00	2.20
CD	3	0.39	3.00	8.60	3.00	3.00	5.00	5.00
	5	4.46	3.20	6.40	3.20	3.60	4.80	1.40
	4	7.48	3.20	7.00	2.00	4.00	5.00	2.40
Averages		***10.27***	***2.20***	***10.15***	***2.17***	***2.32***	***9.30***	***2.25***

Table 8: Location/allocation density by instance-types with no demand correlation

allocation density across each instance set due to reduced demand variability within each scenario set.

One can conclude that the stochastic approach addressed by the proposed PH meta-heuristic is generally more cost-effective for both design and routing decisions. The deterministic formulation approach produces solutions that lack operational efficiency, especially for the second stage, since it does not sufficiently account for uncertainty at the design planning stage. Unless the demand distribution is narrow and low enough, the deterministic formulation approach proves unsuitable for designing distribution networks with uncertain demands, with or without correlation. Therefore, a stochastic approach should be used to warrant an effective distribution system design involving location routing decisions under uncertainty.

7 Conclusions

We introduced the two-Echelon multicommodity location-routing problem with stochastic and correlated Demands (2E-MLRPSCD). The problem is formulated as a two-stage stochastic program where, the location of satellite facilities and the customer-to-satellite allocation decisions are made in the first stage, while the vehicle routes for both echelons are decided in the second stage, when customers demands are observed. To address the proposed two-stage model, we present a specialized PH-based metaheuristic with a series of novel enhancements. These include: 1) population structures of alternative and diverse solutions for the scenario subproblems; 2) strategies to define the reference solutions, which are used to guide the overall search; and 3) a reset procedure that reduces the risk of the method becoming trapped in local optima.

A series of numerical experiments were performed, involving a set of instances with varying characteristics, which computationally showed that the proposed enhancements significantly improved the overall performance of the PH method built for the 2E-MLRPSCD. Moreover, the numerical results also clearly showed the added value of explicitly considering the uncertainty in demand and its interrelations. The solutions obtained by solving the stochastic problem outperformed the ones obtained by applying a deterministic approximation approach.

Several interesting avenues for future research may be identified. There is a need to design novel heuristic and exact methods to more efficiently address the set of scenario subproblems that must be solved at each iteration of the PH metaheuristic. There are also interesting extensions to the considered problem that could be studied. Specifically, solving the problem with additional sources of uncertainty (e.g., travel times uncertainty) would certainly be worthwhile for a wide gamut of applications.

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Appendix 1 - Decomposition strategy for the two-stage stochastic formulation

This section presents the complete steps used to perform the decomposition approach that is applied to the stochastic 2E-MLRPSCD formulation introduced in Section 4.2. This decomposition approach utilizes an augmented Lagrangean strategy.

The decomposition strategy applied on the scenario-based formulation, along the scenarios included in S, requires the first-stage decisions to be reformulated. Specifically, these decisions need to be defined as scenario-dependent. In doing so, constraints (7), (18) and (19) are then reexpressed according to the scenario-specific location and allocation first-stage decisions. Therefore, one starts with the following alternative, but equivalent, formulation:

$$\min \sum_{s\in S} \rho_s \left(\sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} F_i y^s_i + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Delta_{ijk} f^s_{ijk} \right) \tag{59}$$

Subject to

$$(2) - (6)$$
$$(8) - (17)$$

$$\sum_{h\in H^2} \sum_{j\in C} x^s_{ijh} \leq |H^2| y^s_i \quad \forall i \in Z, s \in S \tag{60}$$

$$\sum_{h\in H^1} u^s_{ijkh} = f^s_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{61}$$

$$\sum_{h\in H^2} v^s_{zD(k)h} = \sum_{p\in P} f^s_{pzk} \quad \forall z \in Z, k \in K, s \in S \tag{62}$$

$$y^s_i = \bar{y}_i \quad \forall i \in Z, s \in S \tag{63}$$

$$f^s_{ijk} = \bar{f}_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{64}$$

$$y^s_i \in \{0,1\} \quad \forall i \in Z, s \in S \tag{65}$$

$$f^s_{ijk} \in \{0,1\} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{66}$$

$$\bar{y}_i \in \{0,1\} \quad \forall i \in Z \tag{67}$$

$$\bar{f}_{ijk} \in \{0,1\} \quad \forall i \in P, j \in Z, k \in K \tag{68}$$

This reformulation now explicitly includes the set of non-anticipativity constraints which prevent the first-stage decision variables to be set to different scenario-specific values (i.e., the first-stage decisions must be implementable). Constraints (60)-(62) link the facility allocation variables with the vehicle allocation variables. Constraints (63) and (64) ensure that the first-stage solutions will be the same for all the scenarios (also known as the non-anticipativity constraints), where variables $\bar{y}_i$ and $\bar{f}_{ijk}$ serve as the reference first-stage variables. The latter ensure that a single set of facility location and allocation decisions are made for all the scenarios (thus preventing tailored scenario-specific decisions to be made). Then, following the decomposition scheme, originally proposed by Rockafellar and Wets (1991), constraints (63) and (64) are relaxed using an augmented Lagrangean method, which results in the following objective function:

$$\begin{aligned}\min \sum_{s\in S} \rho_s \Big(& \sum_{h\in H^1} \sum_{(i,j)\in A^1} \zeta_{ij} x^s_{ijh} + \sum_{h\in H^2} \sum_{(i,j)\in A^2} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} F_i y^s_i \\ & + \sum_{i\in Z} \lambda^s_i (y^s_i - \bar{y}_i) + \frac{1}{2} \sum_{i\in Z} \gamma (y^s_i - \bar{y}_i)^2 + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Delta_{ijk} f^s_{ijk} \\ & + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \mu^s_{ijk} (f^s_{ijk} - \bar{f}_{ijk}) + \frac{1}{2} \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \gamma (f^s_{ijk} - \bar{f}_{ijk})^2 \Big)\end{aligned} \tag{69}$$

The objective function now involves the lagrangean multipliers λ^s_i and μ^s_{ijk} for the relaxed constraints (63) and (64), respectively, and a penalty term γ. Given the binary requirements of the location and allocation variables, the objective function can be reduced as follows:

$$\begin{aligned}\min \sum_{s\in S} \rho_s \Big(& \sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} \Big(F_i + \lambda^s_i + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y^s_i \\ & + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Big(\Delta_{ijk} + \mu^s_{ijk} + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f^s_{ijk} + \frac{1}{2} \sum_{i\in Z} \gamma \bar{y}_i - \sum_{i\in Z} \lambda^s_i \bar{y}_i \\ & + \frac{1}{2} \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \gamma \bar{f}_{ijk} + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \mu^s_{ijk} \bar{f}_{ijk} \Big)\end{aligned} \tag{70}$$

Given the objective (70) and the constraint set: (2)-(6), (8)-(17), (60)-(62) and (65)-(68), if the reference point (or solution) $\bar{y}_i$ and $\bar{f}_{ijk}$ is fixed, then the relaxed formulation is decomposed by scenario. Specifically, for each $s \in S$, a deterministic 2E-MLRPSCD subproblem with modified fixed costs is obtained:

$$\begin{aligned}\min \sum_{s\in S} \rho_s \Big(& \sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} \Big(F_i + \lambda^s_i + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y^s_i \\ & + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Big(\Delta_{ijk} + \mu^s_{ijk} + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f^s_{ijk} \Big)\end{aligned} \tag{71}$$

Subject to

$$(2) - (6), (8) - (17), (60) - (62) \text{ and } (65) - (68)\ .$$

As previously stated, the proposed PH algorithm then proceeds by solving the previous scenario subproblems separately, thus obtaining scenario-specific first-stage solutions. These scenario-specific solutions are then used to compute the reference point. Using the reference point, the objective functions (71), for all $s \in S$, are modified to incentivise decision agreement among the subproblems (i.e., consensus). This general process in then repeated until consensus first-stage solution can be found.

Appendix 2 - Complete Result Tables

Instance	$\vert S\vert = \mathbf{10}$		$\vert S\vert = \mathbf{20}$		$\vert S\vert = \mathbf{30}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Ca1-2,3,15	1.52	121.57	0.85	35.95	0.70	24.46
Ca1-3,5,15	2.39	297.35	1.40	101.77	0.94	42.26
Ca1-6,4,15	2.38	216.61	1.02	38.49	0.87	29.87
Ca2-2,3,15	1.04	68.28	0.61	27.66	0.78	30.22
Ca2-3,5,15	1.12	41.96	0.75	20.36	0.61	14.39
Ca2-6,4,15	2.97	466.96	1.11	68.72	0.81	37.49
Ca3-2,3,15	4.37	1447.25	2.05	303.71	1.60	144.58
Ca3-3,5,15	5.37	1406.21	5.93	3351.09	3.55	993.23
Ca3-6,4,15	1.61	200.43	0.67	41.05	0.48	20.11
Ca4-2,3,15	0.62	20.00	2.43	9.75	1.50	17.16
Ca4-3,5,15	0.88	37.02	1.42	120.58	2.22	252.29
Ca4-6,4,15	3.22	547.49	1.59	142.76	0.94	44.33
Ca5-2,3,15	0.62	23.44	0.84	44.53	0.60	23.96
Ca5-3,5,15	4.35	1008.95	3.47	735.45	2.06	243.41
Ca5-6,4,15	2.03	236.19	1.22	89.12	0.73	26.19
Cb1-2,3,15	0.40	15.57	0.20	4.39	0.13	2.05
Cb1-3,5,15	0.09	0.37	0.06	0.15	0.08	0.25
Cb1-6,4,15	0.14	1.16	0.10	0.53	0.03	0.05
Cb2-2,3,15	0.41	10.15	0.22	3.21	0.17	1.83
Cb2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cb2-6,4,15	0.64	12.51	0.28	2.92	0.16	0.81
Cb3-2,3,15	0.08	0.58	0.04	0.10	0.03	0.04
Cb3-3,5,15	0.18	1.12	0.12	0.47	0.05	0.09
Cb3-6,4,15	0.58	17.82	0.36	7.65	0.11	0.46
Cb4-2,3,15	0.78	28.85	0.48	12.22	0.34	6.38
Cb4-3,5,15	0.00	0.00	0.02	0.01	0.01	0.00
Cb4-6,4,15	0.11	0.54	0.02	0.03	0.01	0.01
Cb5-2,3,15	1.17	95.45	0.64	40.17	0.43	16.60
Cb5-3,5,15	0.00	0.00	0.18	1.73	0.08	0.33
Cb5-6,4,15	0.00	0.00	0.19	2.65	0.08	0.49
Cc1-2,3,15	4.45	3469.30	2.66	1206.38	1.65	467.20
Cc1-3,5,15	1.43	244.96	1.41	317.74	1.01	180.20
Cc1-6,4,15	4.38	4084.98	2.15	1037.43	1.71	657.59
Cc2-2,3,15	2.12	776.30	1.33	372.10	1.95	755.08
Cc2-3,5,15	4.22	2998.10	4.83	2131.22	2.09	869.33
Cc2-6,4,15	4.92	6358.39	1.94	833.12	1.28	362.06
Cc3-2,3,15	4.84	5836.54	2.64	1756.38	1.36	472.44
Cc3-3,5,15	7.12	5331.21	3.62	1524.46	1.82	394.82
Cc3-6,4,15	2.54	944.92	2.35	255.76	1.75	85.37

Instance	$\|S\| = \mathbf{10}$		$\|S\| = \mathbf{20}$		$\|S\| = \mathbf{30}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Cc4-2,3,15	3.48	2034.46	1.50	380.47	2.00	604.30
Cc4-3,5,15	8.58	15222.89	3.14	1245.38	1.90	393.44
Cc4-6,4,15	3.73	2509.57	1.41	327.63	1.40	573.11
Cc5-2,3,15	3.24	2604.01	2.75	1834.62	2.48	1944.30
Cc5-3,5,15	1.89	752.12	0.84	166.96	0.83	97.15
Cc5-6,4,15	4.24	2797.43	3.15	1473.54	3.52	1547.38
Cd1-2,3,15	0.90	57.21	0.00	8.58	0.38	12.02
Cd1-3,5,15	0.41	7.93	0.08	0.27	0.12	0.83
Cd1-6,4,15	2.22	3.04	1.58	13.41	0.75	0.42
Cd2-2,3,15	0.00	0.00	0.03	0.04	0.09	0.31
Cd2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd2-6,4,15	0.46	13.54	0.15	0.91	0.13	0.64
Cd3-2,3,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd3-3,5,15	0.13	0.72	0.21	2.23	0.04	0.05
Cd3-6,4,15	0.31	4.59	0.36	5.77	0.01	0.00
Cd4-2,3,15	0.94	47.77	0.01	0.01	0.35	5.79
Cd4-3,5,15	0.01	0.00	0.06	0.24	0.01	0.00
Cd4-6,4,15	0.54	13.37	0.00	0.00	0.15	1.16
Cd5-2,3,15	0.50	22.22	0.32	7.09	0.19	3.02
Cd5-3,5,15	0.01	0.00	0.30	7.66	0.03	0.06
Cd5-6,4,15	0.82	53.81	0.46	20.86	0.11	0.78
Max.	***8.58***	***15222.89***	***5.93***	***3351.09***	***3.55***	***5457.38***
Averages	***1.79***	***1041.89***	***1.13***	***335.62***	***0.82***	***265.20***

Table 9: Stability tests. Relative difference and variance for each instance and scenario set with 10, 20 and 30 scenarios.

Instance	$\|S\| = \mathbf{40}$		$\|S\| = \mathbf{50}$		$\|S\| = \mathbf{100}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Ca1-2,3,15	0.69	37.87	0.98	77.01	0.59	23.07
Ca1-3,5,15	0.22	2.13	0.12	0.70	0.08	0.35
Ca1-6,4,15	0.95	41.30	0.97	34.73	0.66	19.61
Ca2-2,3,15	0.23	5.28	0.09	0.71	0.07	0.33
Ca2-3,5,15	0.25	3.19	0.26	2.62	0.15	1.47
Ca2-6,4,15	1.04	58.61	0.49	18.64	0.32	5.33
Ca3-2,3,15	0.76	35.98	1.08	82.10	0.80	59.33
Ca3-3,5,15	1.75	127.10	1.07	45.46	0.88	33.46
Ca3-6,4,15	0.10	0.73	0.16	2.07	0.06	0.22
Ca4-2,3,15	1.1	50.22	0.82	10.88	0.4	9.54
Ca4-3,5,15	1.86	120.55	1.3	95.5	0.78	80.12
Ca4-6,4,15	0.7	78.45	0.35	35.44	0.11	22.1

Instance	$\|S\| = \mathbf{40}$		$\|S\| = \mathbf{50}$		$\|S\| = \mathbf{100}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Ca5-2,3,15	0.47	12.99	0.22	8.31	0.09	5.5
Ca5-3,5,15	1.95	122.28	1.05	99.75	0.83	70.47
Ca5-6,4,15	0.66	11.96	0.44	20.56	0.21	5.1
Cb1-2,3,15	0.27	6.17	0.22	4.07	0.12	1.23
Cb1-3,5,15	0.08	0.38	0.07	0.24	0.03	0.06
Cb1-6,4,15	0.05	0.18	0.03	0.06	0.02	0.02
Cb2-2,3,15	0.05	0.10	0.04	0.06	0.03	0.03
Cb2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cb2-6,4,15	0.10	0.44	0.08	0.33	0.04	0.07
Cb3-2,3,15	0.00	0.00	0.00	0.00	0.00	0.00
Cb3-3,5,15	0.02	0.02	0.02	0.02	0.02	0.01
Cb3-6,4,15	0.05	0.12	0.05	0.09	0.03	0.05
Cb4-2,3,15	0.23	2.82	0.19	1.82	0.12	0.65
Cb4-3,5,15	0.05	0.12	0.04	0.05	0.01	0.00
Cb4-6,4,15	0.12	0.72	0.09	0.49	0.05	0.13
Cb5-2,3,15	0.15	1.87	0.12	1.27	0.06	0.32
Cb5-3,5,15	0.02	0.02	0.03	0.05	0.02	0.02
Cb5-6,4,15	0.07	0.35	0.05	0.17	0.02	0.04
Cc1-2,3,15	0.85	137.51	0.92	163.63	0.56	59.25
Cc1-3,5,15	1.88	575.19	1.28	252.90	0.79	96.69
Cc1-6,4,15	1.11	258.47	0.94	192.33	0.5	55.07
Cc2-2,3,15	1.31	345.14	1.07	228.73	0.58	65.14
Cc2-3,5,15	1.58	510.21	1.17	281.18	0.69	98.23
Cc2-6,4,15	2.25	1142.38	2.01	933.34	1.44	525.93
Cc3-2,3,15	1.39	346.35	1.15	236.46	0.74	94.22
Cc3-3,5,15	1.77	428.80	1.42	269.09	0.9	121.36
Cc3-6,4,15	1.37	500.51	1.07	319.98	0.6	102.52
Cc4-2,3,15	1.54	579.72	1.12	355.86	0.76	140.25
Cc4-3,5,15	0.83	81.46	0.90	172.08	0.23	11.19
Cc4-6,4,15	0.87	173.46	0.35	30.19	0.20	10.51
Cc5-2,3,15	1.18	417.88	0.49	90.28	0.47	88.57
Cc5-3,5,15	0.88	155.78	0.96	272.79	0.58	76.07
Cc5-6,4,15	0.63	95.11	0.83	126.08	0.61	64.69
Cd1-2,3,15	0.27	6.17	0.22	4.07	0.12	1.23
Cd1-3,5,15	0.08	0.38	0.07	0.24	0.03	0.06
Cd1-6,4,15	0.05	0.18	0.03	0.06	0.02	0.02
Cd2-2,3,15	0.05	0.10	0.04	0.06	0.03	0.03
Cd2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd2-6,4,15	0.10	0.44	0.08	0.33	0.04	0.07
Cd3-2,3,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd3-3,5,15	0.02	0.02	0.02	0.02	0.02	0.01

Instance	$\lvert S \rvert = \mathbf{40}$		$\lvert S \rvert = \mathbf{50}$		$\lvert S \rvert = \mathbf{100}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Cd3-6,4,15	0.05	0.12	0.05	0.09	0.03	0.05
Cd4-2,3,15	0.23	2.82	0.19	1.82	0.12	0.65
Cd4-3,5,15	0.05	0.12	0.04	0.05	0.01	0.00
Cd4-6,4,15	0.12	0.72	0.09	0.49	0.05	0.13
Cd5-2,3,15	0.15	1.87	0.12	1.27	0.06	0.32
Cd5-3,5,15	0.02	0.02	0.03	0.05	0.02	0.02
Cd5-6,4,15	0.07	0.35	0.05	0.17	0.02	0.04
Max.	***2.25***	***1142.38***	***2.01***	***933.34***	***1.44***	***525.93***
Averages	***0.58***	***108.05***	***0.45***	***74.61***	***0.28***	***32.56***

Table 10: Stability tests. Relative difference and variance for each instance and scenario set with 40, 50 and 100 scenarios.

Instance	—S—	CPLEX			PH																				LB
					Classic (AC)					Probability (AC)					Social (AC)					Cluster (AC)					
		UB	OG (%)	TIME	UB	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	GAP (%)	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	
Ca1-2,3,15	30	2984.10	19.10	7200.00	2445.2	21	1.28	7200.00	-22.04	2455.73	47	1.70	2678.67	-21.52	2424.70	10	0.44	2674.34	-23.07	2424.70	10	0.44	1826.92	-23.07	2414.00
Ca1-3,5,15	30	2863.63	28.19	7200.00	3098.63	10	33.64	7200.00	7.58	2062.03	40	0.28	3443.61	-38.87	2076.70	19	0.98	2893.36	-37.89	2076.70	19	0.98	1238.20	-37.89	2056.31
Ca1-6,4,15	30	2453.70	18.57	7200.00	3246.67	11	38.46	7200.00	24.42	2250.23	50	11.21	1984.56	-9.04	2206.40	20	9.45	2467.88	-11.21	2034.00	25	1.77	2560.43	-20.63	1997.93
Ca2-2,3,15	30	3175.90	35.07	7200.00	2288.27	19	9.89	7200.00	-38.79	2287.67	50	9.86	2486.71	-38.83	2184.50	15	5.61	2151.20	-45.38	2110.00	15	2.27	2147.58	-50.52	2062.04
Ca2-3,5,15	30	2927.17	37.28	7200.00	3442.67	10	46.67	7200.00	14.97	2734.90	13	32.87	1311.36	-7.03	1870.10	23	1.83	3552.67	-56.52	1870.10	19	1.83	1904.20	-56.52	1835.92
Ca2-6,4,15	30	2798.63	15.81	7200.00	3678.33	10	35.94	7200.00	23.92	2456.70	30	4.09	3044.98	-13.92	2388.40	20	1.35	1841.73	-17.18	2388.40	18	1.35	1412.05	-17.18	2356.17
Ca3-2,3,15	30	4290.00	43.59	7200.00	3502.47	7	30.91	7200.00	-22.48	2506.60	44	3.46	6328.41	-71.15	2617.20	10	7.54	2298.98	-63.92	2485.50	17	2.64	1243.50	-72.60	2419.99
Ca3-3,5,15	30	3513.17	53.96	7200.00	5039	10	67.90	7200.00	30.28	1629.00	11	0.70	1353.02	-115.66	1886.10	10	14.23	1908.30	-86.27	1629.10	10	0.70	1784.63	-115.65	1617.63
Ca3-6,4,15	30	3056.97	18.69	7200.00	2679.8	7	7.25	7200.00	-14.07	2679.80	20	7.25	2525.70	-14.07	2546.53	22	2.39	2232.41	-20.04	2546.53	12	2.39	1338.94	-20.04	2485.62
Ca4-2,3,15	30	3069.33	22.73	7200.00	2456.93	21	3.47	7200.00	-24.93	2456.93	20	3.47	3630.18	-24.93	2456.93	15	3.47	1238.29	-24.93	2377.38	15	0.24	1410.76	-29.11	2371.67
Ca4-3,5,15	30	2579.53	22.58	7200.00	3303.4	15	39.55	7200.00	21.91	2133.80	18	6.41	2137.43	-20.89	2179.53	10	8.37	3315.26	-18.35	2133.22	15	6.38	1798.50	-20.92	1997.04
Ca4-6,4,15	30	2295.33	16.46	7200.00	2905.87	11	34.01	7200.00	21.01	2896.67	20	33.80	1616.07	20.76	2124.33	26	9.74	2149.35	-8.05	2108.10	20	9.04	2462.50	-8.88	1917.52
Ca5-2,3,15	30	2689.60	20.44	7200.00	3221	10	33.57	7200.00	16.50	2213.17	13	3.32	1234.94	-21.53	2212.03	17	3.27	1301.36	-21.59	2212.03	15	3.27	1845.60	-21.59	2139.72
Ca5-3,5,15	30	4403.87	50.30	7200.00	4967.33	12	55.93	7200.00	11.34	3250.80	11	32.67	794.15	-35.47	2209.03	10	0.91	920.19	-99.36	2209.03	10	0.91	1020.78	-99.36	2188.88
Ca5-6,4,15	30	2754.57	27.74	7200.00	2164.9	9	8.05	7200.00	-27.24	2011.00	10	1.02	1333.33	-36.97	2011.00	18	1.02	1733.26	-36.97	2011.00	9	1.02	956.13	-36.97	1990.54
Cb1-2,3,15	30	2490.57	11.55	7200.00	2237.97	8	1.56	6492.91	-11.29	2214.00	11	0.50	1225.45	-12.49	2214.00	10	0.50	1836.89	-12.49	2214.00	10	0.50	1125.32	-12.49	2203.00
Cb1-3,5,15	30	2270.67	15.85	7200.00	1914.40	8	0.19	7200.00	-18.61	1914.40	11	0.19	1482.36	-18.61	1914.40	10	0.19	4457.70	-18.61	1914.40	10	0.19	3731.31	-18.61	1910.70
Cb1-6,4,15	30	2444.10	14.04	7200.00	2107.73	10	0.32	1327.03	-15.96	2107.73	11	0.32	1487.02	-15.96	2107.73	10	0.32	1992.89	-15.96	2107.73	9	0.32	1280.82	-15.96	2101.00
Cb2-2,3,15	30	3310.70	44.71	7200.00	1834.70	15	0.23	3134.15	-80.45	1834.70	11	0.23	1892.26	-80.45	1834.70	10	0.23	2281.38	-80.45	1834.70	10	0.23	2869.50	-80.45	1830.50
Cb2-3,5,15	30	2784.40	32.82	7200.00	1871.00	16	0.02	6747.27	-48.82	1871.00	11	0.02	1052.97	-48.82	1871.00	10	0.02	2411.31	-48.82	1871.00	10	0.02	2566.47	-48.82	1870.54
Cb2-6,4,15	30	2388.83	19.85	7200.00	1922.37	10	0.40	914.35	-24.26	1922.37	11	0.40	993.44	-24.26	1922.37	10	0.40	1338.19	-24.26	1922.37	10	0.40	1366.29	-24.26	1914.60
Cb3-2,3,15	30	3083.47	29.96	7200.00	2214.53	10	2.48	7200.00	-39.24	2176.20	11	0.76	976.29	-41.69	2176.20	10	0.76	2738.49	-41.69	2176.20	14	0.76	2500.10	-41.69	2159.60
Cb3-3,5,15	30	2159.47	21.29	7200.00	1741.23	8	2.38	6283.76	-24.02	1741.23	11	2.38	1866.33	-24.02	1741.23	10	2.38	1401.96	-24.02	1717.23	10	1.02	1613.95	-25.75	1699.76
Cb3-6,4,15	30	2294.77	15.46	7200.00	1958.00	7	0.92	3435.08	-17.20	1958.00	11	0.92	1650.32	-17.20	1958.00	10	0.92	2407.27	-17.20	1958.00	9	0.92	2084.66	-17.20	1939.90
Cb4-2,3,15	30	3545.03	35.82	7200.00	2297.60	12	0.97	2372.62	-54.29	2297.60	11	0.97	1711.90	-54.29	2297.60	10	0.97	1766.00	-54.29	2297.60	10	0.97	1437.97	-54.29	2275.22
Cb4-3,5,15	30	3029.67	39.58	7200.00	1848.53	12	0.97	7200.00	-63.90	1848.53	11	0.97	1218.64	-63.90	1848.53	10	0.97	2258.24	-63.90	1848.53	13	0.97	1864.30	-63.90	1830.68
Cb4-6,4,15	30	2429.53	16.64	7200.00	2043.77	15	0.91	1250.18	-18.87	2043.77	11	0.91	1601.55	-18.87	2043.77	10	0.91	1238.52	-18.87	2043.77	13	0.91	1411.92	-18.87	2025.23
Cb5-2,3,15	30	3009.00	25.98	7200.00	2229.60	13	0.10	3108.03	-34.96	2229.60	11	0.10	1276.73	-34.96	2229.60	10	0.10	1216.53	-34.96	2229.60	10	0.10	1786.83	-34.96	2227.26
Cb5-3,5,15	30	2740.53	21.38	7200.00	2166.17	12	0.54	3162.39	-26.52	2166.17	11	0.54	1014.90	-26.52	2166.17	10	0.54	1081.41	-26.52	2166.17	10	0.54	1614.34	-26.52	2154.58
Cb5-6,4,15	30	3095.37	20.26	7200.00	2488.53	11	0.82	1238.98	-24.39	2488.53	11	0.82	1319.86	-24.39	2488.53	10	0.82	1413.15	-24.39	2488.53	10	0.82	1548.08	-24.39	2468.15
Cc1-2,3,15	30	5603.50	14.19	7200.00	4852.63	30	0.91	4218.65	-15.47	4989.93	38	3.64	1181.42	-12.30	4852.63	22	0.91	4139.52	-15.47	4852.63	20	0.91	2433.63	-15.47	4808.26
Cc1-3,5,15	30	4440.17	8.88	7200.00	4062.17	35	0.40	2780.82	-9.31	4072.67	11	0.65	702.83	-9.02	4062.17	20	0.40	675.83	-9.31	4062.17	42	0.40	1211.85	-9.31	4046.10
Cc1-6,4,15	30	5946.87	20.44	7200.00	5166.13	30	8.41	4824.92	-15.11	4771.87	50	0.85	4242.11	-24.62	4771.87	31	0.85	1746.19	-24.62	4771.87	25	0.85	1805.19	-24.62	4731.53
Cc2-2,3,15	30	6379.13	22.82	7200.00	5030.10	25	2.12	7200.00	-26.82	4939.33	48	0.32	3394.34	-29.15	4939.83	30	0.33	2352.11	-29.14	4939.83	30	0.33	3432.54	-29.14	4923.41
Cc2-3,5,15	30	4915.20	10.80	7200.00	4594.93	30	4.58	4593.78	-6.97	4597.83	52	4.64	2397.99	-6.90	4594.93	34	4.58	2448.40	-6.97	4394.47	10	0.23	2871.15	-11.85	4384.47
Cc2-6,4,15	30	5688.00	17.22	7200.00	4904.73	30	4.00	4255.09	-15.97	4940.77	43	4.70	2377.38	-15.12	4940.77	25	4.70	2289.77	-15.12	4790.83	30	1.72	1481.57	-18.73	4708.53
Cc3-2,3,15	30	6205.03	14.50	7200.00	5378.20	45	1.36	4060.80	-15.37	5335.10	41	0.56	1868.77	-16.31	5372.33	30	1.25	1924.50	-15.50	5311.03	13	0.11	2647.65	-16.83	5305.06
Cc3-3,5,15	30	4768.47	7.40	7200.00	4524.87	42	2.41	4584.09	-5.38	4528.60	45	2.50	768.11	-5.30	4510.87	59	2.11	894.67	-5.71	4510.87	10	2.11	1057.68	-5.71	4415.60
Cc3-6,4,15	30	6054.90	13.79	7200.00	5655.20	55	7.70	4813.03	-7.07	5532.70	49	5.65	3651.34	-9.44	5532.70	26	5.65	3080.84	-9.44	5457.60	21	4.35	1954.79	-10.94	5219.93
Cc4-2,3,15	30	5404.10	14.82	7200.00	4616.20	40	0.28	5838.46	-17.07	4640.63	50	0.80	1665.85	-16.45	4640.63	37	0.80	3056.46	-16.45	4616.47	29	0.28	1449.05	-17.06	4603.46
Cc4-3,5,15	30	5874.30	6.85	7200.00	5567.70	35	1.72	3486.46	-5.51	5554.83	20	1.49	1089.74	-5.75	5554.13	20	1.48	892.86	-5.76	5554.13	20	1.48	2635.10	-5.76	5471.91
Cc4-6,4,15	30	6479.77	9.31	7200.00	6160.67	45	4.61	3846.69	-5.18	6160.60	48	4.61	3555.68	-5.18	6160.60	39	4.61	1625.86	-5.18	6109.70	39	3.82	1782.66	-6.06	5876.50
Cc5-2,3,15	30	6772.73	15.05	7200.00	5866.00	20	1.92	7200.00	-15.46	5755.40	33	0.03	1119.42	-17.68	5755.40	38	0.03	3281.59	-17.68	5753.67	22	0.00	2236.85	-17.71	5753.43
Cc5-3,5,15	30	5494.37	7.03	7200.00	5184.57	30	1.47	2933.12	-5.98	5230.00	55	2.33	1112.15	-5.05	5230.00	10	2.33	431.41	-5.05	5193.97	28	1.65	2294.64	-5.78	5108.12
Cc5-6,4,15	30	6062.60	12.43	7200.00	6008.70	25	11.64	4696.87	-0.90	5548.50	49	4.32	2302.72	-9.27	5548.50	32	4.32	3669.92	-9.27	5530.21	27	4.00	2084.36	-9.63	5309.02
Cd1-2,3,15	30	5729.57	19.76	7200.00	4759.13	40	3.40	6715.33	-20.39	4641.17	38	0.94	1928.31	-23.45	4641.17	28	0.94	2017.33	-23.45	4640.20	26	0.92	1420.60	-23.48	4597.35
Cd1-3,5,15	30	5421.03	9.26	7200.00	5019.83	33	2.01	5573.19	-7.99	5019.83	28	2.01	1233.64	-7.99	5019.83	21	2.01	2769.83	-7.99	5048.30	20	2.56	2183.80	-7.38	4919.04
Cd1-6,4,15	30	5354.53	8.68	7200.00	5365.00	50	8.86	5331.32	0.20	5066.50	50	3.49	2739.24	-5.68	5066.50	30	3.49	2630.84	-5.68	4985.60	22	1.92	1274.32	-7.40	4889.76
Cd2-2,3,15	30	4632.90	11.49	7200.00	4142.27	55	1.00	6875.08	-11.84	4126.57	30	0.63	1172.88	-12.27	4126.57	17	0.63	1055.86	-12.27	4126.57	18	0.63	2059.15	-12.27	4100.72
Cd2-3,5,15	30	4358.60	9.40	7200.00	4377.50	33	9.79	3482.54	0.43	4048.03	40	2.45	1059.64	-7.67	3982.87	40	0.85	2322.06	-9.43	3982.87	20	0.85	1620.93	-9.43	3948.84
Cd2-6,4,15	30	6549.37	23.40	7200.00	5607.30	25	10.53	4300.29	-16.80	5111.67	37	1.85	1498.48	-28.13	5100.00	22	1.63	1296.91	-28.42	5100.00	25	1.63	2244.71	-28.42	5017.04
Cd3-2,3,15	30	5597.53	11.50	7200.00	5035.73	55	1.63	4389.55	-11.16	5062.50	56	2.15	1329.84	-10.57	5035.73	31	1.63	1666.06	-11.16	4962.37	22	0.18	1175.30	-12.80	4953.63
Cd3-3,5,15	30	3760.10	8.48	7200.00	3509.07	25	1.93	3273.71	-7.15	3777.47	17	8.90	626.67	0.46	3509.07	34	1.93	3030.22	-7.15	3509.07	20	1.93	2332.00	-7.15	3441.24
Cd3-6,4,15	30	5108.20	9.94	7200.00	4605.23	50	0.11	3037.11	-10.92	4833.93	48	4.84	1716.41	-5.67	4680.30	42	1.71	2510.27	-9.14	4605.23	15	0.11	1772.61	-10.92	4600.21
Cd4-2,3,15	30	5671.47	9.47	7200.00	5151.37	40	0.33	3065.34	-10.10	5135.57	35	0.03	854.77	-10.44	5151.60	34	0.34	884.78	-10.09	5145.37	38	0.22	1366.18	-10.22	5134.20
Cd4-3,5,15	30	4684.00	6.32	7200.00	4435.50	20	1.08	4091.75	-5.60	4435.50	33	1.08	910.18	-5.60	4423.23	18	0.80	816.58	-5.90	4423.23	18	0.80	1757.45	-5.90	4387.76
Cd4-6,4,15	30	6216.10	13.44	7200.00	5675.07	18	5.19	2730.26	-9.53	5677.73	54	5.23	3471.61	-9.48	5643.77	36	4.66	3729.45	-10.14	5489.63	22	1.99	1332.61	-13.23	5380.66
Cd5-2,3,15	30	4816.20	8.25	7200.00	5074.80	20	12.93	7095.28	5.10	4444.10	40	0.57	1953.31	-8.37	4448.40	32	0.66	1585.84	-8.27	4448.40	35	0.66	2887.52	-8.27	4418.86
Cd5-3,5,15	30	5586.57	10.55	7200.00	5201.83	20	3.93	6513.27	-7.40	5203.53	37	3.97	854.99	-7.36	5233.80	22	4.52	1830.50	-6.74	5233.17	15	4.51	569.55	-6.75	4997.19
Cd5-6,4,15	30	5921.73	16.56	7200.00	5733.47	25	13.82	5670.99	-3.28	5173.13	51	4.49	2789.67	-14.47	5173.13	23	4.49	2777.04	-14.47	5172.83	23	4.48	2731.32	-14.48	4941.09
Averages			***19.46***	***7200.00***		***22.93***	***9.82***	***5175.74***	***-12.74***		***29.80***	***4.03***	***1870.71***	***-21.66***		***21.30***	***2.42***	***2099.20***	***-23.97***		***18.20***	***1.49***	***1864.32***	***-25.36***	

Table 11: Complete results on instances with demand correlation

Instance	—S—	CPLEX			PH																				LB
					Classic (AC)					Probability (AC)					Social (AC)					Cluster (AC)					
		UB	OG (%)	TIME	UB	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	GAP (%)	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	
Ca1-2,3,15	30	2984.10	20.65	7200.00	2451.10	60	3.39	6247.8	-21.75	2451.1	10	3.39	1048.00	-21.75	2450.90	12	3.38	832.38	-21.76	2450.03	11	3.35	3082.30	-21.80	2367.96
Ca1-3,5,15	30	2364.10	15.60	7200.00	3196.97	11	37.59	6387.9	26.05	2256.73	10	11.58	1431.70	-4.76	2103.23	9	5.13	810.493	-12.40	2081.47	5	4.14	1409.11	-13.58	1995.30
Ca1-6,4,15	30	2400.70	9.80	7200.00	3874.17	6	44.11	4746.7	38.03	3874.83	15	44.12	1310.30	38.04	2400.70	9	9.80	683.951	0.00	2362.37	5	8.34	2889.59	-1.62	2165.38
Ca2-2,3,15	30	2923.40	22.43	7200.00	2288.97	60	0.93	6160.03	-27.72	2288.97	10	0.93	970.05	-27.72	2288.97	9	0.93	688.635	-27.72	2288.97	5	0.93	2157.66	-27.72	2267.69
Ca2-3,5,15	30	2930.10	36.84	7200.00	3700.10	42	49.98	6369.7	20.81	1945.47	9	4.87	827.00	-50.61	1945.47	9	4.87	661.852	-50.61	1937.60	12	4.49	1942.69	-51.22	1850.65
Ca2-6,4,15	30	4874.30	53.05	7200.00	3507.93	12	34.76	2365.7	-38.95	3512.17	10	34.84	1610.40	-38.78	2507.93	9	8.75	693.671	-94.36	2398.77	12	4.59	1047.22	-103.20	2288.58
Ca3-2,3,15	30	3766.53	33.33	7200.00	3359.80	11	25.26	6782.5	-12.11	3394.2	15	26.01	1708.20	-10.97	2572.43	10	2.38	707.498	-46.42	2568.27	13	2.22	1608.87	-46.66	2511.24
Ca3-3,5,15	30	2781.53	41.60	7200.00	2774.10	60	41.45	6217.32	-0.27	2780.03	10	41.57	1585.20	-0.05	1743.37	12	6.83	1604.34	-59.55	1703.87	10	4.67	2486.38	-63.25	1624.29
Ca3-6,4,15	30	2845.43	13.25	7200.00	3271.10	10	24.54	3426.41	13.01	3303.47	12	25.27	1462.20	13.87	2660.53	8	7.22	938.761	-6.95	2535.10	13	2.63	3406.30	-12.24	2468.54
Ca4-2,3,15	30	4978.20	52.31	7200.00	2983.53	12	20.43	1117.6	-66.86	2996.9	10	20.78	1760.30	-66.11	2495.70	9	4.87	995.521	-99.47	2395.37	10	0.89	1082.71	-107.83	2374.11
Ca4-3,5,15	30	2566.80	31.34	7200.00	2575.70	14	31.58	6247.8	0.35	2587.93	9	31.90	1242.30	0.82	2022.37	8	12.86	1031.23	-26.92	1911.30	9	7.79	3170.40	-34.30	1762.33
Ca4-6,4,15	30	2627.07	24.28	7200.00	2124.27	27	6.36	4253.8	-23.67	2148.9	10	7.43	1009.50	-22.25	2124.27	8	6.36	988.307	-23.67	2061.13	8	3.49	1006.32	-27.46	1989.18
Ca5-2,3,15	30	3514.90	39.77	7200.00	2270.13	16	6.74	2210.4	-54.83	2270.57	10	6.76	1555.90	-54.80	2248.23	10	5.84	993.08	-56.34	2220.20	13	4.65	2413.50	-58.31	2117.05
Ca5-3,5,15	30	4072.13	43.48	7200.00	5410.80	11	57.46	2291.9	24.74	5388.13	10	57.28	1587.10	24.42	2383.57	9	3.44	914.942	-70.84	2341.20	11	1.69	2375.60	-73.93	2301.60
Ca5-6,4,15	30	2773.77	25.87	7200.00	3373.43	18	39.04	2046.8	17.78	3365.73	9	38.90	746.79	17.59	3350.13	7	38.62	730.2	17.20	2108.87	14	2.49	4349.70	-31.53	2056.32
Cb1-2,3,15	30	2659.87	19.27	7200.00	2244.00	18	4.31	6936.7	-18.53	2244	8	4.31	2780.35	-18.53	2220.17	8	3.28	2414.88	-19.80	2214	8	3.01	3353.50	-20.14	2147.32
Cb1-3,5,15	30	2433.03	22.69	7200.00	1911.77	60	1.61	7014.12	-27.27	1911.77	8	1.61	2669.96	-27.27	1911.77	9	1.61	1132.95	-27.27	1911.77	8	1.61	1069.26	-27.27	1881.06
Cb1-6,4,15	30	2604.30	20.95	7200.00	2104.97	10	2.19	2179.68	-23.72	2104.97	10	2.19	1093.86	-23.72	2104.97	10	2.19	560.01	-23.72	2104.97	8	2.19	3409.17	-23.72	2058.82
Cb2-2,3,15	30	3532.13	48.26	7200.00	1838.90	11	0.62	3078.91	-92.08	1838.9	11	0.62	1059.20	-92.08	1838.90	8	0.62	1112.68	-92.08	1838.90	8	0.62	1061.65	-92.08	1827.43
Cb2-3,5,15	30	2196.27	15.93	7200.00	1875.53	12	1.55	3409.16	-17.10	1875.53	9	1.55	1409.11	-17.10	1875.53	8	1.55	2738.97	-17.10	1875.53	9	1.55	956.43	-17.10	1846.39
Cb2-6,4,15	30	2367.47	18.99	7200.00	1918.80	10	0.05	1842.186	-23.38	1918.8	9	0.05	839.76	-23.38	1918.80	9	0.05	408.065	-23.38	1918.80	10	0.05	3367.30	-23.38	1917.89
Cb3-2,3,15	30	2640.33	18.67	7200.00	2176.73	60	1.35	7178.6	-21.30	2176.73	8	1.35	1774.89	-21.30	2176.73	9	1.35	1157.3	-21.30	2176.73	9	1.35	1079.95	-21.30	2147.26
Cb3-3,5,15	30	2115.83	18.66	7200.00	1743.07	10	1.26	6263.76	-21.39	1743.07	8	1.26	1316.96	-21.39	1743.07	10	1.26	1581.79	-21.39	1743.07	9	1.26	728.67	-21.39	1721.06
Cb3-6,4,15	30	2282.67	15.48	7200.00	1948.83	9	1.00	1654.399	-17.13	1948.83	10	1.00	659.10	-17.13	1948.83	10	1.00	482.61	-17.13	1948.83	9	1.00	684.88	-17.13	1929.39
Cb4-2,3,15	30	4082.93	44.37	7200.00	2305.00	12	1.45	2374.48	-77.13	2305	11	1.45	1377.58	-77.13	2305.00	7	1.45	690.084	-77.13	2305.00	8	1.45	571.43	-77.13	2271.50
Cb4-3,5,15	30	2715.50	32.41	7200.00	1852.50	11	0.92	3409.12	-46.59	1852.5	12	0.92	1490.20	-46.59	1852.50	6	0.92	2107.93	-46.59	1852.50	10	0.92	319.97	-46.59	1835.40
Cb4-6,4,15	30	2463.40	19.20	7200.00	2048.60	9	2.84	1050.87	-20.25	2048.6	8	2.84	1059.52	-20.25	2048.60	5	2.84	412.606	-20.25	2043.77	9	2.62	462.02	-20.53	1990.32
Cb5-2,3,15	30	2324.17	4.85	7200.00	2224.17	10	0.57	2817.061	-4.50	2224.17	10	0.57	816.73	-4.50	2224.17	5	0.57	303.057	-4.50	2224.17	8	0.57	646.21	-4.50	2211.42
Cb5-3,5,15	30	3454.23	38.37	7200.00	2165.00	11	1.67	3070.23	-59.55	2165	9	1.67	1055.36	-59.55	2165.00	9	1.67	598.016	-59.55	2165.00	9	1.67	505.96	-59.55	2128.80
Cb5-6,4,15	30	2944.10	16.78	7200.00	2491.70	10	1.67	1252.35	-18.16	2491.7	10	1.67	852.59	-18.16	2491.70	9	1.67	526.508	-18.16	2491.70	9	1.67	383.21	-18.16	2450.21
Cc1-2,3,15	30	6755.67	28.89	7200.00	5905.97	35	18.66	4072.39	-14.39	5909.63	15	18.71	4027.86	-14.32	5852.17	15	17.91	586.5	-15.44	4853.07	20	1.01	4340.70	-39.20	4803.93
Cc1-3,5,15	30	4686.40	10.63	7200.00	4589.47	25	8.74	3282.42	-2.11	4590.87	12	8.77	2923.61	-2.08	4639.50	12	9.72	8426.5	-1.01	4197.03	18	0.21	2638.90	-11.66	4188.36
Cc1-6,4,15	30	6546.80	26.27	7200.00	5920.73	50	18.48	5031.55	-10.57	5985.27	22	19.36	5078.81	-9.38	5613.07	10	14.01	797.03	-16.63	4844.63	15	0.37	1864.10	-35.14	4826.78
Cc2-2,3,15	30	6371.13	21.31	7200.00	5465.37	60	8.26	6953.86	-16.57	5465.37	35	8.26	6871.83	-16.57	5380.20	15	6.81	743	-18.42	5021.93	12	0.16	1571.10	-26.87	5013.76
Cc2-3,5,15	30	5375.17	19.61	7200.00	4913.13	30	12.05	3379.43	-9.40	4913.13	22	12.05	3421.97	-9.40	5641.23	10	23.40	813.3	4.72	4394.47	14	1.67	1226.08	-22.32	4320.91
Cc2-6,4,15	30	6074.37	18.78	7200.00	5673.33	25	13.04	3652.01	-7.07	5638.43	21	12.50	3344.92	-7.73	5267.87	9	6.34	641.23	-15.31	4790.83	15	-2.98	2474.56	-26.79	4933.71
Cc3-2,3,15	30	5471.47	12.88	7200.00	4950.30	28	3.71	3548.62	-10.53	4950.3	20	3.71	3084.99	-10.53	4965.90	25	4.01	2301.35	-10.18	4888.47	10	2.49	2886.60	-11.93	4766.77
Cc3-3,5,15	30	4498.60	10.71	7200.00	4199.70	20	4.36	3728.62	-7.12	4199.7	15	4.36	3434.44	-7.12	4316.53	15	6.95	2488.85	-4.22	4178.30	20	3.87	3680.40	-7.67	4016.67
Cc3-6,4,15	30	5279.00	13.17	7200.00	5113.90	44	10.36	5278.12	-3.23	5083.4	15	9.83	5160.15	-3.85	4891.13	20	6.28	2974.21	-7.93	4763.67	11	3.77	2292.40	-10.82	4583.89
Cc4-2,3,15	30	5052.40	16.35	7200.00	4459.47	50	5.23	5291.41	-13.30	4358.07	22	3.02	5385.40	-15.93	4391.10	24	3.75	1059.7	-15.06	4358.90	15	3.04	3492.23	-15.91	4226.27
Cc4-3,5,15	30	4933.27	11.74	7200.00	4542.53	25	4.15	2868.98	-8.60	4542.53	20	4.15	2703.38	-8.60	5313.87	20	18.06	2897.4	7.16	4516.53	12	3.60	2104.56	-9.23	4354.14
Cc4-6,4,15	30	6616.47	15.65	7200.00	5959.33	60	6.35	4797.01	-11.03	5898.47	15	5.38	4069.75	-12.17	5898.47	16	5.38	4256.09	-12.17	5819.23	15	4.09	1948.19	-13.70	5581.06
Cc5-2,3,15	30	7474.70	23.35	7200.00	6552.07	52	12.55	7143.51	-14.08	6552.47	16	12.56	6323.33	-14.07	6461.73	15	11.33	1438.5	-15.68	5753.67	15	0.42	1890.50	-29.91	5729.47
Cc5-3,5,15	30	5987.80	14.84	7200.00	5626.80	48	9.37	6167.79	-6.42	5642.6	25	9.63	3015.36	-6.12	5939.43	25	14.14	1262.6	-0.81	5193.97	25	1.82	1649.15	-15.28	5099.44
Cc5-6,4,15	30	5791.60	17.16	7200.00	5695.13	20	15.75	4579.63	-1.69	5695.13	18	15.75	4813.79	-1.69	5055.53	23	5.09	4204.06	-14.56	5084.57	20	5.64	2584.92	-13.91	4797.96
Cd1-2,3,15	30	5258.73	18.66	7200.00	4531.03	30	5.60	6976.99	-16.06	4438.47	30	3.63	6325.37	-18.48	4459.93	12	4.09	1340.35	-17.91	4430.83	20	3.46	2351.80	-18.68	4277.49
Cd1-3,5,15	30	5331.43	11.50	7200.00	4861.63	25	2.94	4736.16	-9.66	4861.63	25	2.94	4393.57	-9.66	4893.50	16	3.58	915.922	-8.95	4861.43	20	2.94	2369.30	-9.67	4718.51
Cd1-6,4,15	30	5055.70	13.15	7200.00	4802.90	28	8.57	3436.45	-5.26	4804.97	21	8.61	3477.84	-5.22	4538.73	21	3.25	840.004	-11.39	4538.73	26	3.25	3791.40	-11.39	4391.08
Cd2-2,3,15	30	4655.93	14.05	7200.00	4151.63	22	3.61	5507.9	-12.15	4151.63	26	3.61	5158.61	-12.15	4152.63	15	3.63	580.424	-12.12	4128.23	15	3.06	2146.96	-12.78	4001.89
Cd2-3,5,15	30	4236.83	15.49	7200.00	3677.03	36	2.63	5657.06	-15.22	3677.03	27	2.63	2612.48	-15.22	3711.37	18	3.53	558.477	-14.16	3677.03	24	2.63	2859.20	-15.22	3580.45
Cd2-6,4,15	30	5619.10	19.41	7200.00	4872.67	30	7.07	4119.82	-15.32	4872.67	25	7.07	3574.08	-15.32	4777.10	18	5.21	511.523	-17.63	4777.37	10	5.21	983.80	-17.62	4528.24
Cd3-2,3,15	30	5592.87	12.24	7200.00	5093.23	22	3.63	4248.46	-9.81	5093.23	22	3.63	4049.79	-9.81	5013.23	21	2.09	570.811	-11.56	4962.37	25	1.09	1226.70	-12.71	4908.38
Cd3-3,5,15	30	4549.10	18.81	7200.00	4255.67	15	13.21	3730.45	-6.90	4228.13	15	12.65	3451.91	-7.59	4219.13	20	12.46	499.507	-7.82	3758.07	15	1.72	1683.16	-21.05	3693.29
Cd3-6,4,15	30	4693.47	12.33	7200.00	4325.80	27	4.87	3301.1	-8.50	4340.1	22	5.19	3942.10	-8.14	4359.63	21	5.61	475.383	-7.66	4325.80	15	4.87	1263.61	-8.50	4114.99
Cd4-2,3,15	30	5252.93	13.47	7200.00	5305.13	35	14.32	5221.69	0.98	5305.13	33	14.32	5664.35	0.98	4720.30	21	3.70	407.409	-11.28	4720.30	20	3.70	2519.15	-11.28	4545.53
Cd4-3,5,15	30	4919.30	10.14	7200.00	4638.87	33	4.71	3384.91	-6.05	4676.53	33	5.48	2776.62	-5.19	4795.10	15	7.81	1921.65	-2.59	4467.63	25	1.06	2510.20	-10.11	4420.37
Cd4-6,4,15	30	5189.00	17.41	7200.00	4790.93	22	10.55	2688.71	-8.31	4790.93	25	10.55	2706.05	-8.31	4824.40	20	11.17	1723.3	-7.56	4638.17	22	7.60	2914.40	-11.88	4285.58
Cd5-2,3,15	30	4668.23	17.77	7200.00	4495.93	18	14.62	4938.49	-3.83	4495.9	22	14.62	4979.94	-3.83	4495.67	22	14.61	1928.7	-3.84	3970.97	25	3.33	1700.50	-17.56	3838.75
Cd5-3,5,15	30	5189.83	15.06	7200.00	4718.13	46	6.57	4384.32	-10.00	4718.13	35	6.57	4237.30	-10.00	5241.27	15	15.90	2039.1	0.98	4708.97	13	6.39	2415.90	-10.21	4408.03
Cd5-6,4,15	30	6991.40	27.29	7200.00	6251.17	45	18.69	5474.89	-11.84	6251.17	37	18.69	5808.90	-11.84	5916.33	15	14.08	2961.55	-18.17	5172.83	25	1.73	2278.80	-35.16	5083.11
Averages			***21.94***	***7200.00***		***27.32***	***11.99***	***4299.78***	***-14.46***		***16.52***	***10.78***	***2782.14***	***-16.17***		***13.07***	***6.79***	***1361.11***	***-22.61***		***13.65***	***2.65***	***2044.79***	***-27.98***	

Table 12: Complete results on instances with no demand correlation

Instance	CPLEX			PH		
	ub	time (s)	OSU	best ub	dif. ub	OSU
Ca1-2,3,15	2445.13	7.90	0	2424.70	0.84	0
Ca1-3,5,15	3081.47	30.41	144	2062.03	33.08	0
Ca1-6,4,15	3246.93	25.16	124	2034.00	37.36	0
Ca2-2,3,15	2287.67	9.71	0	2110.00	7.77	0
Ca2-3,5,15	3435.97	9.32	171	1870.10	45.57	0
Ca2-6,4,15	3657.30	1.01	151	2388.40	34.69	0
Ca3-2,3,15	3506.63	1.41	133	2485.50	29.12	0
Ca3-3,5,15	5027.43	12.93	372	1629.00	67.60	0
Ca3-6,4,15	5165.10	5.13	334	2546.53	50.70	0
Ca4-2,3,15	2456.93	7.64	0	2377.38	3.24	0
Ca4-3,5,15	3314.70	21.64	169	2133.22	35.64	0
Ca4-6,4,15	2900.53	6.92	116	2108.10	27.32	0
Ca5-2,3,15	3214.30	7.99	110	2212.03	31.18	0
Ca5-3,5,15	4952.87	11.26	224	2209.03	55.40	0
Ca5-6,4,15	3767.30	8.77	195	2011.00	46.62	0
Cb1-2,3,15	2214.00	11.46	0	2214.00	0.00	0
Cb1-3,5,15	1914.40	15.62	0	1914.40	0.00	0
Cb1-6,4,15	2107.73	8.33	0	2107.73	0.00	0
Cb2-2,3,15	1834.70	50.00	0	1834.70	0.00	0
Cb2-3,5,15	1871.00	14.76	0	1871.00	0.00	0
Cb2-6,4,15	1922.37	3.92	0	1922.37	0.00	0
Cb3-2,3,15	2176.20	35.62	0	2176.20	0.00	0
Cb3-3,5,15	1741.23	26.19	0	1717.23	1.38	0
Cb3-6,4,15	1958.00	6.72	0	1958.00	0.00	0
Cb4-2,3,15	2297.60	16.06	0	2297.60	0.00	0
Cb4-3,5,15	1848.53	25.20	0	1848.53	0.00	0
Cb4-6,4,15	2043.77	8.72	0	2043.77	0.00	0
Cb5-2,3,15	2229.60	3.60	0	2229.60	0.00	0
Cb5-3,5,15	2166.17	27.23	0	2166.17	0.00	0
Cb5-6,4,15	2488.53	9.18	0	2488.53	0.00	0
Cc1-2,3,15	4989.93	6.14	161	4852.63	2.75	140
Cc1-3,5,15	4102.73	2.15	54	4062.17	0.99	50
Cc1-6,4,15	5278.93	12.45	128	4771.87	9.61	138
Cc2-2,3,15	4939.33	7.90	115	4939.33	0.00	115
Cc2-3,5,15	4627.83	1.40	120	4394.47	5.04	119
Cc2-6,4,15	5468.07	6.05	123	4790.83	12.39	161
Cc3-2,3,15	5335.10	11.25	114	5311.03	0.45	114
Cc3-3,5,15	4663.03	4.46	147	4510.87	3.26	201
Cc3-6,4,15	5532.70	3.85	232	5457.60	1.36	232
Cc4-2,3,15	4640.63	5.04	129	4616.20	0.53	202
Cc4-3,5,15	5554.83	2.36	306	5554.13	0.01	313

Table 13 continued from previous page

Cc4-6,4,15	6168.00	2.28	175	6109.70	0.95	272
Cc5-2,3,15	5755.40	17.99	127	5753.67	0.03	127
Cc5-3,5,15	5238.67	6.54	151	5184.57	1.03	178
Cc5-6,4,15	5548.50	6.55	253	5530.21	0.33	263
Cd1-2,3,15	4641.17	7.76	171	4640.20	0.02	171
Cd1-3,5,15	5048.30	4.73	171	5019.83	0.56	196
Cd1-6,4,15	5356.30	6.10	215	4985.60	6.92	218
Cd2-2,3,15	4127.40	3.96	136	4126.57	0.02	135
Cd2-3,5,15	4048.03	3.77	172	3982.87	1.61	180
Cd2-6,4,15	5205.17	3.37	174	5100.00	2.02	181
Cd3-2,3,15	5962.50	2.60	262	4962.37	16.77	173
Cd3-3,5,15	3777.47	7.15	108	3509.07	7.11	116
Cd3-6,4,15	4879.17	3.45	204	4605.23	5.61	215
Cd4-2,3,15	5152.50	2.67	315	5135.57	0.33	313
Cd4-3,5,15	4446.83	1.89	189	4423.23	0.53	181
Cd4-6,4,15	5941.80	10.41	231	5489.63	7.61	229
Cd5-2,3,15	4467.23	2.29	151	4444.10	0.52	147
Cd5-3,5,15	5203.53	5.15	184	5201.83	0.03	180
Cd5-6,4,15	5173.13	6.50	173	5172.83	0.01	173
Averages		***9.97***	***123.90***		***9.93***	***90.55***

Table 13: Complete results stochastic vs deterministic problem on instances with demand correlation

Instance	**CPLEX**			**PH**		
	ub	**time (s)**	**outsourced**	**Best UB**	**dif. Ub**	**outsourPH**
Ca1-2,3,15	2450.9	15.34	0	2450.03	0.04	0
Ca1-3,5,15	3239.7	86.33	159	2081.47	35.75	0
Ca1-6,4,15	3883.57	85.67	266	2362.37	39.17	0
Ca2-2,3,15	2288.97	18.03	0	2288.97	0.00	0
Ca2-3,5,15	3731.47	17.23	201	1937.6	48.07	0
Ca2-6,4,15	3532.57	2.58	172	2398.77	32.10	0
Ca3-2,3,15	3378.13	9.1	124	2568.27	23.97	0
Ca3-3,5,15	2768.27	29.68	132	1703.87	38.45	0
Ca3-6,4,15	3311.83	12.78	122	2535.1	23.45	0
Ca4-2,3,15	2451.23	11.01	0	2395.37	2.28	0
Ca4-3,5,15	2066.7	85.67	0	1911.3	7.52	0
Ca4-6,4,15	3569.27	12.84	236	2061.13	42.25	0
Ca5-2,3,15	5149.63	41.29	402	2220.2	56.89	0
Ca5-3,5,15	5368.9	13.69	401	2341.2	56.39	0
Ca5-6,4,15	3428.33	15.95	198	2108.87	38.49	0
Cb1-2,3,15	2220.17	27.38	0	2214	0.28	0

Table 14 continued from previous page

Cb1-3,5,15	1911.77	85.62	0	1911.77	0.00	0
Cb1-6,4,15	2104.97	21.01	0	2104.97	0.00	0
Cb2-2,3,15	1838.9	36.11	0	1838.9	0.00	0
Cb2-3,5,15	1875.53	85.63	0	1875.53	0.00	0
Cb2-6,4,15	1918.8	10.2	0	1918.8	0.00	0
Cb3-2,3,15	2176.73	85.62	0	2176.73	0.00	0
Cb3-3,5,15	1743.07	63.37	0	1743.07	0.00	0
Cb3-6,4,15	1948.83	24.48	0	1948.83	0.00	0
Cb4-2,3,15	2305	32.37	0	2305	0.00	0
Cb4-3,5,15	1852.5	85.63	0	1852.5	0.00	0
Cb4-6,4,15	2048.6	17.1	0	2043.77	0.24	0
Cb5-2,3,15	2224.17	8.27	0	2224.17	0.00	0
Cb5-3,5,15	2165	51.73	0	2165	0.00	0
Cb5-6,4,15	2491.7	16.19	0	2491.7	0.00	0
Cc1-2,3,15	5861.53	28.63	375	4853.07	17.20	350
Cc1-3,5,15	4329.97	4.44	199	4197.03	3.07	227
Cc1-6,4,15	6052.33	31.72	298	4844.63	19.95	375
Cc2-2,3,15	5380.2	25.48	326	5021.93	6.66	296
Cc2-3,5,15	4958.03	7.9	260	4394.47	11.37	200
Cc2-6,4,15	5737.77	8.97	210	4790.83	16.50	190
Cc3-2,3,15	4897.57	25.42	205	4888.47	0.19	170
Cc3-3,5,15	4347.07	11.99	122	4178.3	3.88	172
Cc3-6,4,15	4923.67	9.05	131	4763.67	3.25	176
Cc4-2,3,15	4359.77	9.35	128	4358.07	0.04	118
Cc4-3,5,15	4531.3	4.88	209	4516.53	0.33	129
Cc4-6,4,15	6085.07	5.88	193	5819.23	4.37	256
Cc5-2,3,15	6514.1	34.44	315	5753.67	11.67	288
Cc5-3,5,15	5672.73	11.12	284	5193.97	8.44	236
Cc5-6,4,15	5185.87	13.11	207	5055.53	2.51	222
Cd1-2,3,15	4434.77	16.01	245	4430.83	0.09	224
Cd1-3,5,15	4893.4	14.17	165	4861.43	0.65	162
Cd1-6,4,15	4877.63	13.84	222	4538.73	6.95	200
Cd2-2,3,15	4152.9	9.18	165	4128.23	0.59	223
Cd2-3,5,15	3741.43	5.23	132	3677.03	1.72	147
Cd2-6,4,15	4892.3	6.69	291	4777.1	2.35	147
Cd3-2,3,15	5022.37	8.3	238	4962.37	1.19	342
Cd3-3,5,15	4389.13	9.91	256	3758.07	14.38	260
Cd3-6,4,15	4636.83	7.91	239	4325.8	6.71	234
Cd4-2,3,15	4722.3	5.71	273	4720.3	0.04	234
Cd4-3,5,15	4682.87	4.59	85	4467.63	4.60	239
Cd4-6,4,15	5017.27	26.39	145	4638.17	7.56	230
Cd5-2,3,15	3972	5.92	193	3970.97	0.03	228

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Cd5-3,5,15	4754.73	18	285	4708.97	0.96	141
Cd5-6,4,15	6002.07	13.2	51	5172.83	13.82	300
Averages		***25.09***	***147.67***		***10.27***	***111.93***

Table 14: Complete results stochastic vs deterministic problem on instances with no demand correlation

Instance	PH + DCS																				
	Weak correlation							Moderate correlation							Strong correlation						
	UB	Pos. ratio	Neg. ratio	FE routes	SE routes	STD	Time	UB	Pos. ratio	Neg. ratio	FE routes	SE routes	STD	Time	UB	Pos. ratio	Neg. ratio	FE routes	SE routes	STD	Time
Ca1-2,3,15	2449.60	136	74	2	9	6.55	2990.59	2424.70	100	110	2	8	6.53	1826.92	2557.70	98	112	2	7	6.03	3293.56
Ca1-3,5,15	2066.20	124	86	3	9	6.42	1554.31	2062.03	104	106	3	8	6.48	1238.2	2066.20	110	100	3	9	7.00	1234.78
Ca1-6,4,15	2277.10	96	114	6	8	6.09	1057.88	2034.00	90	120	6	9	6.05	2560.43	2277.10	98	112	6	8	6.03	1522.21
Ca2-2,3,15	2362.13	106	104	3	8	6.35	1330.85	2110.00	100	110	2	9	6.47	2147.58	2362.13	102	108	3	9	6.40	1851.35
Ca2-3,5,15	2016.20	112	98	3	7	6.69	1499.48	1870.10	116	94	3	8	5.98	1904.2	2016.20	102	108	3	10	5.71	3964.49
Ca2-6,4,15	2491.93	94	116	6	8	6.50	1259.47	2388.40	94	116	6	8	5.89	1412.05	2588.10	96	114	6	8	6.82	1721.10
Ca3-2,3,15	2580.20	112	98	2	8	6.31	2593.90	2485.50	106	104	2	8	6.31	1243.5	2580.20	112	98	3	8	6.52	4003.78
Ca3-3,5,15	1775.10	100	110	3	8	6.32	1327.36	1629.00	138	72	3	9	5.98	1784.63	1775.10	104	106	3	9	6.69	3111.69
Ca3-6,4,15	2692.27	86	124	6	7	6.96	1235.40	2546.53	98	112	6	9	6.50	1338.94	2696.77	110	100	6	9	6.65	1265.71
Ca4-2,3,15	2697.73	106	104	2	9	6.81	1580.40	2377.38	108	102	3	8	6.99	1410.76	2697.73	100	110	2	8	6.68	1513.40
Ca4-3,5,15	2134.07	106	104	3	9	6.35	3406.26	2133.22	86	124	3	9	6.21	1798.5	2299.20	100	110	3	9	6.13	3924.03
Ca4-6,4,15	2130.10	110	100	6	9	6.00	2498.39	2108.10	104	106	6	8	7.08	2462.5	2324.00	90	120	6	8	6.39	1623.20
Ca5-2,3,15	2300.17	112	98	2	8	6.39	4027.12	2212.03	86	124	3	8	6.54	1845.6	2300.17	136	74	3	12	7.46	3099.32
Ca5-3,5,15	2329.67	82	128	3	8	6.65	1373.53	2209.03	104	106	3	8	5.90	1020.78	2433.93	102	108	3	9	6.74	1463.44
Ca5-6,4,15	2100.67	104	106	6	8	6.68	4161.28	2011.00	102	108	6	10	6.23	956.13	2100.67	90	120	6	8	6.82	2500.69
Cb1-2,3,15	2223.43	124	86	2	6	1.47	6667.56	2214.00	100	110	2	5	6.66	1125.32	2223.43	104	106	2	5	1.62	1339.94
Cb1-3,5,15	1911.63	108	102	3	6	1.43	4458.90	1914.40	78	132	3	5	1.42	3731.31	1911.63	100	110	3	5	1.33	1697.85
Cb1-6,4,15	2203.17	120	90	6	6	1.49	2055.68	2107.73	102	108	6	5	1.47	1280.82	2203.17	104	106	6	6	1.28	1956.18
Cb2-2,3,15	1840.40	98	112	2	5	1.48	2655.82	1834.70	112	98	2	6	1.40	2869.5	1842.37	94	116	2	5	1.58	3069.73
Cb2-3,5,15	1873.50	98	112	3	5	1.45	1470.18	1871.00	98	112	3	6	1.41	2566.47	1922.27	100	110	3	6	1.54	4744.53
Cb2-6,4,15	1918.53	100	110	6	5	1.43	1265.05	1922.37	100	110	6	5	1.50	1366.29	1924.67	98	112	6	5	1.48	1038.16
Cb3-2,3,15	2196.23	108	102	2	6	1.44	2086.94	2176.20	104	106	2	6	1.47	2500.1	2194.40	114	96	2	6	1.45	2344.10
Cb3-3,5,15	1822.80	110	100	3	6	1.48	4127.96	1717.23	110	100	3	6	1.43	1613.95	1739.93	92	118	3	5	1.45	1258.83
Cb3-6,4,15	1948.47	116	94	6	5	1.43	1107.00	1958.00	148	62	6	6	1.31	2084.66	1953.80	120	90	6	5	1.47	1513.00
Cb4-2,3,15	2492.00	110	100	2	6	1.30	4495.03	2297.60	116	94	2	6	1.56	1437.97	2492.50	96	114	2	6	1.55	3884.29
Cb4-3,5,15	1898.70	106	104	3	6	1.55	4396.04	1848.53	110	100	3	6	1.53	1864.3	1849.60	100	110	3	5	1.33	3042.99
Cb4-6,4,15	2047.10	116	94	6	5	1.47	1410.57	2043.77	110	100	6	5	1.49	1411.92	2158.80	94	116	6	6	1.45	1967.91
Cb5-2,3,15	2233.57	94	116	2	5	1.47	1383.41	2229.60	110	100	2	5	1.45	1786.83	2301.43	102	108	2	6	1.50	1246.63
Cb5-3,5,15	2162.07	104	106	3	5	1.41	1160.06	2166.17	104	106	3	5	1.53	1614.34	2314.60	102	108	3	6	1.47	4808.29
Cb5-6,4,15	2639.90	124	86	6	6	1.51	3982.77	2488.53	136	74	6	6	1.45	1548.08	2592.37	112	98	6	6	1.50	2735.81
Cc1-2,3,15	4927.00	92	118	4	12	10.77	2928.64	4852.63	90	120	4	12	1.54	2433.63	4551.63	108	102	4	12	10.48	1910.62
Cc1-3,5,15	4509.97	96	114	3	13	11.32	1433.41	4062.17	102	108	3	12	11.25	1211.85	4410.90	118	92	4	14	10.26	1502.79
Cc1-6,4,15	4777.73	108	102	6	14	11.04	1447.51	4771.87	124	86	6	13	9.89	1805.19	5374.40	126	84	6	13	11.86	1569.37
Cc2-2,3,15	5377.07	112	98	4	13	11.49	2947.28	4939.33	104	106	4	13	10.65	3432.54	4855.77	98	112	4	11	10.45	5082.88
Cc2-3,5,15	4590.10	96	114	4	12	10.89	2243.06	4394.47	112	98	4	12	10.64	2871.15	4574.43	118	92	3	12	10.89	3664.53
Cc2-6,4,15	4829.57	94	116	6	12	11.02	1365.84	4790.83	100	110	6	12	10.70	1481.57	5220.73	112	98	6	12	11.60	1331.52
Cc3-2,3,15	5156.67	98	112	4	13	11.34	1618.84	5311.03	102	108	3	12	10.94	2647.65	4796.73	102	108	4	11	11.37	1809.02
Cc3-3,5,15	4370.40	82	128	3	12	10.44	1487.46	4510.87	110	100	3	13	11.24	1057.68	4632.27	110	100	4	12	12.30	1750.34
Cc3-6,4,15	5454.53	110	100	6	13	11.79	1321.72	5457.60	106	104	6	13	11.43	1954.79	4958.77	94	116	6	11	10.71	2179.35
Cc4-2,3,15	4584.37	122	88	4	13	11.61	3158.38	4616.20	98	112	4	13	11.79	1449.05	4640.90	102	108	4	13	11.24	2912.30
Cc4-3,5,15	4847.30	86	124	3	14	11.35	1395.87	5554.13	102	108	3	12	11.14	2635.1	5397.37	88	122	3	11	11.80	1325.87
Cc4-6,4,15	5594.83	90	120	6	13	10.96	1332.62	6109.70	100	110	6	13	12.97	1782.66	5518.60	98	112	6	10	10.70	1342.42
Cc5-2,3,15	5715.43	94	116	4	13	11.14	3378.53	5753.67	114	96	4	13	11.65	2236.85	5678.80	94	116	4	12	10.47	3896.42
Cc5-3,5,15	5130.43	112	98	4	13	11.04	1761.48	5184.57	106	104	4	12	11.17	2294.64	4753.43	100	110	3	11	10.36	1719.76
Cc5-6,4,15	5006.67	86	124	6	13	10.50	1688.32	5530.21	120	90	6	14	10.66	2084.36	5146.73	112	98	6	12	10.87	1273.28
Cd1-2,3,15	3648.87	100	110	3	13	2.54	4263.50	4640.20	88	122	4	11	12.22	1420.6	3671.37	102	108	3	12	2.67	4173.17
Cd1-3,5,15	4232.77	114	96	3	14	2.54	1694.95	5019.83	98	112	3	12	11.01	2183.8	4222.90	110	100	3	12	2.74	3559.50
Cd1-6,4,15	3959.30	108	102	6	12	2.46	3743.82	4985.60	106	104	6	12	11.05	1274.32	3993.30	100	110	6	13	2.59	1812.28
Cd2-2,3,15	3428.17	112	98	3	15	2.59	4403.14	4126.57	116	94	4	12	11.90	2059.15	3409.60	112	98	4	13	2.71	4532.82
Cd2-3,5,15	3216.90	90	120	3	14	2.58	1432.32	3982.87	96	114	3	11	11.14	1620.93	3148.77	114	96	3	13	2.65	1341.84
Cd2-6,4,15	4294.80	110	100	6	13	2.59	1658.52	5100.00	112	98	6	14	10.92	2244.71	4304.47	104	106	6	12	2.53	3608.00
Cd3-2,3,15	3595.80	98	112	3	14	2.42	4694.39	4962.37	94	116	4	13	11.38	10175.3	3389.50	118	92	4	13	2.61	5471.30
Cd3-3,5,15	3029.57	102	108	3	12	2.52	1399.17	3509.07	106	104	4	12	11.84	2332	3104.07	100	110	3	12	2.68	1668.14
Cd3-6,4,15	3426.73	110	100	6	14	2.62	1455.39	4605.23	96	114	6	11	10.48	1772.61	3383.33	92	118	6	12	2.53	3764.34
Cd4-2,3,15	3742.10	102	108	4	13	2.58	2203.21	5135.57	114	96	4	12	11.74	1366.18	3780.17	120	90	4	13	2.45	2960.23
Cd4-3,5,15	3404.57	86	124	3	13	2.50	1556.87	4423.23	112	98	4	13	12.44	1757.45	3417.13	100	110	3	13	2.37	1705.61
Cd4-6,4,15	4157.40	110	100	6	12	2.50	2156.82	5489.63	96	114	6	11	11.77	1332.61	4126.60	102	108	6	12	2.61	3694.76
Cd5-2,3,15	3486.73	110	100	3	13	2.52	2924.11	4444.10	116	94	4	14	12.26	2887.52	3464.07	110	100	4	13	2.53	2018.44
Cd5-3,5,15	4094.13	98	112	3	14	2.52	3448.59	5201.83	120	90	4	14	11.18	569.55	4080.83	96	114	3	12	2.71	2555.24
Cd5-6,4,15	4296.77	100	110	6	12	2.53	3635.68	5172.83	116	94	6	15	11.50	2731.32	4300.37	118	92	6	13	2.53	2285.55
		104.17	105.83			5.39	2413.31		105.83	104.17			7.55	2014.32		104.33	105.67			5.41	2519.38

Table 15: Complete result table for instances with different correlation levels.