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A Two-Stage Stochastic Integrated Unit Commitment and Distribution Problem

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Abstract. We address an integrated unit commitment and distribution problem within a two-stage stochastic framework. The problem involves multiple production units that manufacture a single product and distribute it to clients over a discrete and finite planning horizon. The clients face a stochastic demand. A given client is served by at most a single production unit during a given period of time. Production units are subject to minimum up/down time constraints. We develop two deterministic models for the problem and extend one to a two-stage stochastic model, incorporating demand scenarios to account for uncertainty. Due to the computational complexity of solving this model using commercial solvers, we propose three heuristic approaches aimed at reducing the number of second stage binary variables, making the problem more tractable. Numerical experiments demonstrate the quality of the heuristics in finding good first stage decisions.

Keywords: unit commitment, production-distribution optimization, two-stage stochastic optimization

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1. Introduction

A supply chain's (SC) purpose is to transform raw materials into finished items and bring them to customers. A standard SC is divided into several independent parts, such as the item manufacturers and the distributors. Even if each part within the SC makes optimal decisions regarding production, inventory, and distribution, the overall outcome can still be suboptimal. For decision makers controlling different parts of the SC, it can be beneficial to simultaneously optimize the decisions of these parts. In particular, optimizing at once the production and distribution of a SC has raised high interest over the past few years (see Darvish et al. [9], Fahimnia et al. [12] and Hrabec et al. [20]).

Production may be handled by several factories, referred to as production units. Determining the best production decisions in terms of timing and quantities to be produced is challenging even with a single unit. Production planning, also referred to in the literature as lot sizing, involves determining the optimal production level for each period over a time horizon to satisfy a customer demand. Jans and Degraeve [21] and Brahimi et al. [6] provide an extensive review of deterministic single-level dynamic lot sizing problems.

In the lot sizing literature, demand is often considered deterministic. As shown by Adulyasak et al. [2] and Guarnaschelli et al. [17], this can lead to poor decisions and high costs since generally there is a lot of uncertainty. Consequently, accounting for this uncertainty in the decision-making process is desirable, even though it makes the problems more complex. Demand scenarios are a common way to represent uncertainty. Their use helps keep the problem tractable, which may not be the case when working with demand distributions. Gruson et al. [16] use demand scenarios in their study of a stochastic three-level lot sizing and distribution problem.

Unit commitment aims to minimize the costs of a production schedule for several production units over a planning horizon. The specificity of this problem is that production units have to commit to a certain production state for several periods.

We study a two-level integrated unit commitment and distribution problem in a two-stage stochastic setting. A general manufacturing company has multiple production units, which manufacture a single type of item over several periods and distribute it to clients. The production units can store items and are not interconnected, meaning that they cannot deliver items to each other. Each period, a unit can be either up or down. If a unit is up, it has a fixed production level, which is a parameter of the problem; if it is down, the production level is zero. This specificity shares similarities with discrete lot-sizing and scheduling problems, where in a given period, either there is no production or the whole capacity is used (Fleischmann [14]). Each unit has minimum up/down time constraints: it must remain in the same state for a given number of periods. In the unit commitment literature, these constraints are referred to as *minimum up/down time constraints* [4], a term that will be used throughout this paper. Distribution to clients is such that a client can only be served by one unit during a given period. However, this unit-client assignment can change over the planning horizon. Each client is served using direct shipments.

The objective is to minimize the total costs, which are composed of the production cost, the distribution cost, the holding cost at the production units, and the lost sales cost. Lost sales costs occur when demand cannot be met. We aim to compute an optimal production and distribution plan for each unit.

Moreover, clients have stochastic and dynamic demands. The distribution of the demand of each client is known and uncertainty is represented by demand scenarios. The clients' demand over the entire planning horizon is revealed once the first stage decisions are made. These first stage decisions involve the production state of the units (up or down) over certain periods at the beginning of the planning horizon. The second stage decisions include the unit states for the remaining periods, the distribution decisions and the resulting inventory decisions over the whole planning horizon.

Even simple versions of the unit commitment problem (UCP), which can be reduced to the studied problem, have been proven to be NP-hard (see Tseng [32]). It implies that the problem under study is itself NP-hard.

To the best of our knowledge, our work is the first to integrate the consideration of minimum up/down time constraints in the production and dynamic unit-client assignment for the transportation within an uncertain demand context.

Our paper makes three contributions:

- **Extension of existing literature:** We extend the existing production-distribution literature by considering dynamic unit-client assignment decisions and minimum up/down time constraints, proposing two deterministic models for this problem.
- **Two-stage stochastic model:** We extend one of the deterministic models to a two-stage stochastic version, with uncertainty modeled using scenarios.
- **Development and evaluation of heuristic solution approaches:** Given the inefficiency of using a commercial Mixed Integer Linear Programming (MILP) solver for the stochastic problem, we propose three heuristics and conduct extensive numerical experiments to assess their performance. These heuristics relax the studied problem by reducing the number of second stage binary variables, using different approaches to achieve this.

The paper is organized as follows. After a literature review related to our work presented in Section 2, we present the studied problem in Section 3. In Section 4, we propose two mathematical models of the deterministic version of the problem and extend one of them to a two-stage stochastic version using scenarios for uncertainty modeling. Section 5 presents the proposed heuristics, which find promising first stage decisions. Studied instances and extensive numerical experiments are presented in Section 6. Finally, Section 7 provides concluding remarks and perspectives of our work.

2. Literature review

In this section a literature review related to the studied problem is provided. It focuses on two-stage stochastic optimization, integrated production-distribution and unit commitment.

2.1. Two-stage stochastic optimization

Stochastic optimization aims to optimize an objective function by considering the probability distributions of uncertain parameters. Typically, these models optimize the expected value of the objective function. This makes stochastic optimization particularly useful in situations where the model's decisions are repeatedly taken, resulting in an average cost close to the computed expectation. Ruszczyński and Shapiro [27] and Birge and Louveaux [5] provide details on stochastic optimization.

In some applications it is possible to make decisions before and after the uncertainty realization. The former are called "here-and-now" or "first stage" decisions, and the latter are called "wait-and-see" or "second stage" decisions. "Here-and-now" decisions must be identical for all realizations of uncertainty, while "wait-and-see" decisions can depend on the realization of uncertainty. Two-stage stochastic optimization comes as a natural modeling approach for such problems. The relevance of this approach is shown by Grass and Fischer [15] who provide a review of two-stage stochastic optimization in disaster management. Shapiro et al. [28] provide a comprehensive overview of two-stage stochastic optimization.

Methods based on Bender's decomposition are often used to solve two-stage stochastic optimization problems [5]. This approach requires significant work, especially in case of integer second stage variables [29].

2.2. Integrated production-distribution

The simultaneous optimization of production and distribution processes in SCs has garnered significant attention in both research and industry [20]. The complexity of this integrated production-distribution problem (PDP) is influenced by the number of levels and the size of each level within the SC. Typical levels include suppliers, production units, warehouses, and clients. Additionally, factors such as the number of transportation links between these levels, the possibility of routes between elements at the same level, the potential for clients to be served by one or more origins, the variety of manufactured items, and the length of the planning horizon contribute to the overall difficulty of the problem [12].

Several studies have addressed different aspects of this problem. Haq et al. [19], and Hamedi et al. [18] consider a single-product model. Dhaenens-Flipo and Finke [10] study a multi-unit, multi-product, multi-warehouse, multi-period, multi-client industrial problem considering direct shipments, formulating and modeling the deterministic PDP and using a commercial solver for its solution. Amorim and Almada-Lobo [3] examine a multi-objective PDP within the context of perishable products, comparing sequential and integrated approaches to production and distribution optimization. Fahimnia et al. [12] offer a thorough and detailed review of deterministic PDP, covering a broad spectrum of problems, methodologies, and practical applications within this domain. Engebrethsen and Dauzère-Pérès [11] provide a review of inventory models with multiple transportation modes.

PDP optimization in the context of uncertain client demand has naturally extended deterministic works. Guarnaschelli et al. [17] address a two-level, two-stage stochastic PDP within the context of the dairy industry, with supply and demand uncertainty. In this case, production, units' inventory and delivery decisions are made in the first stage, while warehouse inventory decisions are handled in the second stage, based on demand and supply uncertainties. The authors compared the deterministic and stochastic models, showing the importance of taking uncertainty into account while modeling the problem. Gruson et al. [16] study a three-level, two-stage stochastic PDP, employing a Benders decomposition and a Benders-based branch-and-cut algorithm to solve the problem. In their work, the setup decisions are made in the first stage, while the production, transportation, and inventory decisions are handled in the second stage. While most studies typically consider direct deliveries to clients, other studies have included routing decisions. For instance, Adulyasak et al. [2] introduce a two-stage production routing problem with uncertain demand, where the first stage decisions involve setup and routing, and the second stage decisions concern the produced and delivered quantities. Kermani et al. [22] consider a two-stage production routing problem with uncertain demand, where the first stage involves production setup decisions, and the second stage concerns routing decisions.

Our work extends the existing production-distribution literature by considering minimum up/down time constraints in the production environment.

2.3. Unit commitment

The unit commitment problem (UCP) has been widely treated by the research community since the 1940s. Abdou and Tkiouat [1] present a chronological review of advancements in the study of this problem. This problem is typically encountered in the context of energy production. Usually, the costs are composed of the production costs, unit start up and shut down costs [7]. The production cost can be either fixed or proportional to the produced quantity.

The standard constraints of this problem are the following. Given a discrete and finite planning horizon, a certain demand has to be met in each period. Each unit can be either up or down in each period and must satisfy minimum up/down-time constraints [4]. Other constraints in the unit commitment literature include the consideration of operating ramp-up rates for certain types of units [30]. These constraints control how quickly a production unit can increase its output over a given period.

Usually in the UCP, the transmission network is considered as a static element, meaning there are no distribution decisions [33]. However, a certain class of unit commitment problems exists, in which some network decisions are incorporated. Indeed, sometimes opening or closing a certain electrical line may result in overall energy re-routing and allows to use less expensive energy sources [13].

UCP is difficult in practice. Van Ackooij et al. [33] present an overview of the methodologies used to solve the problem and separate them into four classes: dynamic programming [24], MILP approaches [23], decomposition approaches [8] and heuristic approaches [36]. Recently, machine learning techniques, such as random forests, neural networks, and support vector machines, have been increasingly combined with MILP approaches to address the UCP. Yang and Wu [35] provide an overview of these methods.

In the UCP there are several sources of uncertainty. One can cite uncertainty in energy demand, on energy prices or on unit output. Van Ackooij et al. [33] provide an extensive review of the stochastic version of UCP, categorizing the approaches to deal with it. The two major ones are robust and stochastic optimization. The robust optimization is based on a set-based uncertainty model, where solutions must remain feasible for all possible uncertainty realizations, with the objective typically being to optimize the worst-case. Wang et al. [34] propose a robust optimization model solving a case of UCP with uncertainty belonging to an interval. Stochastic optimization is based on the probability distribution of uncertain parameters and typically aims to optimize the expected value of the objective function. Rahmani et al. [25] propose a two-stage stochastic model solving a particular case of UCP.

Our work extends the existing UCP literature by considering distribution decisions.

3. Problem formulation

In this section we present the formulation of the deterministic version of the integrated unit commitment and distribution problem (UCDP) as well as the description of the stochastic parameters.

3.1. Deterministic version

A general manufacturing company operates a set U of units, which produce and distribute a single type of item to a set C of clients over a discrete planning horizon $T = \{1, \ldots, |T|\}$.

At each period t, a unit u can be in one of two states: up or down. If the unit is up, its production is P_u; if it is down its production is zero. Each unit must satisfy minimum up/down time constraints: a unit u must remain in the same state for at least $\hat{T}$ periods after a state change. For each unit u, *initial conditions* $(\tilde{w}_u, \tilde{t}_u)$ are known, where $\tilde{w}_u$ is the state of the unit u before period 1 and $\tilde{t}_u \leqslant \hat{T}$ is the number of periods for which the state must remain unchanged (if $\tilde{t}_u = 0$, the state can be chosen freely at the beginning of the planning horizon).

Each unit u incurs three types of production costs: a fixed production cost c_u^{p} each period the unit is up; a turn-up cost c_u^{up} each period the unit is turned on; and a turn-down cost c_u^{d} each period the unit is turned off. During each period each client can be served by only one unit, although this unit-client assignment may change over the planning horizon. The cost of distributing a unit of item from unit u to client c during period t is $k_{u,c}^t$. Unit u can store produced items in its inventory, incurring a holding cost h_u^t per unit of item during period t. An initial stock s_u^0 is available at the unit u at the beginning of the planning horizon. The demand of client c in period t is d_c^t. Any unit of unsatisfied demand for client c in period t incurs a lost sale cost l_c^t. The objective is to minimize the total production, inventory, state change, delivery and lost sales costs over the planning horizon.

The decisions to be made in each period include selecting the state of each unit and making distribution decisions, which involve unit-client assignments and determining the quantities delivered from units to clients.

3.2. Uncertain parameters

In the stochastic version of UCDP the parameter d_c^t, representing the demand of client c in period t, is uncertain. State decisions for units must be made for the next $\hat{T}$ periods in advance, while state decisions after period $\hat{T}$ and delivery decisions over the whole horizon can be made after the realization of demand uncertainty for each period.

4. Modeling

In this section, we present two formulations for the deterministic problem and a two-stage stochastic model for the studied problem. The two deterministic models were used in preliminary experiments to determine the best one to extend to a two-stage stochastic version, where uncertainty is modeled using demand scenarios.

4.1. Deterministic models

In this section we provide two deterministic models for the studied problem. We call a *state profile* a sequence of states over the whole planning horizon, which respects the minimum up/down time constraints. Let M be the set of all possible state profiles common to all units. Some of the state profiles might not be valid for a given unit because of its initial conditions. A given state profile m has a total operating cost of g_m, which includes the production and the state change costs over the planning horizon. The binary parameter w_m^t indicates the state of the unit in period t according to the state profile m. If w_m^t is equal to 1 (resp. 0), the unit is up (resp. down) in period t. The cardinality of the set M depends on the values of parameters $|T|$ and $\hat{T}$. Table 1 gives the cardinality of the set M depending on the parameters, which was computed using recursion. While the number of profiles is manageable for small instances, it increases rapidly as the problem size grows.

$\hat{T}$ \ $\lvert T\rvert$	12	18	24
2	288	5168	92736
3	82	812	8046
4	38	262	1814

Table 1: Number of possible state profiles

The first model, which we call natural, uses binary variables to represents states of the units, while the second one, which we call profile, takes as input the set of all possible state profiles and uses binary variables to assign a given profile to a unit.

4.1.1. Natural model

In this section we present a first natural model for UCDP. Table 2 summarizes the variables of this natural model.

Continuous variables	
$x_{u,c}^t$	quantity of item delivered from unit u to client c during period t
s_u^t	stock of item at unit u at the end of period t
p_u^t	quantity of item produced by unit u during period t
r_c^t	unmet demand of client c in period t
Binary variables	
w_u^t	up/down unit u state in period t, taking the value 1 if unit u is in the up state during period t and 0 otherwise
n_u^t	unit u turn up state at the beginning of period t, taking the value 1 if unit u is turned up at the beginning of period t and 0 otherwise
o_u^t	unit u turn down state at the beginning of period t, taking the value 1 if unit u is turned down at the beginning of period t and 0 otherwise
$a_{u,c}^t$	unit u assignment to client c during period t, taking the value 1 if unit u is assigned to client c in period t and 0 otherwise

Table 2: $\mathrm{UCDP}_{\mathrm{nat}}$ variables

The problem UCDP is formulated in its natural form as follows:

$$(\mathrm{UCDP}_{\mathrm{nat}})\quad \begin{aligned}
\min \quad & \sum_{t\in T}\left(\sum_{u\in U} c^{\mathrm{p}} w_u^t + \sum_{u\in U}\sum_{c\in C} k_{u,c}^t x_{u,c}^t + \sum_{u\in U} h_u^t s_u^t + \sum_{u\in U}(c_u^{\mathrm{up}} n_u^t + c_u^{\mathrm{d}} o_u^t) + \sum_{c\in C} l_c^t r_c^t\right) && (1)\\
\text{s.t.} \quad & w_u^t = \tilde{w}_u && \forall u \in U, 1 \leqslant t \leqslant \tilde{t}_u && (2)\\
& n_u^1 \geqslant w_u^1 - \tilde{w}_u && \forall u \in U && (3)\\
& o_u^1 \geqslant \tilde{w}_u - w_u^1 && \forall u \in U && (4)\\
& n_u^t \geqslant w_u^t - w_u^{t-1} && \forall u \in U, 2 \leqslant t \leqslant |T| && (5)\\
& o_u^t \geqslant w_u^{t-1} - w_u^t && \forall u \in U, 2 \leqslant t \leqslant |T| && (6)\\
& n_u^t \leqslant w_u^{t'} && \forall u \in U, t \in T, t \leqslant t' \leqslant t + \hat{T} - 1 && (7)\\
& 1 - o_u^t \geqslant w_u^{t'} && \forall u \in U, t \in T, t \leqslant t' \leqslant t + \hat{T} - 1 && (8)\\
& p_u^t = P_u w_u^t && \forall u \in U, t \in T && (9)\\
& \sum_{u\in U} a_{u,c}^t = 1 && \forall c \in C, t \in T && (10)\\
& x_{u,c}^t \leqslant d_c^t a_{u,c}^t && \forall c \in C, u \in U, t \in T && (11)\\
& \sum_{u\in U} x_{u,c}^t + r_c^t = d_c^t && \forall c \in C, t \in T && (12)\\
& s_u^t = s_u^{t-1} + p_u^t - \sum_{c\in C} x_{u,c}^t && \forall t \in T, u \in U && (13)\\
& p, x, s, r \geqslant 0 && && (14)\\
& a, w, n, o \in \{0, 1\}. && && (15)
\end{aligned}$$

The objective function (1) minimizes the sum of the production cost, the distribution cost, the holding cost, the cost of changing the state of units, and the lost sales cost.

Constraint (2) models the initial state conditions. Constraints (3), (4), (5), and (6) are linking the state of the unit and the change-over variables (i.e., turn up or turn down decisions). Constraints (7) and (8) are minimum up/down time constraints. Constraint (9) links the produced quantity to the unit state. Constraint (10) imposes that every client is served by only one unit in each period and constraint (11) links the unit-client assignment and the delivered quantities. Constraint (12) represents the demand satisfaction for each client c. Constraint (13) models the inventory balance at each unit. Constraints (14) and (15) define the domains of the variables.

Remark 4.1. It is well known that variables o may be expressed as a linear combination of variables w and n [26], but we choose to present them separately for the sake of readability.

Remark 4.2. As shown by Rajan et al. [26], constraints (7) and (8) can be rewritten as:

$$\begin{aligned} \sum_{t'=\max(1,t-\hat{T}+1)}^{t} n_u^t \leqslant w_u^t \quad & \forall u \in U, t \in T \\ \sum_{t'=\max(1,t-\hat{T}+1)}^{t} o_u^t \leqslant 1 - w_u^t \quad & \forall u \in U, t \in T\,. \end{aligned} \tag{16}$$

In our preliminary studies, using either (16) or (7) and (8) had no impact on the resolution time.

4.1.2. Profile model

In this section we present a second model that uses state profiles. We define a binary variable $z_{u,m}$, which is equal to 1 if unit u follows the state profile m over the planning horizon. Each unit can choose only one state profile. The problem UCDP is formulated in its profile form as follows:

$$(\mathrm{UCDP}_{\mathrm{prof}}) \quad \begin{aligned} \min \quad & \sum_{u\in U}\sum_{m\in M} g_m z_{u,m} + \sum_{u\in U}\sum_{c\in C} k_{u,c}^t x_{u,c}^t + \sum_{u\in U} h_u^t s_u^t + \sum_{c\in C} l_c^t r_c^t && (17)\\ \text{s.t.} \quad & (10) \text{ to } (14) && \\ & \sum_{m\in M} z_{u,m} = 1 \qquad \forall u \in U && (18)\\ & \sum_{m\in M} w_m^t z_{u,m} = \tilde{w}_u \qquad \forall u \in U, 1 \leqslant t \leqslant \tilde{t}_u && (19)\\ & p_u^t = P_u \sum_{m\in M} w_m^t z_{u,m} \qquad \forall u \in U, t \in T && (20)\\ & a, z \in \{0,1\}\,. && (21) \end{aligned}$$

The objective function (17) models the same costs as (1), using the state profile variables. Constraint (18) imposes the uniqueness of the state profile assigned to each unit. Constraints (19) to (21) are the adapted version of constraints (2), (9) and (15) respectively.

Remark 4.3. Rajan et al. [26] proved that when constraints (16) are used in UCDP_{nat} the objective values of the relaxed versions of UCDP_{nat} and $\text{UCDP}_{\text{prof}}$ are equal. The validity of this result remains an open question when using constraints (7) and (8) in UCDP_{nat}.

4.2. Stochastic model

Preliminary experiments showed that solving UCDP_{nat} was faster than solving $\text{UCDP}_{\text{prof}}$ (see Section 6). In this section we present the two-stage stochastic extension of UCDP_{nat}. As mentioned previously, the clients' demand d_u^t is now an uncertain parameter. Therefore, a random vector $\mathbf{d} = (\tilde{d}_1^1, \ldots, \tilde{d}_{|C|}^T)$ is considered, where $\tilde{d}_c^t$ represents the random demand of client c in period t. We denote by $w^{\leqslant \hat{T}}$ the vector of unit state first stage decision variables $(w^1, \ldots, w^{\hat{T}})$, by $n^{\leqslant \hat{T}}$ the vector of unit turn up first stage decision variables $(n^1, \ldots, n^{\hat{T}})$, and by $o^{\leqslant \hat{T}}$ the vector of unit turn down first stage decision variables $(o^1, \ldots, o^{\hat{T}})$. The second stage decision variables are the unit-client assignment a, the delivered quantities x, the unit state variables w^t for period indices $t > \hat{T}$, the unit turn up variables n^t for period indices $t > \hat{T}$, the unit turn down variables o^t for period indices $t > \hat{T}$, stock variables s, lost sales variables r and production variables q. Note that the latter are completely determined by the unit states and could also be separated in first and second stage variables, but it has no impact on the resolution of the problem. We assume that the objective is to minimize the expected value of the previously introduced cost function (1). We can write the two-stage stochastic programming model as:

$$
\text{(2S-UCDP)}\quad
\begin{aligned}
\min \quad & \sum_{t=1}^{\hat{T}} \sum_{u \in U} (c_u^{\text{p}} w_u^t + c_u^{\text{up}} n_u^t + c_u^{\text{d}} o_u^t) + \mathbb{E}_{\mathbf{d}}[Q(w^{\leqslant \hat{T}}, o^{\leqslant \hat{T}}, n^{\leqslant \hat{T}}, \mathbf{d})] && (22)\\
\text{s.t.} \quad & (2) \text{ to } (4) \\
& n_u^t \geqslant w_u^t - w_u^{t-1} && \forall u \in U, t \leqslant \hat{T} \quad (23)\\
& o_u^t \geqslant w_u^{t-1} - w_u^t && \forall u \in U, t \leqslant \hat{T} \quad (24)\\
& n_u^t \leqslant w_u^{t'} && \forall u \in U, t \leqslant \hat{T}, t \leqslant t' \leqslant \hat{T} \quad (25)\\
& 1 - o_u^t \geqslant w_u^{t'} && \forall u \in U, t \leqslant \hat{T}, t \leqslant t' \leqslant \hat{T} \quad (26)\\
& w, n, o \in \{0, 1\}, && (27)
\end{aligned}
$$

where, for a specific realization $\mathbf{d}$, the quantity $Q(w^{\leqslant \hat{T}}, o^{\leqslant \hat{T}}, n^{\leqslant \hat{T}}, \mathbf{d})$ is the optimal value of the following second stage problem:

$$
\text{(Q)}\quad
\begin{aligned}
\min \quad & \sum_{t=\hat{T}+1}^{|T|} \sum_{u \in U} (c_u^{\text{p}} w_u^t + c_u^{\text{up}} n_u^t + c_u^{\text{d}} o_u^t) + \sum_{t=1}^{|T|} (\sum_{u \in U} \sum_{c \in C} k_{u,c}^t x_{u,c}^t + \sum_{u \in U} h_u^t s_u^t + \sum_{c \in C} l_c^t r_c^t) && (28)\\
\text{s.t.} \quad & (5) \text{ to } (10) \text{ and } (13) \text{ to } (15) \\
& x_{u,c}^t \leqslant \tilde{d}_c^t a_{u,c}^t && \forall c \in C, u \in U, t \in T \quad (29)\\
& \sum_{u \in U} x_{u,c}^t + r_c^t = \tilde{d}_c^t && \forall c \in C, t \in T. \quad (30)
\end{aligned}
$$

Constraints (23) to (26) are the same as constraints (5) to (8), with time indices adapted for first stage variables. Constraints (29) and (30) are the stochastic version of constraints (11) and (12). Because it contains random variables, the two-stage stochastic programming model 2S-UCDP is intractable. To overcome it, we assume that there is a finite number of possible demand scenarios. We denote by Ω the set of all considered demand scenarios. Let π_ω be the probability of realization of scenario ω and $d^t_{c,\omega}$ be the demand of client c during period t for scenario ω. We add an index ω to the second stage variables, i.e., the distribution, stock, lost sales variables for the whole planning horizon and the unit state variables for all periods except the first $\hat{T}$ periods. The scenario-based two-stage stochastic problem is:

$$
(\text{2S-UCDP}_\Omega)\quad
\begin{aligned}
\min \quad & \sum_{t=1}^{\hat{T}} \sum_{u \in U} (c^{\text{p}}_u w^t_u + c^{\text{up}}_u n^t_u + c^{\text{d}}_u o^t_u) + \sum_{\omega \in \Omega} \pi_\omega \Bigg(\sum_{t=\hat{T}+1}^{|T|} \sum_{u \in U} (c^{\text{p}}_u w^t_{u,\omega} + c^{\text{up}}_u n^t_{u,\omega} + c^{\text{d}}_u o^t_{u,\omega}) + \\
& \sum_{t=1}^{|T|} \Big(\sum_{u \in U} \sum_{c \in C} k^t_{u,c} x^t_{u,c,\omega} + \sum_{u \in U} h^t_u s^t_{u,\omega} + \sum_{c \in C} l^t_c r^t_{c,\omega} \Big) \Bigg) && (31) \\
\text{s.t.} \quad & (2) \text{ to } (4), (14), (15) \text{ and } (23) \text{ to } (26) \\
& n^{\hat{T}+1}_{u,\omega} \geqslant w^{\hat{T}+1}_{u,\omega} - w^{\hat{T}}_u && \forall u \in U, \omega \in \Omega && (32) \\
& o^{\hat{T}+1}_{u,\omega} \geqslant w^{\hat{T}}_u - w^{\hat{T}+1}_{u,\omega} && \forall u \in U, \omega \in \Omega && (33) \\
& n^t_{u,\omega} \geqslant w^t_{u,\omega} - w^{t-1}_{u,\omega} && \forall u \in U, \hat{T}+2 \leqslant t \leqslant |T|, \omega \in \Omega && (34) \\
& o^t_{u,\omega} \geqslant w^{t-1}_{u,\omega} - w^t_{u,\omega} && \forall u \in U, \hat{T}+2 \leqslant t \leqslant |T|, \omega \in \Omega && (35) \\
& n^t_u \leqslant w^{t'}_{u,\omega} && \forall u \in U, 2 \leqslant t \leqslant \hat{T} < t' \leqslant \hat{T}+t-1, \omega \in \Omega && (36) \\
& 1 - o^t_u \geqslant w^{t'}_{u,\omega} && \forall u \in U, 2 \leqslant t \leqslant \hat{T} < t' \leqslant \hat{T}+t-1, \omega \in \Omega && (37) \\
& n^t_{u,\omega} \leqslant w^{t'}_{u,\omega} && \forall u \in U, \hat{T} < t \leqslant t' \leqslant \hat{T}+t-1, \omega \in \Omega && (38) \\
& 1 - o^t_{u,\omega} \geqslant w^{t'}_{u,\omega} && \forall u \in U, \hat{T} < t \leqslant t' \leqslant \hat{T}+t-1, \omega \in \Omega && (39) \\
& p^t_u = P_u w^t_u && \forall u \in U, t \leqslant \hat{T} && (40) \\
& p^t_{u,\omega} = P_u w^t_{u,\omega} && \forall u \in U, \hat{T} < t \leqslant |T|, \omega \in \Omega && (41) \\
& \sum_{u \in U} a^t_{u,c,\omega} = 1 && \forall c \in C, t \in T, \omega \in \Omega && (42) \\
& x^t_{u,c,\omega} \leqslant d^t_{c,\omega} a^t_{u,c,\omega} && \forall c \in C, u \in U, t \in T, \omega \in \Omega && (43) \\
& \sum_{u \in U} x^t_{u,c,\omega} + r^t_{c,\omega} = d^t_{c,\omega} && \forall c \in C, t \in T, \omega \in \Omega && (44) \\
& s^t_{u,\omega} = s^{t-1}_{u,\omega} + p^t_u - \sum_{c \in C} x^t_{u,c,\omega} && \forall u \in U, t \leqslant \hat{T}, \omega \in \Omega && (45) \\
& s^t_{u,\omega} = s^{t-1}_{u,\omega} + p^t_{u,\omega} - \sum_{c \in C} x^t_{u,c,\omega} && \forall u \in U, \hat{T} < t \leqslant |T|, \omega \in \Omega . && (46)
\end{aligned}
$$

Constraints (32) to (46) are the scenario version constraints of UCDP_{nat}.

5. Heuristics

Using commercial MILP solvers to solve 2S-UCDP$_{\Omega}$ is not efficient in practice (see Section 6, in particular Figure 5). In this section, we propose three heuristics to address this inefficiency. The goal of these heuristics is to find promising first stage decisions. The quality of these decisions is evaluated by fixing their values in the resolution of 2S-UCDP$_{\Omega}$. The main difficulty of 2S-UCDP$_{\Omega}$ is the high number of second stage binary variables. The common idea behind all the proposed heuristics is to reduce this number. Two different approaches are explored to achieve this. The first one is relaxing some of the second stage binary variables. The second one is fixing the value of some of the second stage binary variables. Additionally, a combination of these approaches is studied.

Remark 5.1. Given the form of 2S-UCDP$_{\Omega}$, in particular the fact that lost sales are allowed, every first stage decision is feasible. Thus, feasibility is not a concern in the proposed heuristics.

5.1. First heuristic — relaxing second stage binary variables

The first heuristic takes as a parameter $\tilde{T}$ ($\hat{T} < \tilde{T} < |T|$) and consists in solving a modified version of 2S-UCDP$_{\Omega}$, in which the second stage binary variables for the periods t, $\tilde{T} \leqslant t \leqslant |T|$, are relaxed. Constraint (15) is replaced by

$$a^t, w^t, n^t, o^t \in \{0, 1\} \qquad \forall t < \tilde{T} \tag{47}$$

$$a^t, w^t, n^t, o^t \in [0, 1] \qquad \forall t \geqslant \tilde{T}\,. \tag{48}$$

We call this heuristic the period-based relaxing heuristic (RH). Figure 1 illustrates the first and second stage variables for RH, with $\tilde{T} = 10$ and $\hat{T} = 3$. Periods highlighted in green represent constraint (47), while those in red correspond to constraint (48).

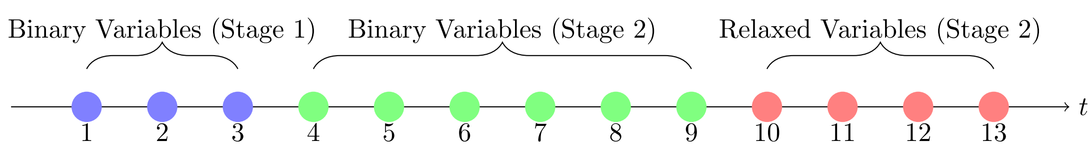

Figure 1: Scheme for first and second stage binary variables for RH

5.2. Second heuristic — fixing second stage binary variables

As we want to lower the number of second stage binary variables, one way to do this is to forbid some unit-client assignments. In particular we want each client to be assigned to one of its q closest units, where q is a parameter. We call this heuristic the proximity-based fixing heuristic (FH). For client c we define U_c^q as the set of its q closest units. The second heuristic consists in solving a modified version of 2S-UCDP$_{\Omega}$, in which we add the following constraint:

$$a^t_{u,c} = 0 \qquad \forall c \in C, u \notin U_c^q, t \in T\,. \tag{49}$$

5.3. Third heuristic — relaxing and fixing second stage binary variables

The combined relaxing and fixing heuristic (RFH) is a combination of the first and second heuristics. It takes as parameters $\tilde{T}$ and q previously introduced. We simultaneously relax the second stage binary variables for the periods t, $\tilde{T} \leqslant t \leqslant T$, and allow clients to be assigned only to one of its q closest units.

6. Numerical experiments

In this section, after introducing the studied instances, we present the numerical results obtained when solving UCDP_{nat}, $\text{UCDP}_{\text{prof}}$ and 2S-UCDP_{Ω}. We also evaluate the quality of first stage decisions found by the heuristics presented in Section 5 and those found by the mean value approach (MVA), which consists in solving UCDP_{nat}, using the average demand values

6.1. Instances and setting

As this problem is newly introduced, no benchmark instances are available in the literature. Therefore, we generate our instances as follows. The U.S. map was used to position the units and clients, and to calculate delivery costs. Specifically, the number of clients is set to 49, representing the 48 contiguous U.S. states and the District of Columbia. For each geographical area, we assume that the client is located at its center of gravity. The number of production units $|U|$ is set to 5, with each unit located at the center of gravity of one of the following U.S. cities: Boise, Santa Fe, St. Paul, Nashville, and Albany. The planning horizon T is composed of 18 periods, with a required minimum up/down period $\hat{T}$ of 3 periods. The overall demand at the clients is generated for each period from $U([500, 5000])$. The individual demand for each client is then computed by multiplying the proportion of the U.S. population residing in the geographical area corresponding to the client by the overall demand. Cost for changing states (c_u^{up} and c_u^{d}) are generated from $U([7500, 13500])$ (with $c_u^{\text{up}}{=}c_u^{\text{d}}$), and the production fixed cost c_u^{p} is set to $\frac{1}{4}c_u^{\text{up}}$.

For all units u and periods t the holding cost h_u^t is 0.5 and for all clients c the lost sales unit cost l_c^t is 1000. Delivery costs $k_{u,c}^t$, ranging between 0.1 and 7.8, are calculated by multiplying the distance between the unit and the client by 0.002, representing the cost per kilometer per transported unit of item. Figure 2 shows a histogram of the delivery costs. The initial stock s_u^0 for all units is set to 500. The initial state conditions $(\tilde{w}_u, \tilde{t}_u)$ for all units are $(0, 0)$, meaning units start in the down state and are free to change state at the beginning of the planning horizon. Production capacities P_u are set equal for all units to either 750, 875, or 1000. We generated 5 deterministic instances of demands and costs, changed the production capacity among the stated values, yielding 15 deterministic instances in total.

To transform the deterministic instances into stochastic instances, we introduce the amplitude of uncertainty τ. A demand scenario is generated by independently drawing the demand $d_{c,\omega}^t$ for each client c and period t uniformly from the interval $[d_c^t(1-\tau), d_c^t(1+\tau)]$. For each τ value in $\{0.1, 0.3, 0.5\}$ we generate 10 demand scenarios. The scenarios are assumed to be equiprobable. This procedure results in 45 stochastic instances derived from 15 deterministic ones.

For the experiments, we used Gurobi 10.0.0 and a Python wrapper *gurobipy*, developed by Gurobi. All experiments were conducted on 6 cores of an Intel Xeon 2.10GHz computer, with a default MILP optimality gap of 10^{-4}. Time limits for numerical experiments are specified in their respective sections.

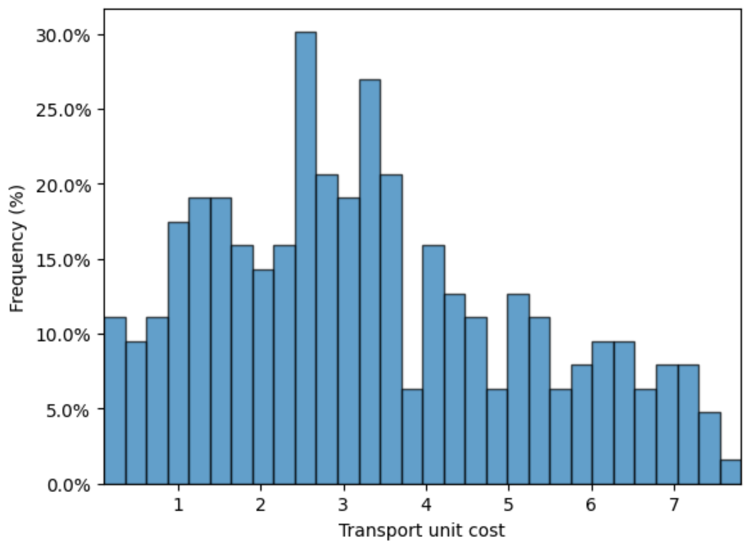

Figure 2: Transport costs histogram

6.2. Results for UCDP$_{nat}$ and UCDP$_{prof}$

To assess the performances of UCDP$_{\text{nat}}$ and UCDP$_{\text{prof}}$, the 15 deterministic instances have been solved using Gurobi with a one-hour time limit for both models. Figure 3 shows the solution times for both models for all the instances, as well as the average solution times. Both models were solved to optimality within the time limit for all instances. On average UCDP$_{\text{nat}}$ was solved almost twice as fast as UCDP$_{\text{prof}}$ and for 12 instances out of 15, UCDP$_{\text{nat}}$ was solved faster.

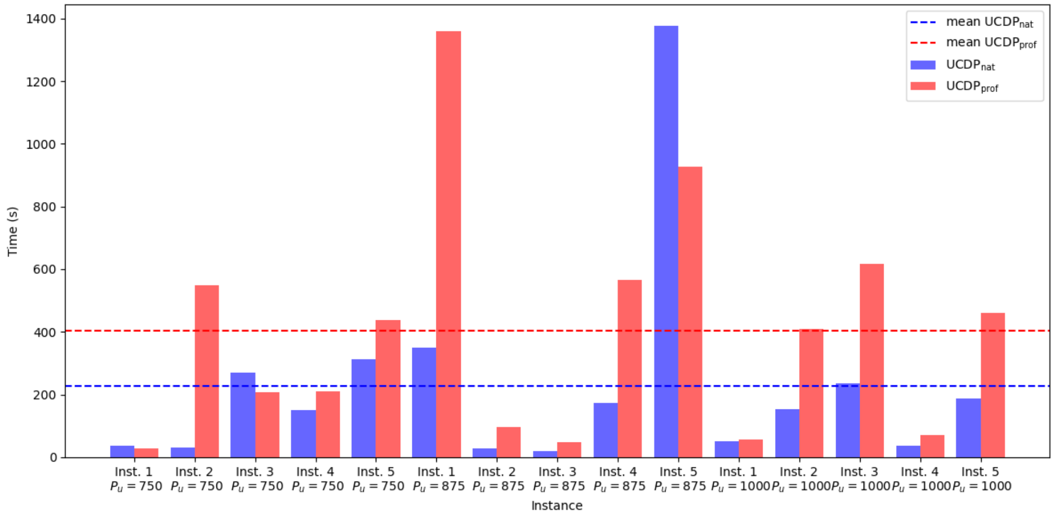

Figure 3: $UCDP_{nat}$ and $UCDP_{prof}$ solution times

We also analyzed the unit to which each client was assigned. To that end, we define the proximity index, which indicates the rank in terms of distance of the unit to which the client is assigned, with 1 being the closest and $|U|$ the farthest. Figure 4 presents a histogram of the proximity index of the unit assigned to the client among the 13230 unit-client assignments resulting from the optimal solutions for the 15 deterministic instances. It indicates that in nearly 70% of cases, clients are assigned to their closest unit.

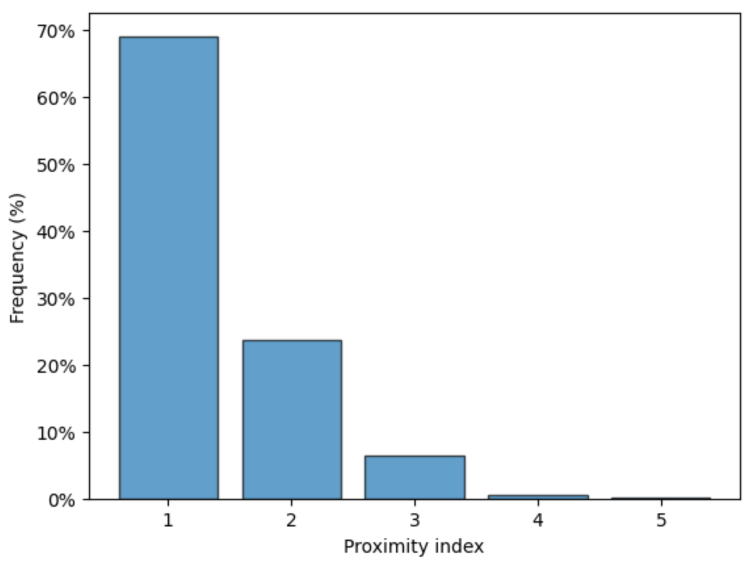

Figure 4: Frequency of proximity index for client-unit assignment (deterministic case)

6.3. Results for 2S-UCDP$_\Omega$

We use Gurobi to solve 2S-UCDP$_\Omega$ for the 45 stochastic instances, with a two-hour time limit. No instance was solved to optimality within the time limit. Figure 5, which is a histogram of the optimality gap (%) for these experiments, shows that the gap remained significant after two hours of solving time. The average optimality gap for the instances when the time limit is achieved is 10%.

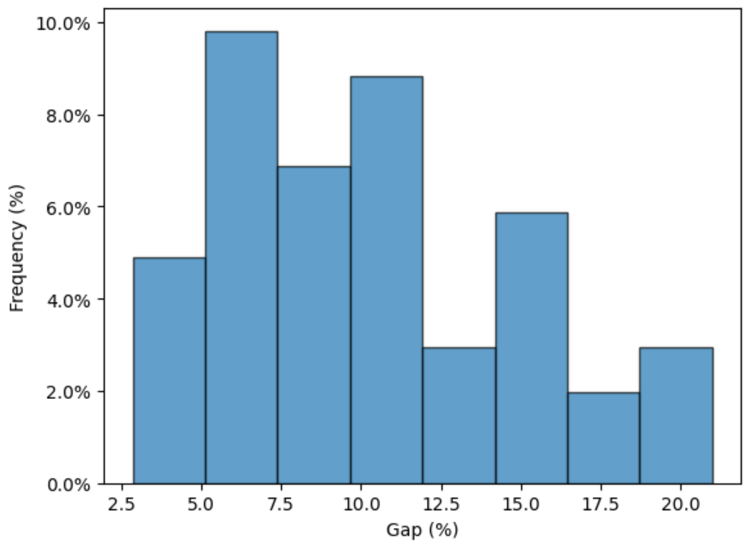

Figure 5: Optimality gap histogram for 2S-UCDP$_\Omega$

As the optimal solution of 2S-UCDP$_\Omega$ was not found for all the instances, we were not able to measure the value of the stochastic solution, which measures the expected gain from solving the stochastic model rather than the deterministic one (Birge and Louveaux [5]).

6.4. Results for heuristics

We evaluate the performance of the three heuristics introduced in Section 5 and of the MVA (which is a standard heuristic) on the 45 stochastic instances. As mentioned in Remark 5.1, the heuristics always provide feasible solutions. Next, we will compare the quality of the obtained solutions.

For each method (three new heuristics and MVA) and instance, we use Gurobi to solve the instance using the heuristic to get first stage decisions, with a one-hour time limit. Heuristic parameters $(\tilde{T}, q)$ were set to $(10, 2)$, based on preliminary experiments showing that smaller values performed poorly. Higher values were not tested due to the increasing problem complexity. Figure 4 shows that in more than 90% of the 13230 unit-client assignments for the 15 deterministic instances, the client was assigned to one of its two closest units, supporting the choice of parameter q. The obtained first stage decisions are then subsequently fixed in 2S-UCDP$_\Omega$ and this reduced problem is solved using Gurobi with a one-hour time limit.

Table 3 provides a performance summary of these experiments. The first column is the production capacity P_u and the second one is the uncertainty amplitude τ. The third column presents the average relative performance (%) $\frac{v-v^e_{\text{MVA}}}{v}$ of MVA for the studied instances. Here, v is the best found objective value of 2S-UCDP$_\Omega$ within a two-hour time limit and v^e_{MVA} is the objective value of the solution of 2S-UCDP$_\Omega$ with fixed first stage decisions from MVA. The fourth, fifth, and sixth columns present the relative performance (%) values for RH, FH, and RFH. Positive values indicate an improvement in performance when using the method (heuristic or MVA) compared to the stochastic model 2S-UCDP$_\Omega$.

Table 4 provides a solution time summary of these experiments. The first column is the production capacity P_u and the second one is the uncertainty amplitude τ. Column three is divided into three subcolumns, presenting for MVA: the average time (s) to find first stage decisions, the average time (s) to solve 2S-UCDP$_\Omega$ with fixed first stage decisions from MVA, and the total average time (s), which is the sum of the previous two. The fourth, fifth, and sixth columns present the same solution time values for the RH, FH, and RFH heuristics, respectively.

We refer the reader to Tables A.5, A.6, A.7, and A.8 in the appendix for more detailed results.

		MVA	RH	FH	RFH
P_u	τ	$\frac{v-v^e_{\text{MVA}}}{v}$ (%)	$\frac{v-v^e_{\text{RH}}}{v}$ (%)	$\frac{v-v^e_{\text{FH}}}{v}$ (%)	$\frac{v-v^e_{\text{RFH}}}{v}$ (%)
750	0.5	2.85	2.93	2.14	2.80
	0.3	2.89	2.55	1.15	2.45
	0.1	3.09	3.01	2.07	2.62
	avg.	2.94	2.83	1.79	2.62
875	0.5	1.00	3.56	3.49	3.49
	0.3	4.58	5.15	5.41	5.50
	0.1	2.85	2.45	2.38	2.81
	avg.	3.72	0.91	3.76	3.93
1000	0.5	5.79	6.77	5.99	6.64
	0.3	3.45	3.82	3.51	3.85
	0.1	2.59	3.49	3.74	3.75
	avg.	3.94	4.69	4.41	4.74
avg.		3.23	3.75	3.32	3.76

Table 3: Average relative performance of the heuristics compared to 2S-UCDP$_\Omega$

		MVA			RH			FH			RFH		
P_u	τ	1st st.	eval.	total	1st st.	eval.	total	1st st.	eval.	total	1st st.	eval.	total
750	0.5	161	2175	2336	3223	2055	5278	3600	1912	5512	1892	2006	3899
	0.3		2318	2479	3223	2055	5278	3600	2700	6300	2048	2516	4565
	0.1		2172	2233	3600	1579	5179	3600	1848	5448	1954	1545	3499
	avg.	161	2222	2383	3355	1872	5227	3600	2154	5754	1965	2023	3988
875	0.5	389	2771	3160	3600	2569	6169	3600	2496	6096	2227	2481	4708
	0.3		2691	3080	2998	2668	5666	3600	1921	5521	2291	2802	5093
	0.1		2490	2879	2965	2469	5435	3600	1875	5475	2812	2575	5387
	avg.	389	2651	3040	3188	2569	5757	3600	2098	5698	2443	2619	5062
1000	0.5	133	2170	2303	3600	1803	5403	3600	2243	5843	2318	2008	4325
	0.3		2333	2466	3600	2845	6445	3600	1861	5461	2210	2244	4455
	0.1		2449	2582	3600	2690	6290	3600	2856	6456	2382	2827	5209
	avg.	133	2318	2451	3600	2446	6046	3600	2320	5920	2303	2360	4663
avg.		228	2397	2625	3381	2296	5677	3600	2191	5791	2237	2334	4571

Table 4: Average solution times (s)

All the heuristics and MVA have better a performance on average than 2S-UCDP$_\Omega$. RFH has the best relative average performance, while MVA has the worst one. MVA has the best average total solution time, which is due to the fact that this method finds first stage decisions much quicker than the other heuristics. There is no significant difference in the average solution time of 2S-UCDP$_\Omega$ with fixed first stage decisions, depending on the used heuristic to obtain these first stage decisions.

In most instances, we observe very low lost sales costs, except in certain cases where the first stage decisions are fixed using the MVA. This highlights another advantage of the heuristics, accounting for uncertainty explicitly.

7. Concluding remarks

We formulate the integrated unit commitment and distribution problem and provide two deterministic models and one two-stage stochastic model. After comparing the efficiency of the two deterministic models we conduct numerical experiments for the stochastic case. As the direct use of the commercial solver is not efficient to solve the studied stochastic instances, we introduce three heuristics, finding first stage decisions for the studied problem. The common idea behind all the proposed heuristics is to reduce the number of second stage binary variables. Extensive numerical experiments show that the proposed heuristics are more efficient in practice than the resolution of the deterministic model to find first stage decisions. On average, the best-performing heuristic outperforms the deterministic model, although it takes more time.

Two research directions can be further explored. The first one aims at comparing the long term effects of the use of introduced heuristics. This study is noteworthy as an effective short-term optimization can potentially have detrimental effects on long-term outcomes. This can be achieved through a rolling horizon scheme and scenario simulation. The second one seeks to study the impact of optimization if some service level constraints (see Tempelmeier

[31]) are introduced in the problem. It can allow the studied problem to be closer to real industrial cases.

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Appendix A. Complete numerical results

Tables A.5, A.6, A.7, and A.8 summarize the experiments for MVA, RH, FH, and RFH, respectively. The first three columns are common across these tables: instance, uncertainty amplitude τ, and best found objective value v of 2S-UCDP$_{\Omega}$ within the two-hour time limit. The remaining columns provide results for method meth. Column four gives the solving time (s) t_{meth} or the optimality gap (%) g_{meth} if the one-hour time limit is reached. Column five gives the objective value of 2S-UCDP$_{\Omega}$ with fixed first stage decision from meth v^e_{meth}. Column six presents the solving time (s) of the latter t^e_{meth} or its optimality gap (%) g^e_{meth} if the one-hour time limit is reached. The last column shows the relative performance (%) $\frac{v-v^e_{\text{meth}}}{v}$.

Table A.5: Performance of MVA first stage solution

inst.	τ	v	MVA t_{MVA} (g_{MVA} (%))	MVA first stage v^e_{MVA}	MVA first stage t^e_{MVA} (g^e_{MVA} (%))	$\frac{v-v^e_{\text{MVA}}}{v}$ (%)
$P_u = 750$						
inst.1	0.5	291988	36	285161	781	2.34
	0.3	288069		280822	1461	2.52
	0.1	301618		276901	386	8.19
inst.2	0.5	292059	32	273482	999	6.36
	0.3	289793		270936	835	6.51
	0.1	276486		270151	1136	2.29
inst.3	0.5	341220	270	329541	(0.04)	3.42
	0.3	344188		330308	(0.05)	4.03
	0.1	340553		329972	(0.07)	3.11
inst.4	0.5	332204	151	329881	1893	0.70
	0.3	322845		320990	2096	0.57
	0.1	324947		322832	2137	0.65
inst.5	0.5	317541	314	312997	(0.03)	1.43
	0.3	317135		314568	(0.02)	0.81
	0.1	317692		313931	(0.01)	1.18
$P_u = 875$						
inst.1	0.5	281303	349	297363	(0.05)	-5.71
	0.3	295438		276840	(0.06)	6.30
	0.1	292374		266212	(0.10)	8.95
inst.2	0.5	243873	28	242277	512	0.65
	0.3	244668		241725	323	1.20
	0.1	240855		240856	463	0.00
inst.3	0.5	321399	20	293627	3081	8.64
	0.3	329152		304242	2443	7.57
	0.1	296609		302505	1188	-1.99
inst.4	0.5	316989	173	322326	3064	-1.68
	0.3	327423		312515	3489	4.55
	0.1	330708		311070	(0.01)	5.94
inst.5	0.5	299137	1376	289804	(0.04)	3.12
	0.3	300404		290568	(0.03)	3.27
	0.1	293131		289175	(0.04)	1.35
$P_u = 1000$						
inst.1	0.5	248285	52	244285	387	1.61
	0.3	245968		241099	677	1.98
	0.1	244074		241188	604	1.18
inst.2	0.5	267162	154	235953	757	11.68
	0.3	249984		235039	395	5.98
	0.1	259765		235092	843	9.50
inst.3	0.5	321181	235	281783	(0.01)	12.27
	0.3	305371		283731	(0.03)	7.09
	0.1	288699		276672	(0.02)	4.17
inst.4	0.5	293546	36	295286	(0.01)	-0.59
	0.3	288138		284801	3395	1.16
	0.1	285201		295086	(0.02)	-3.47
inst.5	0.5	275629	188	264586	2508	4.01
	0.3	267045		264229	(0.01)	1.05
	0.1	267364		263227	(0.03)	1.55

Table A.6: Performance assessment of RH first stage solution

inst.	τ	v	RH t_{RH} $(g_{\mathrm{RH}}$ (%))	RH first stage v^e_{RH}	RH first stage t^e_{RH} $(g^e_{\mathrm{RH}}$ (%))	$\frac{v-v^e_{\mathrm{RH}}}{v}$ (%)
$P_u = 750$						
inst.1	0.5	291988	(4.35)	286924	198	1.73
	0.3	288069	(5.62)	285048	158	1.05
	0.1	301618	(2.76)	277338	352	8.05
inst.2	0.5	292059	(1.78)	272124	1439	6.83
	0.3	289793	(3.04)	271412	759	6.34
	0.1	276486	(1.23)	269705	854	2.45
inst.3	0.5	341220	(3.70)	330613	(0.03)	3.11
	0.3	344188	(1.78)	330282	(0.04)	4.04
	0.1	340553	(2.78)	329894	(0.04)	3.13
inst.4	0.5	332204	(2.24)	327178	1436	1.51
	0.3	322845	(2.24)	321243	1786	0.50
	0.1	324947	(1.62)	323756	1395	0.37
inst.5	0.5	317541	1717	312957	(0.02)	1.44
	0.3	317135	1801	314557	(0.02)	0.81
	0.1	317692	(0.70)	314415	1696	1.03
$P_u = 875$						
inst.1	0.5	281303	(5.99)	268373	(0.02)	4.60
	0.3	295438	(7.37)	268045	(0.02)	9.27
	0.1	292374	(7.15)	268100	(0.02)	8.30
inst.2	0.5	243873	(4.17)	242277	501	0.65
	0.3	244668	(4.48)	241725	309	1.20
	0.1	240855	(4.15)	240856	453	0.00
inst.3	0.5	321399	(1.70)	293627	3036	8.64
	0.3	329152	590	304242	2327	7.57
	0.1	296609	426	302505	1094	-1.99
inst.4	0.5	316989	(4.75)	312964	2108	1.27
	0.3	327423	(5.83)	310966	(0.01)	5.03
	0.1	330708	(7.12)	314103	(0.02)	5.02
inst.5	0.5	299137	(6.27)	291278	(0.03)	2.63
	0.3	300404	(7.29)	292334	3504	2.69
	0.1	293131	(6.80)	290424	(0.04)	0.92
$P_u = 1000$						
inst.1	0.5	248285	(4.90)	244285	373	1.61
	0.3	245968	(5.11)	241099	648	1.98
	0.1	244074	(6.20)	241188	589	1.18
inst.2	0.5	267162	(9.01)	236526	2463	11.47
	0.3	249984	(9.75)	235477	3219	5.80
	0.1	259765	(10.61)	238637	2505	8.13
inst.3	0.5	321181	(8.01)	281571	1610	12.33
	0.3	305371	(5.98)	284389	(0.03)	6.87
	0.1	288699	(4.04)	276820	(0.01)	4.11
inst.4	0.5	293546	(1.39)	280602	2805	4.41
	0.3	288138	(2.63)	278292	3160	3.42
	0.1	285201	(1.97)	278215	3156	2.45
inst.5	0.5	275629	(2.24)	264586	1765	4.01
	0.3	267045	(1.29)	264232	(0.01)	1.05
	0.1	267364	(1.90)	263229	(0.03)	1.55

Table A.7: Performance assessment of FH first stage solution

inst.	τ	v	FH t_{FH} (g_{FH} (%))	FH first stage v^e_{FH}	FH first stage t^e_{FH} (g^e_{FH} (%))	$\frac{v-v^e_{\text{FH}}}{v}$ (%)
$P_u = 750$						
inst.1	0.5	291988	(5.81)	290637	1248	0.46
	0.3	288069	(8.26)	286721	(0.05)	0.47
	0.1	301618	(5.38)	280850	728	6.89
inst.2	0.5	292059	(7.77)	274093	896	6.15
	0.3	289793	(11.28)	270064	1599	6.81
	0.1	276486	(10.15)	276203	2033	0.10
inst.3	0.5	341220	(7.68)	331153	(0.05)	2.95
	0.3	344188	(10.06)	332198	(0.06)	3.48
	0.1	340553	(9.98)	331278	(0.07)	2.72
inst.4	0.5	332204	(1.89)	328637	1410	1.07
	0.3	322845	(0.86)	321848	1103	0.31
	0.1	324947	(0.02)	324029	1297	0.28
inst.5	0.5	317541	(5.40)	317353	2408	0.06
	0.3	317135	(7.79)	333923	(0.04)	-5.29
	0.1	317692	(1.28)	316535	1582	0.36
$P_u = 875$						
inst.1	0.5	281303	(8.91)	268372	(0.02)	4.60
	0.3	295438	(11.56)	268045	(0.02)	9.27
	0.1	292374	(10.63)	268100	(0.02)	8.30
inst.2	0.5	243873	(1.30)	242277	490	0.65
	0.3	244668	(4.41)	241725	298	1.20
	0.1	240855	(3.53)	240856	436	0.00
inst.3	0.5	321399	(5.79)	293627	2872	8.64
	0.3	329152	(5.21)	296294	192	9.98
	0.1	296609	(3.96)	296574	176	0.01
inst.4	0.5	316989	(8.68)	312964	2077	1.27
	0.3	327423	(10.56)	311211	1916	4.95
	0.1	330708	(11.62)	319172	2025	3.49
inst.5	0.5	299137	(6.98)	292277	3443	2.29
	0.3	300404	(7.53)	295548	(0.01)	1.62
	0.1	293131	(7.60)	292782	3138	0.12
$P_u = 1000$						
inst.1	0.5	248285	(6.59)	252447	(0.02)	-1.68
	0.3	245968	(3.80)	241958	286	1.63
	0.1	244074	(3.14)	241188	590	1.18
inst.2	0.5	267162	(13.55)	238148	1660	10.86
	0.3	249984	(14.24)	237040	1133	5.18
	0.1	259765	(11.83)	235332	3380	9.41
inst.3	0.5	321181	(9.48)	281571	1533	12.33
	0.3	305371	(9.34)	286289	1228	6.25
	0.1	288699	(5.95)	276820	(0.01)	4.11
inst.4	0.5	293546	(5.34)	280602	2783	4.41
	0.3	288138	(5.87)	278292	3058	3.42
	0.1	285201	(5.24)	278215	3109	2.45
inst.5	0.5	275629	(4.37)	264586	1641	4.01
	0.3	267045	(4.09)	264229	(0.01)	1.05
	0.1	267364	(3.73)	263227	(0.03)	1.55

Table A.8: Performance assessment of RFH first stage solution

inst.	τ	v	RFH	RFH first stage		$\frac{v-v^e_{\mathrm{RFH}}}{v}$ (%)
			t_{RFH} (g_{RFH} (%))	v^e_{RFH}	t^e_{RFH} (g^e_{RFH} (%))	
$P_u = 750$						
inst.1	0.5	291988	2199	286924	201	1.73
	0.3	288069	(1.26)	286719	(0.05)	0.47
	0.1	301618	(1.62)	281482	353	6.68
inst.2	0.5	292059	182	271737	1197	6.96
	0.3	289793	163	268784	1623	7.25
	0.1	276486	71	269705	814	2.45
inst.3	0.5	341220	(2.20)	331203	(0.04)	2.94
	0.3	344188	(1.47)	331639	(0.04)	3.65
	0.1	340553	(0.95)	331497	(0.01)	2.66
inst.4	0.5	332204	3333	328637	1433	1.07
	0.3	322845	2595	322139	1569	0.22
	0.1	324947	2298	324029	1300	0.28
inst.5	0.5	317541	148	313460	(0.01)	1.29
	0.3	317135	284	315095	2189	0.64
	0.1	317692	200	314415	1659	1.03
$P_u = 875$						
inst.1	0.5	281303	(2.89)	268366	(0.02)	4.60
	0.3	295438	(2.45)	268044	(0.02)	9.27
	0.1	292374	(4.02)	268097	(0.02)	8.30
inst.2	0.5	243873	147	242277	479	0.65
	0.3	244668	444	237772	1037	2.82
	0.1	240855	3142	237333	1048	1.46
inst.3	0.5	321399	187	293627	2785	8.64
	0.3	329152	213	304242	2173	7.57
	0.1	296609	119	302505	1028	-1.99
inst.4	0.5	316989	(1.80)	312964	2080	1.27
	0.3	327423	(4.31)	310966	(0.01)	5.03
	0.1	330708	(3.33)	313097	(0.01)	5.33
inst.5	0.5	299137	(1.83)	292277	3460	2.29
	0.3	300404	(2.62)	291988	(0.03)	2.80
	0.1	293131	(2.28)	290421	(0.04)	0.92
$P_u = 1000$						
inst.1	0.5	248285	(0.01)	244285	356	1.61
	0.3	245968	(0.01)	241958	281	1.63
	0.1	244074	(0.02)	241188	582	1.18
inst.2	0.5	267162	(0.51)	238148	1649	10.86
	0.3	249984	(1.34)	237040	1125	5.18
	0.1	259765	(4.19)	235332	3294	9.41
inst.3	0.5	321181	1178	281691	(0.02)	12.30
	0.3	305371	159	281069	3119	7.96
	0.1	288699	1081	276672	(0.02)	4.17
inst.4	0.5	293546	3009	280602	2761	4.41
	0.3	288138	(0.64)	278292	3097	3.42
	0.1	285201	3424	278215	3061	2.45
inst.5	0.5	275629	201	264586	1673	4.01
	0.3	267045	92	264229	(0.01)	1.05
	0.1	267364	204	263227	(0.03)	1.55