CIRRELT

CIRRELT-2026-01

A Chance-constrained Model for a Production Routing Problem with Uncertain Availability of Vehicles

Alline Zanette
Michel Gendreau
Walter Rei

Janvier 2026

Bureau de Montréal

Université de Montréal
C.P. 6128, succ. Centre-Ville
Montréal (Québec) H3C 3J7
Tél : 1-514-343-7575
Télécopie : 1-514-343-7121

Bureau de Québec

Université Laval,
2325, rue de la Terrasse
Pavillon Palasis-Prince, local 2415
Québec (Québec) G1V 0A6
Tél : 1-418-656-2073
Télécopie : 1-418-656-2624

A Chance-constrained Model for a Production Routing Problem with Uncertain Availability of Vehicles

Alline Zanette[1,2], Michel Gendreau[1,2,*], Water Rei[1,3]

1. Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)
2. Dept. of Mathematics and Industrial Engineering, Polytechnique Montréal, Montreal, QC, H3T 1J4, Canada
3. Dept. of Analytics, Operations and Information Technologies, UQÀM, Montreal, QC, H2X 3X2, Canada

Abstract. The Production Routing Problem (PRP) is a complex integrated problem that allows for the achievement of competitive advantages, such as better management of inventory, reduction in operational costs and lead times, improvement in efficiency and customer service, and better response to market changes. Most of the literature on PRP considers only deterministic data, and the papers that take stochastic parameters into account focus mainly on uncertain demand. In this study, we consider a PRP with a single capacitated production facility, a single product type, and a homogeneous fleet of capacitated vehicles. The availability of these vehicles is assumed uncertain and formulated as a stochastic parameter. The problem is modeled using two types of chance-constraints, and the sample approximation approach method is used to linearize the formulations, which are then solved using Benders decomposition (BD) and partial BD (PBD). Results show that PBD outperforms the standard BD method, and it is able to produce good optimality gaps for most instances within two hours of CPU time. In the remaining experiments, which are performed using PBD, results show that the problem becomes significantly more difficult to solve as the number of periods and retailers increase. Moreover, one of the mathematical formulations allows more flexibility to decision makers, resulting in higher feasibility, and smaller costs.

Keywords: Routing, Transportation, Stochastic programming, Production, Inventory, Chance constraints, Sample Approximation

Acknowledgements. The first author has been supported by grants from the Natural Sciences and Engineering Research Council of Canada, MITACS, and Conseil 2.0.

---

* Corresponding author: michel.gendreau@cirrelt.net

Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2026

1 | INTRODUCTION

In the last decades, supply chain planning has become one of the major concerns of companies in the fierce environment that they now face, forcing them to focus on the improvement of their performance. While firms have long been optimizing the functions in the supply chain sequentially and separately [5, 7], it is well known that optimizing the activities of the supply chain separately often prevents the achievement of better solutions overall [3, 27]. One way of coping with this situation is to consider integrated problems, such as the Production Routing Problem (PRP), in which one performs the joint and simultaneous optimization of production, inventory, distribution, and routing decisions. This is a recognized approach, which allows for a greater opportunity to achieve competitive advantages, such as better management of inventory, reduction in operational costs and lead times, improvement in efficiency and customer service, and better response to market changes [9, 10, 27].

Most of the literature on PRP considers only deterministic data. This is a significant concern, as uncertainty is a major issue in supply-chain management, and some of the critical information necessary for decision-making is often only approximated by forecasts. Special attention should be given to risk management in the transportation phase, due to the worldwide capacity shortage [26, 16, 14]. In the specific case of road transportation, this has been caused mainly by the cargo growth, the reduction of the number of vehicles owned by carriers, the drivers shortage, and the reduction of the number of sub-contractors, a situation not expected to improve in the coming years [26, 14, 20, 33]. As a consequence, shippers become more dependent on carriers, and are faced with an uncertain and unreliable transportation process [16]. This aspect has been neglected in the context of the PRP, but it is essential to avoid transportation disruptions and compromises in the operational performance [26].

In this paper, we study a PRP with a discrete and finite time horizon. The production-distribution network consists of a single capacitated production facility that can produce a single type of product, and a set of geographically dispersed retailers that require regular deliveries. A fleet of homogeneous capacitated vehicles is available to perform the latter. In order to characterize the aforementioned source of uncertainty, we assume that the number of readily available vehicles to perform the deliveries at each period is a stochastic parameter. Guaranteeing the feasibility of the problem for all the possible values of the stochastic parameter may result in costly and highly conservative solutions, thus, in many practical applications, it is common to relax this conservativeness up to an admissible risk tolerance by enforcing chance-constraints [25, 21]. We adopt this modelling approach and propose two mathematical chance-constrained formulations for the problem under study, one more conservative (aggregated), which guarantees a minimum feasibility amongst all scenarios, and one more flexible (disaggregated), which imposes the risk level with respect to the periods in each scenario. The sample approximation approach (SAA) method is used, and it was chosen because it deals with the type of chance-constraints under study, which include randomness in the right-hand side and non-linearities, and it is also appropriate for problems in which very small risk levels do not need to be enforced, allowing the decision maker to choose between the competing goals of cost and risk. One of the models is initially solved for a subset of the considered instances, using traditional Benders decomposition (BD) and partial Benders decomposition (PBD) as an acceleration strategy. The results of both methods are compared and the best performing is used for the remaining computational experiments. Finally a sample analysis is performed in order to verify the stability and validity of the samples used in the experiments.

The remainder of the paper is organized as follows. In Section 2, a literature review is performed, followed by the problem description in Section 3. In Section 4, we define the mathematical formulations. Section 5 includes the approaches used to solve the problem, followed by the computational experiments and results in Section 6. Conclusions are drawn in Section 7.

2 | LITERATURE REVIEW

The PRP has received an increasing amount of attention in the last years [2, 5, 34] and it has proved very successful in theory and practice, being able to reduce the logistic costs of several companies. The great majority of the literature focuses on deterministic versions of the problem, and on the use of heuristic algorithms as solution approaches. Many variants of the deterministic PRP have been explored and include, but are not limited to, perishable products, time windows, cross-docking satellites, backlog allowance, environmental and social aspects. More details concerning the existing studies can be found in the work of Adulyasak et al. [5]. When one considers the stochastic variants of the problem, the literature is scarce, and the existing studies focus on uncertain demand. In the remainder of this section, we present a summary of the works on the stochastic PRP and its variations.

The seminal work that considers stochastic parameters in the context of PRP is attributed to Adulyasak et al. [4], who assumed the existence of uncertain demand. The authors study a single-product PRP, with a capacitated production facility, and a homogeneous fleet of vehicles. Two formulations are proposed, namely a two-stage and a multi-stage stochastic program. In the first, scenarios are independent, while in the latter a scenario tree is considered, thus some scenarios have common elements and non-anticipativity constraints are necessary. To solve the models, the authors propose an exact method, a Benders-based branch-and-cut (BBC) algorithm, and incorporate computational enhancements for both formulations. Results show that the regular branch-and-cut (BC) algorithm outperforms the BBC for smaller instances with a small number of scenarios. Nevertheless, for large instances and cases with a large number of scenarios, its enhanced version is better than the BC, both in computational times and results.

An extension of the PRP, also considering uncertain demand, is proposed by Zhang et al. [35] to include reverse logistics and carbon emission regulations. The authors assume the existence of multiple manufacturing and remanufacturing plants, simultaneous pickups and deliveries of worn out items and finished goods, a heterogeneous fleet of vehicles, and carbon cap-and-trade policy. Two formulations for the problem are introduced, a deterministic and a stochastic one. The latter is a two-stage model with stochastic demand of finished and worn-out products. The authors employ the commercial solver Cplex to solve the models and conclude that the carbon price is the parameter with the most significant impact on the total supply chain profit.

A similar work is developed by Shuang et al. [31] aiming at selecting the best carbon emissions control policy while maximizing the total network profit. Two case studies are performed to demonstrate the application of the proposed modelling approaches, which include a deterministic and a two-stage stochastic formulation with uncertain demand and quantity of worn-out items. Both models are solved using Cplex. Similarly to [35], the authors confirmed the effects of carbon prices on the profits of the supply chain.

A commercial solver is used by Ji et al. [19] to solve a PRP with uncertain demand, multiple products, multiple production facilities, a heterogeneous fleet of vehicles, backlog allowance and physical internet (PI) hubs. The developed formulation is solved using Cplex and the authors report that PI hubs allow for a reduction in total costs when compared to traditional supply chains with dynamic configuration and lateral transshipment.

The effectiveness of PI hubs in a context of uncertain demand is also studied by Peng et al. [28], but a heuristic procedure is used to solve the proposed formulation. The authors consider the PRP in a multi-layer network, with multiple production plants, PI hubs, open distribution centers (DCs), transshipments and en-route consolidation. The authors also consider the occurrence of disruption events at the plants and/or DCs, which can compromise the production, storage and handling capacities, as well as some links of the network. The problem is modelled with a two-stage formulation, and two disruption management strategies are evaluated. A two-level heuristic algorithm is proposed to solve the model, and its results are generally better than those of the commercial solver Gurobi. The authors also conclude that the total cost and the service level are substantially improved when mitigation strategies are considered in the problem.

Another heuristic procedure is used to solve a PRP with uncertain demand, and backlog allowance [6]. The authors propose a two-stage formulation and use two different approaches to solve the problem. In the first one, called static, a sample average approximation procedure is applied in combination with branch-and-bound (BB). The problem is solved independently for different sample sets in order to generate candidate solutions. A time limit is imposed in the solution process, and the best solution is chosen. In the adjustable approach, the sample average approximation is applied and the same heuristic is used to produce a set of candidate solutions; the latter are then used to fix part of the first-stage solution, and the new restricted problem is solved again for a new larger sample. To improve the generated solutions, two alternatives are used in conjunction with both solution strategies: iterative local search, and a simplification of the model by the predefinition of routes, making the restricted model easier to solve. Results show that the adjustable approach produces better quality solutions, and using the enhancement strategies helps to improve the results and computational times, mainly for larger-sized instances.

A PRP with multiple perishable products, a heterogeneous fleet of vehicles, distribution with time windows, and stochastic demand is studied by Vahdani et al. [32]. A heuristic based on a large neighborhood search algorithm is presented to solve the proposed problem, as well as the water cycle algorithm, a metaheuristic founded on the observation of the water cycles that occur in nature. The computational results obtained show that the metaheuristic generally outperforms the software LINGO and the proposed heuristic.

Metaheuristics procedures are also adopted to solve a PRP that considers demand uncertainty, multiple perishable products, multiple production sites, production in two different phases, a heterogeneous fleet of vehicles, time windows, and gas emissions in the supply chain [29]. The authors propose a non-linear formulation and solve it using a cuckoo search algorithm (CSA) and the flower pollination algorithm (FPA). Results show that the CSA outperforms the FPA both in solution quality and computational times.

Workforce planning is introduced in the context of PRP by Majidi et al. [24], who also consider price-dependent uncertain demands, transportation emissions, and the existence of multiple perishable products. A non-linear multi-objective formulation is proposed, aiming at maximizing profits, and minimizing emissions and workforce fluctuations. The model is linearized using continuous prices and discrete demands, and it is initially solved using an ϵ-constraint method. This approach allows one to obtain up to 50% of feasible solutions in small instances and none in the larger ones, within a time limit of one hour of CPU time. As an alternative, the authors use the metaheuristic NSGA-II, which is enhanced by dynamic mutation, dynamic crossover and fuzzy domination. The results obtained by the enhanced NSGA-II are generally better than those of the traditional version of the algorithm.

A problem with the same characteristics, with the exception of the gas emissions, is studied by Farghadani-Chaharsooghi et al. [15]. In addition to the stochastic demands, the authors also consider uncertain travel times. A two-stage scenario-based stochastic formulation is developed, and a matheuristic that combines a progressive hedging algorithm, BD and a genetic algorithm is used to solve it. Results show that, in comparison with a formulation that considers all the scenarios of a scenario tree, the model decomposed by scenario produced solutions in shorter time and with low optimality gap. The proposed approach also outperforms Cplex.

An extension of the PRP, the collection disassembly problem, is proposed by Habibi et al. [18]. In this problem, a single end-of-life product, composed of multiple components, must be collected by a homgeneous fleet of vehicles at several centers. Then, the products are taken to a plant, where they are disassembled to release the requested components. Three parameters are considered as stochastic by the authors: the quantity of products returned to the collection centers; the quantity of components in each item and; the demand for the components. The problem is modeled as a two-stage stochastic program and solved using and adaptation of the two-phase iterative heuristic of Absi et al. [1], which is known for its good performance in solving the PRP. To deal with the large number of scenarios, the SAA is also applied. The computational results show that obtaining the perfect information is too costly for managers, since the expected value of the perfect information is, on average, at least 39%. On the other hand, solving the problem using the average value of the uncertain parameter can make the managers pay at least 1.4% and up to 10.6% more than by solving the proposed stochastic formulation.

As it is possible to see, there are many opportunities to be explored while contemplating uncertainty in the supply chain. In this study, we investigate the PRP with uncertainty in the availability of vehicles. This is a problem setting not considered by previous studies, but commonly found in industrial environments.

3 | PROBLEM DESCRIPTION

The PRP under study is defined on a complete graph $\mathcal{G} = \{\mathcal{N}, \mathcal{A}\}$, where $\mathcal{N} = \{0, \ldots, n\}$ is the set of nodes and $\mathcal{A} = \{(i, j) : i, j \in \mathcal{N}, i \neq j\}$ is the set of arcs. The plant is denoted by the node 0 and $\mathcal{N}_c = \mathcal{N} \setminus \{0\}$ is the set of retailers. A single product is produced by a single plant, with production capacity C, along a finite discrete time horizon $\mathcal{T} = \{1, \ldots, l\}$. When production takes place in a period t, we assume that there are no delays, and products can be fabricated and distributed in the same period; moreover, a fixed setup cost f and a variable production cost u per unit are incurred for the product considered.

The demand of retailer $i \in \mathcal{N}_c$ for the product considered, in each period $t \in \mathcal{T}$ is denoted by d_{it}. The product can be stored by the plant and by the retailers up to an inventory limit of L_i, incurring an inventory holding cost of h_i, $\forall i \in \mathcal{N}$. We observe that the holding costs vary depending on the node.

A fleet of homogeneous vehicles $\mathcal{K}_t = \{0, \ldots, K_t\}$, each with capacity Q, is available at the plant in period $t \in \mathcal{T}$ to distribute the product to the retailers. When a vehicle travels directly from i to j, a cost c_{ij} is incurred, $\forall i, j \in \mathcal{N}$. As the fleet is rented from a third party that cannot always provide the minimum number of trucks required by the company, the actual number of vehicles available in each period is uncertain and is given by the random parameter K_t, which is independent with respect to period $t \in \mathcal{T}$, and follows a given probability distribution. It is also assumed that each tour starts and ends at the plant, no limit is imposed on the duration of the trips, each vehicle can perform at most one trip per period, and split deliveries are not allowed (each retailer can be served by only one vehicle per period).

The goal of the problem is to simultaneously minimize production, inventory, and routing costs at the plant and retailers, satisfying the demands, and respecting inventory limits and the production capacity.

4 | PROBLEM FORMULATION

In order to formulate the problem described in the previous section, we decided to use the chance-constraint approach to model the uncertainty in the number of vehicles available in each period (K_t). This can be done using an aggregated or disaggregated approach with respect to the time horizon. In the first one, the risk level is imposed over all the scenarios, i.e., we require a minimum number of scenarios to be feasible. In the second one, the risk level is imposed with respect to the periods in each scenario, i.e., we check how many periods, over all the scenarios, are feasible. These two approaches are respectively presented in subsections 4.1 and 4.2.

4.1 | Aggregated Chance-Constraint

In order to model an aggregated chance-constraint with respect to the time horizon, we require that $\bar{k}_t$ vehicles be available in all the periods with probability $1-\epsilon$, where $\bar{k}_t$ is the largest integer such that $Pr\{K_t \geq \bar{k}_t, \forall t \in \mathcal{T}\} \geq 1-\epsilon$ and ϵ is the nominal risk level. Considering u_t as the actual number of vehicle routes designed for period t, we have to guarantee that:

$$u_t \leq \bar{k}_t \Rightarrow Pr\{u_t \leq K_t, \forall t \in \mathcal{T}\} \geq 1-\epsilon \tag{1}$$

Assuming the existence of the vector of decision variables $\boldsymbol{u} = (u_1, \ldots, u_l)$, the random vector $\boldsymbol{K} = (K_1, \ldots, K_l)$, and the constant vector $\mathbf{1}_l = (1, \ldots, 1) \in \mathbb{R}^l$ we can rewrite (1) as

$$Pr\{\boldsymbol{u} \leq \boldsymbol{K}\} \geq (1-\epsilon)\mathbf{1}_l \tag{2}$$

and the probabilistic constrained problem [PCP] can then be formulated as:

Parameters

n	number of retailers
l	number of periods
C	production capacity
f	fixed production setup cost
g	unitary production cost
h_i	unitary inventory holding cost at node i
L_i	inventory limit at node i
d_{it}	demand of retailer i in period t
c_{ij}	transportation cost over arc (i, j)
Q	capacity of each vehicle
K_t	number of vehicles available in period t
ϵ	nominal risk level

Decision variables

x_{ijt}	binary variable equal to 1 if node i is visited immediately before node j in period t
y_t	binary variable equal to 1 if a production set up occurs in period t
z_{it}	binary variable equal to 1 if retailer i is visited in period t
z_{0t}	number of vehicles that leave the plant in period t
u_t	number of routes to be performed in period t
p_t	quantity produced in period t
q_{it}	quantity delivered to retailer i in period t
I_{it}	quantity in inventory at node i at the end of period t

$$[PCP] = \min_{\substack{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{u},\\ \boldsymbol{p},\boldsymbol{q},\boldsymbol{I}}} \sum_{t\in\mathcal{T}} \left(f y_t + g p_t + \sum_{i\in\mathcal{N}} h_i I_{it} + \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ijt} \right) \tag{3}$$

$$\text{s.t.} \quad I_{0,t-1} + p_t = I_{0t} + \sum_{i\in\mathcal{N}_c} q_{it} \ , \forall t \in \mathcal{T} \tag{4}$$

$$I_{i,t-1} + q_{it} = I_{it} + d_{it} \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{5}$$

$$p_t \leq B' y_t \ , \forall t \in \mathcal{T} \tag{6}$$

$$I_{0t} \leq L_0 \ , \forall t \in \mathcal{T} \tag{7}$$

$$I_{i,t-1} + q_{it} \leq L_i \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{8}$$

$$q_{it} \leq B'' z_{it} \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{9}$$

$$\sum_{j\in\mathcal{N}} x_{ijt} = z_{it} \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{10}$$

$$\sum_{j\in\mathcal{N}} x_{ijt} + \sum_{j\in\mathcal{N}} x_{jit} = 2z_{it} \ , i \in \mathcal{N}, t \in \mathcal{T} \tag{11}$$

$$z_{it} \leq z_{0t} \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{12}$$

$$Q \sum_{i\in S}\sum_{j\in S} x_{ijt} \leq \sum_{i\in S} (Q z_{it} - q_{it}) \ , \forall S \subseteq \mathcal{N}_c, |S| \geq 2, t \in \mathcal{T} \tag{13}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijt} \leq |S| - 1 \ , \forall S \subseteq \mathcal{N}_c, |S| \geq 2, t \in \mathcal{T} \tag{14}$$

$$Pr\{\boldsymbol{u} \leq \boldsymbol{K}\} \geq (1-\epsilon)\mathbf{1}_l \tag{15}$$

$$z_{0t} = u_t \ , \forall t \in \mathcal{T} \tag{16}$$

$$x_{ijt} \in \{0,1\} \ , \forall (i,j) \in \mathcal{A}, t \in \mathcal{T} \tag{17}$$

$$y_t \in \{0,1\} \ , \forall t \in \mathcal{T} \tag{18}$$

$$z_{it} \in \{0,1\} \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{19}$$

$$z_{0t} \in \{0,\ldots,K_t\} \ , \forall t \in \mathcal{T} \tag{20}$$

$$u_t \in \{0,\ldots,K_t\} \ , \forall t \in \mathcal{T} \tag{21}$$

$$p_t \geq 0 \ , \forall t \in \mathcal{T} \tag{22}$$

$$q_{it} \geq 0 \ , \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{23}$$

$$I_{it} \geq 0 \ , \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{24}$$

where $B' = \min\{C, \sum_{t'=t}^{l} \sum_{i\in\mathcal{N}_c} d_{it'}\}$ and $B'' = \min\{L_i, \sum_{t'=t}^{l} d_{it'}, Q\}$.

The objective function (3) minimizes the sum of production, inventory and transportation costs. Constraints (4) and (5) are the inventory balance at the production facility and the retailers, respectively. The inequalities (6) are responsible to force a setup if there is production in period t and to limit the production to the minimum between the production capacity and the total demand in the remaining periods. The inventory limit at the production facility is imposed by constraints (7) and the quantity of products at the retailers cannot exceed their inventory capacity after the deliveries are made, as imposed with constraints (8). Constraints (9) limit the quantity of products that can be loaded in each vehicle, and a positive delivery to retailer i in period t is allowed only if this node is visited in that period. The latter constraints also impose the delivered quantity to node i to be the minimum between the inventory limit at the retailer, the remaining demand at the node, and the vehicle capacity. Moreover, each retailer can be visited at most one time per period (10) and the degree constraints are represented by (11). Inequalities (12) are used to strengthen the formulation, and they guarantee that if a retailer is visited in period t, at least a vehicle leaves the plant. Constraints (13) ensure the respect of vehicle capacity and also work as subtour elimination constraints (SECs). In order to speed up the solution process, another SEC was introduced (14), as it is stronger than (13). The probabilistic constraint (15) states that a sufficient number of vehicles needs to be available to perform the designed routes, in all periods, with a probability of $1-\epsilon$. Additionally, we ensure that the number of vehicles used in a period is equal to the number of designed routes (16). Constraints (17) - (24) impose the non-negativity and integrality requirements on the decision variables of the optimization model.

4.2 | Disaggregated Chance-Constraint

Alternatively, one can see the probabilistic constraint (15) in a disaggregated fashion, looking at periods separately, and producing a less conservative formulation. In this case, one needs to enforce that in each period we have enough vehicles to perform the schedule routes with a probability of $1-\epsilon$. This is guaranteed by substituting inequality (15) by

$$Pr\{u_t \leq K_t\} \geq 1-\epsilon\ , \forall t \in \mathcal{T} \tag{25}$$

The [PCP] that considers the period-based probabilistic constraints is called [PCP-D], and is defined by (3)-(14) and (16)-(25).

Due to the difficulty of calculating $Pr\{\boldsymbol{u} \leq \boldsymbol{K}\}$ and $Pr\{u_t \leq K_t\}$, both the [PCP] and [PCP-D] models need to be converted into manageable forms. This can be done by using information about the underlying distribution of the random parameter K_t, by means of a sampling method [8]. Due to the characteristics of the problem and of the probabilistic constraint, we decided to make use of the SAA proposed by [22], which is discussed in Section 5.

5 | SOLUTION PROCEDURE

In this section we present the procedures used to solve the proposed model. In Subsection 5.1, we introduce the SAA approach, which is applied in order to produce a solvable model through the use of Monte Carlo sampling. The resulting scenario based model is then solved by means of the BD and PBD methods, which are respectively described in Subsections 5.2 and 5.3.

5.1 | Sample Approximation Approach

The SAA aims at solving a sample approximation problem (SAP) using a Monte Carlo sampling to empirically approximate the general distribution of the random vector. Some probabilistic constraints do not need to be strictly enforced ("soft" constraints), which means that the assumed risk level α is positive. In these cases, the decision maker would like the constraints to be respected, but would compromise if the violated constraints lead to a sufficient decrease in the cost of the solution. This allows for more flexibility, once one is now able to choose from the efficient frontier between competing objectives.

The SAA method allows to tackle cases where α is positive and the uncertainty appears in the right-hand side of the constraints, which is the condition in constraints (15) and (25). Moreover, the methodology can be applied to both finite and continuous feasible regions and distributions, and also guarantees that one can obtain lower bounds (LB) to the optimal value of a general probabilistic constrained problem with high confidence, producing good quality feasible solutions [22].

To be able to approximate the original probabilistic constrained problem by solving a SAP, one needs to redefine the feasible region X_ϵ of the former model, where ϵ is the nominal risk parameter. Considering that, for a given deterministic feasible region X, $x \in X$ are the problem variables, ξ is the random vector, and $G(x,\xi)$ is the constraint mapping, the feasible region of the probabilistic constrained problem is given by

$$X_\epsilon = \{x \in X : Pr\{G(x,\xi) \leq \mathbf{0}\} \geq 1-\epsilon\} \tag{26}$$

In the case herein studied, X is the feasible region defined by constraints (4)-(14) and (16)-(24), ξ is the random vector $\boldsymbol{K}$, and $G(x,\xi) = G(\boldsymbol{u},\boldsymbol{K}) = \boldsymbol{u} - \boldsymbol{K}$.

Assuming that we make an independent Monte Carlo sampling of the random vector ξ, then, for a fixed risk level $\alpha \in [0,1)$, the feasible region of the equivalent SAP can be defined as

$$X_\alpha^N = \{x \in X : \frac{1}{N}\sum_{s=1}^{N} \mathbb{I}(g(x) \geq \xi^s) \geq 1-\alpha\} \tag{27}$$

where ξ^s is the random vector in scenario $s \in \{1,\dots,N\}$, $G(x,\xi^s) = \xi^s - g(x)$ is the constraint mapping under scenario s, and $\mathbb{I}(\cdot)$ is the indicator function, which takes value one when $\cdot$ is true and zero otherwise. Making use of the previous considerations, we assume in our case that the random vector in scenario s is given by $\boldsymbol{K}^s = (K_1^s,\dots,K_l^s)$, where K_t^s is the number of vehicles available in period $t \in \mathcal{T}$ under scenario s. Then, we can define the SAP formulation of the [PCP] model as follows:

$$[SAP] = \min_{\substack{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{u}\\ \boldsymbol{p},\boldsymbol{q},\boldsymbol{I}}} \quad \sum_{t\in\mathcal{T}}\left(f y_t + g p_t + \sum_{i\in\mathcal{N}} h_i I_{it} + \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ijt}\right) \tag{28}$$

$$\text{s.t.} \quad (4)-(14), (16)-(19), (22)-(24)$$

$$\frac{1}{N}\sum_{s=1}^{N} \mathbb{I}\left(\boldsymbol{K}^s \leq \boldsymbol{u}\right) \leq \alpha \mathbf{1}_l \tag{29}$$

$$z_{0t} \in \{0,\dots,\max_s K_t^s\}, \forall t \in \mathcal{T}, s \in \{1,\dots,N\} \tag{30}$$

$$u_t \in \{0,\dots,\max_s K_t^s\}, \forall t \in \mathcal{T}, s \in \{1,\dots,N\} \tag{31}$$

To solve the [SAP] as a mixed-integer linear program, constraint (29) still needs to be linearized. Hence, the indicator function $\mathbb{I}(\cdot)$ is replaced by the binary variable w^s, which equals 1 if, for a given scenario s, $\exists t \in \mathcal{T} : K_t^s < u_t$, and 0 otherwise. The constraint (29) then becomes

$$\frac{1}{N}\sum_{s=1}^{N} w^s \leq \alpha \tag{32}$$

$$w^s \in \{0,1\}, \forall s \in \{1,\ldots,N\} \tag{33}$$

To complete the linearization, the following constraints need to be added to the [SAP] model

$$u_t \leq K_t^s + Aw^s, \forall s \in \{1,\ldots,N\}, t \in \mathcal{T} \tag{34}$$

where A is a positive number large enough to make the model feasible whenever $w^s = 1$. Thus, the final [SAP] formulation for the [PCP] is given by (3)-(14), (16)-(19), (22)-(24) and (30)-(34).

Similarly, for the [PCP-D] model, we can define the feasible region of its equivalent SAP as

$$\overline{X}_\alpha^N = \{x \in X : \frac{1}{N}\sum_{s=1}^{N} \mathbb{I}(g(x) \geq \xi_t^s) \geq 1-\alpha, \forall t \in \mathcal{T}\} \tag{35}$$

where ξ_t^s is the component of the random vector associated with period t and scenario s, and $G(x,\xi_t^s) = \xi_t^s - g(x)$ is the constraint mapping under scenario s and period $t, \forall t \in \mathcal{T}, s \in \{1,\ldots,N\}$. Thus, to obtain the SAP version of the [PCP-D] model, we substitute constraint (29) by

$$\frac{1}{IN}\sum_{t=1}^{I}\sum_{s=1}^{N} \mathbb{I}\left(K_t^s \leq u_t\right) \leq \alpha \tag{36}$$

In order to linearize inequality (36), the indicator function $\mathbb{I}(\cdot)$ is replaced by the binary variable w_t^s, which equals 1 if $K_t^s < u_t, \forall t \in \mathcal{T}$, and 0 otherwise. The constraint (36) is then replaced by

$$\frac{1}{IN}\sum_{t=1}^{I}\sum_{s=1}^{N} w_t^s \leq \alpha \tag{37}$$

$$u_t \leq K_t^s + Aw_t^s, \forall t \in \mathcal{T}, s \in \{1,\ldots,N\} \tag{38}$$

$$w_t^s \in \{0,1\}, \forall t \in \mathcal{T}, s \in \{1,\ldots,N\} \tag{39}$$

The SAP version of the [PCP-D] model, here called [SAP-D], is defined by (3)-(14), (16)-(19), (22)-(24), (30)-(31) and (37)-(39).

5.2 | Benders Decomposition

The [SAP] and [SAP-D] formulations can be considered as NP-hard problems, one of the reasons for that being that they contain an exponentially increasing number of capacity constraints and SECs. Given the existence of a large number of decision variables, both discrete and continuous, decomposition methods are appropriate to handle this type of problem, as they allow for large-scale optimization models to be split into smaller parts based on a partition of the decision variables. Therefore, they offer an interesting approach for us to decompose the model in such a way to isolate the complicating factors, which makes the solution procedure easier.

Considering the aforementioned, we decided to apply the BD method to the proposed models, a decomposition approach that is able to handle their special structure, and makes use of BC to deal with the high number of constraints. Furthermore, this approach has been successfully applied to some other versions of the PRP, both deterministic and stochastic [see 15, 36, 4]. We highlight that standard commercial solvers could not be efficiently used to solve the models directly, as they usually require more computational memory and cannot take advantage of the problem structure at hand.

The BD approach is composed of three main phases, namely, projection, outer-linearization and relaxation [17]. In the projection phase, also known as partitioning, we project the model into the subspace defined by the so called complicating variables, those that, when temporarily fixed, make the original problem significantly less complex. The resulting projected formulation is dualized, and its corresponding extreme rays are used to define the feasibility requirements of the complicating variables (feasibility cuts). The dual problem is then outer-linearized, and its extreme points define the projected costs of the complicating variables (optimality cuts). In order to obtain the equivalent formulation to the original problem, we would need to enumerate all the feasibility and optimality cuts, which would largely increase the complexity of the model. In order to avoid this, we relax these inequalities, and add the feasibility and optimality cuts as needed. As a result of this procedure, we obtain a master problem (MP), which is defined by the complicating variables, their related constraints, and the necessary feasibility and optimality cuts. The subproblem (SP) is obtained in the projection phase, and it is defined by the remainder of the variables and constraints from the original problem. Both MP and SPs are iteratively solved until the optimal solution is obtained. In cases where the SP is infeasible, a feasibility cut is added to the MP, otherwise, optimality cuts are generated.

In order to perform the decomposition of both [SAP] and [SAP-D], one needs to define how to partition the decision variables between the MP and the SP. The integer and binary variables are included in the MP. Moreover, preliminary tests indicated that we also need to keep the variables q_{imt} in the MP, in order to allow us to eliminate existent subtours. This means that distribution decisions stay in the MP, while production quantities and inventory plans are decided in the SP.

Once we perform the decomposition of the final [SAP], we obtain the equivalent Benders decomposition to the original formulation, herein called [SAP|BD] model, which is composed of a MP ([SAP|BD|MP]) and a SP ([SAP|BD|SP]). The [SAP|BD|MP] can be formulated as

$$[SAP|BD|MP] = \min_{\substack{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{u}\\ \boldsymbol{q},\boldsymbol{w},\boldsymbol{\eta}}} \sum_{t\in\mathcal{T}} \left(f y_t + \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ijt} \right) + \eta \tag{40}$$

$$\text{s.t.} \quad (9)-(14),(16)-(19),(23),(30)-(34)$$

$$\sum_{i\in\mathcal{N}_c} \left(-\alpha_1^f q_{i1} + \beta_{i1}^f (q_{i1} - d_{i1} + I_{i0}) + \delta_{i1}^f (-q_{i1} + L_i - I_{i0}) + \sum_{t=2}^{l} \left(-\alpha_t^f q_{it} + \beta_{it}^f (q_{it} - d_{it}) + \delta_{it}^f (-q_{it} + L_i) \right) \right) + \sum_{t\in\mathcal{T}} \left(B' y_t \gamma_t^f + L_0 \theta_t^f \right) + I_{00} \alpha_1^f \geq 0 \,, \forall \left(\boldsymbol{\alpha^f}, \boldsymbol{\beta^f}, \boldsymbol{\gamma^f}, \boldsymbol{\theta^f}, \boldsymbol{\delta^f} \right) \in F_\Delta \tag{41}$$

$$\sum_{i\in\mathcal{N}_c} \left(-\alpha_1^e q_{i1} + \beta_{i1}^e (q_{i1} - d_{i1} + I_{i0}) + \delta_{i1}^e (-q_{i1} + L_i - I_{i0}) + \sum_{t=2}^{l} \left(-\alpha_t^e q_{it} + \beta_{it}^e (q_{it} - d_{it}) + \delta_{it}^e (-q_{it} + L_i) \right) \right) + \sum_{t\in\mathcal{T}} \left(B' y_t \gamma_t^e + L_0 \theta_t^e \right) + I_{00} \alpha_1^e + \eta \geq 0 \,, \forall \left(\boldsymbol{\alpha^e}, \boldsymbol{\beta^e}, \boldsymbol{\gamma^e}, \boldsymbol{\theta^e}, \boldsymbol{\delta^e} \right) \in E_\Delta \tag{42}$$

$$\eta \geq 0 \tag{43}$$

where Δ is the polyhedron defined by the constraints of the dual SP, F_Δ is the set of extreme rays of Δ, and E_Δ is the set of extreme points of Δ. The auxiliary variable η establishes a direct link between the SP and the MP, more specifically, it enables the expression of the costs associated with the SP for given solutions of the MP, allowing for the construction of the optimality cuts. Therefore, η characterizes a LB in the objective function of the MP provided by the SP, which prevents the unboundedness of the problem. Furthermore, $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\theta}$ and $\boldsymbol{\delta}$ are the vectors of dual variables of the SP, where $\boldsymbol{\alpha} = (\alpha_t \text{ free} \mid \forall t \in \mathcal{T}), \boldsymbol{\beta} = (\beta_{it} \text{ free} \mid \forall i \in \mathcal{N}_c, \forall t \in \mathcal{T}), \boldsymbol{\gamma} = (\gamma_t \geq 0 \mid \forall t \in \mathcal{T}), \boldsymbol{\theta} = (\theta_t \geq 0 \mid \forall t \in \mathcal{T})$ and $\boldsymbol{\delta} = (\delta_{it} \geq 0 \mid \forall i \in \mathcal{N}_c, \forall t \in \mathcal{T})$, which are used to define the feasibility (41) and optimality cuts (42).

Once the MP is solved, we let the variables $\overline{\boldsymbol{y}}$ and $\overline{\boldsymbol{q}}$ represent respectively the vectors of fixed variables y and q, which allow us to obtain the SP of the [SAP|BD] model, given by

$$[SAP|BD|SP] = \min_{\boldsymbol{p},\boldsymbol{I}} \quad \sum_{t\in\mathcal{T}} \left(g p_t + \sum_{i\in\mathcal{N}} h_i I_{it} \right) \tag{44}$$

$$\text{s.t.} \quad I_{0,t-1} + p_t = I_{0t} + \sum_{i\in\mathcal{N}_c} \overline{\boldsymbol{q}}_{\boldsymbol{it}} \,, \forall t \in \mathcal{T} \tag{45}$$

$$I_{i,t-1} + \overline{\boldsymbol{q}}_{\boldsymbol{it}} = I_{it} + d_{it} \,, \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{46}$$

$$p_t \leq B' \overline{\boldsymbol{y}}_{\boldsymbol{t}} \,, t \in \mathcal{T} \tag{47}$$

$$I_{0t} \leq L_0 \,, \forall t \in \mathcal{T} \tag{48}$$

$$I_{i,t-1} + \overline{\boldsymbol{q}}_{\boldsymbol{it}} \leq L_i \,, \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{49}$$

$$p_t \geq 0 \,, \forall t \in \mathcal{T} \tag{50}$$

$$I_{it} \geq 0 \,, \forall i \in \mathcal{N}, t \in \mathcal{T} \tag{51}$$

The dual version of the [SAP|BD|SP] is then defined as:

$$[SAP|BD|DSP] = \max_{\substack{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}\\ \boldsymbol{\theta},\boldsymbol{\delta}}} \sum_{i\in\mathcal{N}_c} \left(\alpha_1 \overline{\boldsymbol{q}}_{\boldsymbol{i1}} + \beta_{i1}\left(-\overline{\boldsymbol{q}}_{\boldsymbol{i1}} + d_{i1} - I_{i0}\right) + \delta_{i1}\left(\overline{\boldsymbol{q}}_{\boldsymbol{i1}} - L_i + I_{i0}\right)\right.$$
$$\left. + \sum_{t=2}^{l}\left(\alpha_t \overline{\boldsymbol{q}}_{\boldsymbol{it}} + \beta_{it}\left(-\overline{\boldsymbol{q}}_{\boldsymbol{it}} + d_{it}\right) + \delta_{it}\left(\overline{\boldsymbol{q}}_{\boldsymbol{it}} - L_i\right)\right)\right) - \sum_{t\in\mathcal{T}}\left(B'\overline{\boldsymbol{y}}_{\boldsymbol{t}}\gamma_t\right.$$
$$\left. + L_0\theta_t\right) - I_{00}\alpha_1 \tag{52}$$
$$\text{s.t. } (\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\theta},\boldsymbol{\delta}) \in \Delta \tag{53}$$

where the polyhedron Δ can be defined by:

$$\alpha_t - \gamma_t \leq g \;, \forall t \in \mathcal{T} \tag{54}$$
$$-\alpha_t + \alpha_{t+1} - \theta_t \leq h_0 \;, \forall t \in \mathcal{T} \tag{55}$$
$$-\beta_{it} + \beta_{i,t+1} - \delta_{i,t+1} \leq h_i \;, \forall i \in \mathcal{N}_c, t \in \mathcal{T} \tag{56}$$

A similar decomposition can be performed for the [SAP-D] formulation, just by replacing constraints (32)-(34) by (37)-(39). This gives us the [SAP-D|BD] model, and its corresponding MP, [SAP-D|BD|MP], and SP, [SAP-D|BD|SP].

5.3 | Acceleration Strategy

Although the traditional BD is a good alternative to decompose the problem under study, it results in a weak version of the MP. In fact, the MP loses all the important information related to the non-complicating variables that belong to the SP [30, 11, 12]. This tends to lead to poor quality solutions being found at the beginning of the solution process, and an arbitrarily large number of optimality cuts being added in order to converge to an optimal solution. Moreover, BD often suffers from slow running times [12].

To solve these issues, one can make use of PBD as an acceleration strategy, introducing in the MP explicit information from variables of the SP, thus improving the structure of the problem. This should allow for a reduction in the computational times when solving the problem, as showed by [11, 12].

One manner to do the aforementioned is to reformulate the original problem by introducing new redundant aggregated variables to the problem. Then, by manipulating some of the constraints, we generate valid inequalities, which are defined with respect to the new variables. After that, the traditional BD is applied. The resulting formulation is equivalent to the original one, and its optimal solution remains the same; the only significant change is in the complexity of the MP model, which contains a larger number of variables and constraints.

In our case, one can generate the following aggregated constraints, valid for both the [SAP] and the [SAP-D] models:

$$I_{0t} - I_{0,t-1} + I_{N_c t} - I_{N_c,t-1} + \sum_{i \in N_c} d_{it} \leq B' y_t \, , \forall t \in \mathcal{T} \tag{57}$$

$$I_{N_c t} - I_{N_c,t-1} + \sum_{i \in N_c} d_{it} \leq \sum_{i \in N_c} B'' z_{it} \, , \forall t \in \mathcal{T} \tag{58}$$

$$I_{N_c t} - I_{N_c,t-1} + \sum_{i \in N_c} d_{it} = \sum_{i \in N_c} q_{it} \, , \forall t \in \mathcal{T} \tag{59}$$

$$I_{0t} \geq 0 \, , \forall t \in \mathcal{T} \tag{60}$$

$$I_{N_c t} \geq 0 \, , \forall t \in \mathcal{T} \tag{61}$$

where I_{0t} and $I_{N_c t}$ are respectively the total inventory at the production plant and at the retailers at the end of period $t \in \mathcal{T}$.

Moreover, the new aggregated variables can be used to introduce a LB to the variable η, and consequently, to the objective function (40):

$$h_0 I_{0t} + \min_{i \in N_c} \{h_i\} I_{N_c t} \leq \eta \, , \forall t \in \mathcal{T} \tag{62}$$

Once we add the constraints (57)-(62) to the final [SAP] and perform its decomposition, we obtain its partial Benders reformulation [SAP|PBD], which is composed of a MP ([SAP|PBD|MP]) and of a SP ([SAP|PBD|SP]). The [SAP|PBD|MP] model can be defined as

$$[SAP|PBD|MP] = \min_{\substack{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z} \\ \boldsymbol{u},\boldsymbol{q},\boldsymbol{w} \\ \boldsymbol{\eta},\boldsymbol{I_0},\boldsymbol{I_{N_c}}}} \sum_{t \in \mathcal{T}} \left(f y_t + \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ijt} \right) + \eta \tag{63}$$

$$\text{s.t.} \quad (9) - (14), (16) - (19), (23), (30) - (34), (41) - (43), (57) - (62)$$

while the [SAP|PBD|SP] formulation is the same as the [SAP|BD|SP] model. The same procedure can be done for the [SAP-D] model, by substituting constraints (32)-(34) by (37)-(39). This gives us the [SAP-D|PBD] model, and its corresponding MP, [SAP-D|PBD|MP], and SP, [SAP-D|PBD|SP].

6 | COMPUTATIONAL EXPERIMENTS

In this section, we describe the main computational experiments performed, as well as the instances used and results obtained. Initially, we assess the two decomposition approaches, BD and PBD, to identify the most efficient one. This is done by solving the [SAP|BD] and the [SAP|PBD] formulations on a subset of instances. The best solution method is then used for the remaining experiments, which are conducted in order to analyze the characteristics of the solutions when we consider aggregated and disaggregated probabilistic constraints. Finally, we evaluate the accuracy of the finite approximations of the random distributions by the samples that are generated.

The algorithms are coded in C++ on Visual Studio 2017, and Cplex 20.1.0 is applied to solve the linear programs. The experiments are performed on Narval, one of the main supercomputers of the Digital Research Alliance of Canada

[13], and a time limit of 2 hours is imposed. It is important to point out that, in practice, it is not computationally interesting to add to the problem all the Benders cuts or the constraints (13)-(14) for all the existent subsets of retailers, as the model would be too large. Thus, we decided to implement the BBC algorithm, where a single BC tree is constructed and the Benders cuts, SECs, and capacity constraints are added as needed as the tree is explored. In this case, we identify the existence of subtours using the Lysgaard's separation procedure, which uses rounded capacity inequalities [23]. The latter is a well known and used package that makes use of multiple heuristics to detect violated inequalities, improving the LBs and the speed of the solution process. In the cases where subtours are identified, the corresponding inequalities (13)-(14) are added to the MP in order to exclude them from the upcoming solutions. Constraint (13) is also added in the case where real tours are found to guarantee that they respect the vehicle capacity.

In order to further improve the solution process, we set higher branching priorities to the variables w^s and u_t, as they are integer and directly related to the stochastic parameter and the probabilistic constraint. Preliminary tests showed that, in general, we can achieve better solutions if these variables are decided first. Moreover, the experiments were performed 10 times for each instance, using different scenarios, under seven different risk levels α, varying from 0.50 to 0.00.

6.1 | Details of the Instances

As this is the first time that the PRP with uncertain number of vehicles is studied, there are no benchmark instances readily available to conduct the computational experiments. Thus, we generate a set of instances from those used by [2] in the case of deterministic multi-vehicle PRP (MVPRP). Originally, the authors presented four classes of instances: standard, higher unit production costs, higher transportation costs, and no retailer inventory costs. For the purposes of this paper, we use the standard instances, changing only the number of vehicles available, which is our stochastic parameter. We consider that the number of vehicles follows a discrete triangular distribution of probabilities with median value MED, and it can assume a value in the set {MED-2, MED-1, MED, MED+1, MED+2}, with probability (0.1, 0.2, 0.4, 0.2, 0.1), respectively. For most cases, MED is chosen as the number of vehicles of the deterministic benchmark. For cases where preliminary tests showed consistent infeasibilities, we increased this median by one; the only exception to this are the benchmark instances with 2 vehicles, in which MED is always maintained as 2, allowing us to analyze situations in which no vehicles are available.

The stochastic parameters $K_t, \forall t \in \mathcal{T}$ are assumed to be independent from one another. In order to obtain a finite approximation of the random distributions, we generate sets of scenarios by means of a Monte Carlo sampling approach. For each instance, we sample 30, 50 and 100 scenarios.

To represent instance sizes, we use the notation C*a*_P*b*_MED*c* where *a*, *b*, and *c* are the number of retailers, periods, and median of vehicles available, respectively. The instances consider 10 to 35 retailers, 3 to 9 periods, and median of vehicles from 2 to 4, giving us 22 different types of instances. A summary of these stochastic instances can be found in the supplementary material.

6.2 | Performance of the Reformulations

The initial experiments aim at comparing the performance of the BD and PBD solution approaches, and are performed on the [SAP|BD] and [SAP|PBD] models using a subset of the generated instances, 5 out of the 22 available. The chosen instances are those with 10 and 15 retailers and MED = 3. For each instance, we evaluated the value of the objective function ($\mathcal{Z}$), the computational time in seconds (CT), the number of feasibility (Feas) and optimality (Opt) cuts generated, the relative optimality gap (Gap) and the relative difference with respect to the deterministic benchmark (RDD) [3].

In comparison to the standard BD, the use of the PBD allows for a significant improvement in the value of the optimality gaps, on average 50%. This behaviour is even more pronounced in the larger instances, where we can see up to 72% of gap reduction in some cases. The running times are also substantially lower using PBD, mainly in the smaller instances. For those cases not solved to optimality, namely the larger instances, one can also obtain smaller optimality gaps. Furthermore, the PBD approach allows for a major reduction in the number of feasibility cuts generated, on average approximately 18% less when compared to the BD. Results also show that the number of optimality cuts does not present a consistent behaviour, however, on average, [SAP|PBD] produces less optimality cuts than [SAP|BD] as α decreases. In general, the enhanced formulation of the MP, obtained through the application of the PBD, appears to result in a more targeted overall search process focused on high-quality solutions. Thus, in turn, diminishes the necessity to generate optimality cuts to steer the algorithm towards the optimum.

It is possible to see that, in general, the PBD method overperforms the classical BD, thus, we decided to adopt the former in the remaining of this study.

6.3 | Assessment of the Impacts of the Probabilistic Constraints

Given the better performance of the PBD over the standard BD, the [SAP|PBD] and [SAP-D|PBD] reformulations are used to perform the computational experiments using all the 22 generated instances. The results for the [SAP|PBD] model under high ($\alpha \geq 0.10$) and small ($\alpha < 0.10$) risk levels are presented in Tables 1 and 2, respectively, while those for the [SAP-D|PBD] model under high ($\alpha \geq 0.10$) and small ($\alpha < 0.10$) risk levels are presented in Tables 3 and 4, respectively. All the tables display the average values of Z, CT, optimality gaps and RDD. The value of each cell is obtained by calculating the average of all experiments for a given instance, disconsidering the cases of proven infeasibility or where we could not find a feasible solution within the time limit.

From the results of the [SAP|PBD] reported in Tables 1 and 2, one can see that reasonable gaps are produced, mainly for the smaller instances; however, they tend to increase if more retailers and/or periods are considered. Problems become specially difficult to be solved for 9 periods, even those with a small number of customers.

For the instances with 3 periods, we are able to either find a feasible solution or prove infeasibility for all the instances. On average, 37% of the experiments produce optimal solutions, 42% produce feasible solutions, while 21% are infeasible problems. The latter are cases that consider a median of 2 vehicles, which means that sometimes there are no vehicles to perform the distribution in a given period. This leads to a quick increase on the LHS of constraint (32), which is not respected for smaller risk levels, mostly for $\alpha \leq 0.15$. In the cases considering 6 periods, we are able to obtain a smaller percentage of optimal solutions, on average 18% of the experiments, while the average of proven infeasible problems increase to 35%. Similarly to the cases of 3 periods, the latter are experiments that consider a median of 2 vehicles.

TABLE 1 Results for the [SAP|PBD] reformulation, $\alpha \geq 0.10$.

Instance	$\alpha = 0.50$				$\alpha = 0.30$			
	Z	CT	Gap	RDD	Z	CT	Gap	RDD
C10_P3_MED2	13924	0.75	0.0	0.0	13927	0.87	0.0	0.0
C10_P3_MED3	14002	0.95	0.0	0.0	14002	0.88	0.0	0.0
C15_P3_MED2	19488	53.86	0.0	0.0	19499	34.02	0.0	0.1
C15_P3_MED3	19680	208.40	0.0	0.0	19680	234.8	0.0	0.0
C20_P3_MED2	22209	87.98	0.0	0.0	22217	63.14	0.0	0.0
C20_P3_MED3	22375	572.84	0.0	0.0	22375	523.52	0.0	0.0
C25_P3_MED2	26055	921.79	0.0	0.0	26065	627.31	0.0	0.0
C25_P3_MED3	26322	7090.67	0.9	0.2	26295	6422.38	0.6	0.1
C30_P3_MED3	29054	7200.00	2.0	0.6	29116	7200.00	2.3	0.8
C30_P3_MED4	29712	7200.00	5.0	1.9	29698	7200.00	5.0	1.9
C35_P3_MED3	36069	7200.00	4.4	1.6	36023	7200.00	4.1	1.5
C35_P3_MED4	36957	7200.00	17.5	3.1	36983	7200.00	11.3	3.1
C10_P6_MED2	29559	60.46	0.0	0.0	29681	43.82	0.0	0.4
C10_P6_MED3	29897	183.79	0.0	0.0	29897	186.30	0.0	0.0
C15_P6_MED2	41643	6805.74	0.5	0.0	42480	4715.06	0.3	2.0
C15_P6_MED3	42523	7200.00	4.8	0.8	42458	7200.00	2.5	0.6
C20_P6_MED2	47606	7200.00	1.9	0.3	48086	6675.85	0.9	1.3
C20_P6_MED3	48128	7200.00	3.2	0.9	48109	7200.00	3.1	0.9
C25_P6_MED2	57252	7200.00	2.9	0.7	58372	7200.00	2.6	2.6
C25_P6_MED4	58017	7200.00	4.8	1.6	58139	7200.00	5.1	1.9
C10_P9_MED2	51363	6880.50	1.3	0.9	55952	3804.25	0.3	9.9
C10_P9_MED3	51979	7200.00	2.7	0.5	52005	7200.00	2.5	0.5
	$\alpha = 0.15$				$\alpha = 0.10$			
	Z	CT	Gap	RDD	Z	CT	Gap	RDD
C10_P3_MED2	13966	0.58	0.0	0.3	13924	0.37	0.0	0.0
C10_P3_MED3	14002	0.97	0.0	0.0	14002	0.96	0.0	0.0
C15_P3_MED2	19614	33.66	0.0	0.6	19504	4.32	0.0	0.1
C15_P3_MED3	19680	241.32	0.0	0.0	19684	249.48	0.0	0.0
C20_P3_MED2	22323	50.56	0.0	0.5	22209	36.06	0.0	0.0
C20_P3_MED3	22387	742.44	0.0	0.1	22401	1159.58	0.0	0.1
C25_P3_MED2	26203	107.06	0.0	0.6	26055	74.99	0.0	0.0
C25_P3_MED3	26306	6740.04	0.7	0.1	26304	6868.45	0.7	0.1
C30_P3_MED3	29098	7200.00	2.4	0.8	29048	7200.00	2.0	0.6
C30_P3_MED4	29709	7200.00	5.2	1.9	29771	7200.00	5.6	2.1
C35_P3_MED3	36065	7200.00	4.3	1.6	36111	7200.00	4.5	1.7
C35_P3_MED4	36893	7200.00	19.4	2.9	36931	7200.00	8.9	3.0
C10_P6_MED2	30275	25.94	0.0	2.4	30267	10.51	0.0	2.4
C10_P6_MED3	29907	201.72	0.0	0.0	29920	117.79	0.0	0.1
C15_P6_MED2	41627	1912.47	0.0	0.0	-	-	-	-
C15_P6_MED3	42635	7200.00	2.6	1.1	42792	7200.00	2.6	1.4
C20_P6_MED2	49380	5148.84	0.3	4.0	49657	3856.94	0.1	4.6
C20_P6_MED3	48248	7200.00	3.1	1.2	48296	7200.00	3.2	1.3
C25_P6_MED2	57063	7200.00	2.2	0.3	-	-	-	-
C25_P6_MED4	58016	7200.00	4.4	1.6	57913	7200.00	4.2	1.5
C10_P9_MED2	-	-	-	-	-	-	-	-
C10_P9_MED3	53749	7200.00	4.2	3.9	54223	7200.00	4.1	4.8

For the cases without data, all the solutions are proven infeasible.

TABLE 2 Results for the [SAP|PBD] Reformulation, $\alpha < 0.10$.

	$\alpha = 0.05$				$\alpha = 0.01$				$\alpha = 0.00$			
Instance	Z	CT	Gap	RDD	Z	CT	Gap	RDD	Z	CT	Gap	RDD
C10_P3_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C10_P3_MED3	14002	0.84	0.0	0.0	14002	0.77	0.0	0.0	14002	0.68	0.0	0.0
C15_P3_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C15_P3_MED3	19688	189.86	0.0	0.0	19689	304.91	0.0	0.0	19689	243.37	0.0	0.0
C20_P3_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C20_P3_MED3	22445	2666.64	0.0	0.3	22460	5192.44	0.1	0.4	22460	6580.50	0.1	0.4
C25_P3_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C25_P3_MED3	26317	6475.41	0.9	0.2	26316	7145.56	1.0	0.2	26330	7200.00	1.1	0.2
C30_P3_MED3	29019	7200.00	1.8	0.5	29043	7200.00	2.2	0.6	29047	7200.00	2.3	0.6
C30_P3_MED4	29746	7200.00	5.4	2.0	29859	7200.00	5.8	2.4	29846	7200.00	5.9	2.4
C35_P3_MED3	35977	7200.00	4.4	1.3	36047	7200.00	4.7	1.5	36041	7200.00	4.6	1.5
C35_P3_MED4	36892	7200.00	15.4	2.9	36795	7200.00	6.3	2.6	36717	7200.00	6.0	2.4
C10_P6_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C10_P6_MED3	29936	94.75	0.0	0.1	29937	98.74	0.0	0.1	29937	111.83	0.0	0.1
C15_P6_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C15_P6_MED3	44323	7200.00	2.4	5.1	45192	7200.00	2.9	7.1	45259	7200.00	3.2	7.3
C20_P6_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C20_P6_MED3	48552	7200.00	3.5	1.8	48738	7200.00	4.8	2.2	48911	7200.00	5.8	2.6
C25_P6_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C25_P6_MED4	58005	7200.00	4.4	1.6	58372	7200.00	5.9	2.3	58591	7200.00	7.0	2.7
C10_P9_MED2	-	-	-	-	-	-	-	-	-	-	-	-
C10_P9_MED3	54749	7200.00	3.7	5.8	54840	7200.00	3.5	6.0	54873	7200.00	3.6	6.1

For the cases without data, all the solutions are proven infeasible

TABLE 3 Results for the [SAP-D|PBD] reformulation, $\alpha \geq 0.10$.

	$\alpha = 0.50$				$\alpha = 0.30$			
Instance	Z	CT	Gap	RDD	Z	CT	Gap	RDD
C10_P3_MED2	13924	0.63	0.0	0.0	13924	0.64	0.0	0.0
C10_P3_MED3	14002	0.68	0.0	0.0	14002	0.96	0.0	0.0
C15_P3_MED2	19488	36.94	0.0	0.0	19488	35.95	0.0	0.0
C15_P3_MED3	19680	171.78	0.0	0.0	19680	220.20	0.0	0.0
C20_P3_MED2	22209	91.77	0.0	0.0	22209	63.13	0.0	0.0
C20_P3_MED3	22375	808.72	0.0	0.0	22375	741.33	0.0	0.0
C25_P3_MED2	26055	823.52	0.0	0.0	26055	879.33	0.0	0.0
C25_P3_MED3	26298	6718.42	0.6	0.1	26299	6691.73	0.7	0.1
C30_P3_MED3	29085	7200.00	2.3	0.7	29081	7200.00	2.2	0.7
C30_P3_MED4	29779	7200.00	5.4	2.1	29712	7200.00	5.0	1.9
C35_P3_MED3	36016	7200.00	4.0	1.4	36079	7200.00	4.4	1.6
C35_P3_MED4	36907	7200.00	17.2	2.9	36879	7200.00	11.0	2.9
C10_P6_MED2	29559	17.93	0.0	0.0	29559	18.13	0.0	0.0
C10_P6_MED3	29897	139.31	0.0	0.0	29897	137.45	0.0	0.0
C15_P6_MED2	41655	7003.06	0.7	0.1	41651	7200.00	0.7	0.1
C15_P6_MED3	42350	7200.00	2.2	0.4	42369	7200.00	2.3	0.4
C20_P6_MED2	47651	7200.00	2.0	0.4	47658	7200.00	2.0	0.4
C20_P6_MED3	48104	7200.00	2.8	0.9	48076	7200.00	2.8	0.8

TABLE 3 Continued.

Instance	α = 0.50				α = 0.30			
	Z	CT	Gap	RDD	*Z*	CT	Gap	RDD
C25_P6_MED2	57274	7200.00	3.3	0.7	57207	7200.00	3.1	0.6
C25_P6_MED4	57950	7200.00	4.2	1.5	57927	7200.00	4.2	1.5
C10_P9_MED2	51047	6753.42	1.1	0.2	51028	6695.20	1.1	0.2
C10_P9_MED3	52018	7200.00	2.8	0.5	52000	7200.00	2.7	0.5
	α = 0.15				α = 0.10			
	Z	CT	Gap	RDD	*Z*	CT	Gap	RDD
C10_P3_MED2	13924	0.75	0.0	0.0	13930	0.91	0.0	0.0
C10_P3_MED3	14002	0.89	0.0	0.0	14002	1.03	0.0	0.0
C15_P3_MED2	19489	49.10	0.0	0.0	19507	43.80	0.0	0.1
C15_P3_MED3	19680	186.46	0.0	0.0	19680	269.31	0.0	0.0
C20_P3_MED2	22209	70.75	0.0	0.0	22225	54.00	0.0	0.1
C20_P3_MED3	22375	625.23	0.0	0.0	22375	778.17	0.0	0.0
C25_P3_MED2	26055	822.52	0.0	0.0	26076	707.66	0.0	0.1
C25_P3_MED3	26303	7103.59	0.7	0.1	26285	6670.25	0.5	0.1
C30_P3_MED3	29060	7200.00	2.0	0.6	29056	7200.00	2.2	0.6
C30_P3_MED4	29742	7200.00	5.3	2.0	29722	7200.00	7.0	1.9
C35_P3_MED3	36091	7200.00	8.8	1.7	36060	7200.00	6.4	1.6
C35_P3_MED4	37048	7200.00	13.8	3.3	37029	7200.00	22.2	3.3
C10_P6_MED2	29559	29.67	0.0	0.0	29559	22.80	0.0	0.0
C10_P6_MED3	29897	178.26	0.0	0.0	29897	166.50	0.0	0.0
C15_P6_MED2	41654	7124.21	0.7	0.1	41637	6524.70	0.5	0.0
C15_P6_MED3	42359	7200.00	2.3	0.4	42367	7200.00	2.3	0.4
C20_P6_MED2	47579	7200.00	1.9	0.2	47606	7200.00	1.8	0.3
C20_P6_MED3	48168	7200.00	3.2	1.0	48062	7200.00	3.0	0.8
C25_P6_MED2	57273	7200.00	3.1	0.7	57221	7200.00	3.1	0.6
C25_P6_MED4	57925	7200.00	4.1	1.5	57992	7200.00	4.6	1.6
C10_P9_MED2	51022	6406.21	1.0	0.2	51081	6306.78	1.0	0.3
C10_P9_MED3	51991	7200.00	2.7	0.5	52032	7200.00	2.8	0.6

For the cases without data, all the solutions are proven infeasible.

Results show that, for the same risk level, problems with a larger number of periods are generally more difficult to solve, leading to larger optimality gaps. Similarly, problems with smaller risk levels are more difficult, leading to more proven infeasibilities. For the instances with 3 periods, when a feasible solution is found, the risk level does not seem to have significant impact on the objective function or on the optimality gap, which means that one can take the advantage of solving the problem with a significantly small α. For instances with 6 and 9 periods, on the other hand, small values of α are usually associated with higher values of Z and RDD. In these cases, it is more difficult to enforce the tighter chance constraints over all the periods in a scenario, which leads to an increase in value of the objective function.

The results of [SAP-D|PBD] reformulation reported in Tables 3 and 4 are similar to those of the [SAP|PBD]. A major difference, however, is that with the [SAP-D|PBD] model we are able to obtain solutions for smaller risk levels in the instances with MED = 2. This occurs because the disaggregated chance constraints are easier to be enforced, which allows one to impose lower risk levels and still obtain feasible or optimal solutions. As a consequence, there is a reduction in the percentage of proven infeasible problems and an increase in the number of optimal solutions, which represent 13% and 46% of the experiments for instances with 3 periods, and 15% and 24% for those with 6 periods, respectively.

When solving the [SAP-D|PBD] model, the use of the less restrictive chance constraints also allows for a reduction on the value of the objective function, followed by a significant reduction of the RDD (on average 40%). This behavior is even more evident on the instances with 9 periods, which are more difficult to solve. These results show that a disaggregated chance constraint has a tendency to produce solutions closer to the deterministic ones, in which we

TABLE 4 Results for the [SAP-D|PBD] reformulation, $\alpha < 0.10$.

	$\alpha = 0.05$				$\alpha = 0.01$				$\alpha = 0.00$			
Instance	Z	CT	Gap	RDD	Z	CT	Gap	RDD	Z	CT	Gap	RDD
C10_P3_MED2	13959	0.54	0.0	0.2	-	-	-	-	-	-	-	-
C10_P3_MED3	14002	1.15	0.0	0.0	14002	0.83	0.0	0.0	14002	0.71	0.0	0.0
C15_P3_MED2	19594	29.50	0.0	0.5	-	-	-	-	-	-	-	-
C15_P3_MED3	19680	257.59	0.0	0.0	19689	213.47	0.0	0.0	19689	219.33	0.0	0.0
C20_P3_MED2	22303	48.68	0.0	0.4	-	-	-	-	-	-	-	-
C20_P3_MED3	22387	655.18	0.0	0.1	22454	3043.05	0.0	0.4	22460	6578.72	0.2	0.4
C25_P3_MED2	26177	100.13	0.0	0.5	-	-	-	-	-	-	-	-
C25_P3_MED3	26301	7048.43	0.6	0.1	26341	6867.83	0.9	0.3	26391	7200.00	1.6	0.5
C30_P3_MED3	29072	7200.00	2.0	0.7	29059	7200.00	2.1	0.6	29028	7200.00	2.1	0.5
C30_P3_MED4	29784	7200.00	5.4	2.1	29787	7200.00	5.5	2.2	29705	7200.00	5.5	1.9
C35_P3_MED3	36071	7200.00	4.4	1.6	36067	7200.00	9.3	1.6	35903	7200.00	4.2	1.1
C35_P3_MED4	36971	7200.00	11.4	3.1	36982	7200.00	24.2	3.1	36758	7200.00	6.2	2.5
C10_P6_MED2	29880	27.69	0.0	1.1	-	-	-	-	-	-	-	-
C10_P6_MED3	29897	141.82	0.0	0.0	29934	95.06	0.0	0.1	29937	120.16	0.0	0.1
C15_P6_MED2	43775	4670.52	0.1	5.2	-	-	-	-	-	-	-	-
C15_P6_MED3	42401	7200.00	2.4	0.5	43980	7200.00	2.5	4.2	45245	7200.00	3.2	7.2
C20_P6_MED2	48404	6015.11	0.5	2.0	-	-	-	-	-	-	-	-
C20_P6_MED3	48234	7200.00	3.5	1.2	48459	7200.00	3.3	1.6	48989	7200.00	5.2	2.7
C25_P6_MED2	60636	7200.00	4.4	6.6	-	-	-	-	-	-	-	-
C25_P6_MED4	58128	7200.00	4.9	1.8	57919	7200.00	4.4	1.5	58556	7200.00	7.0	2.6
C10_P9_MED2	53791	5392.16	0.4	5.6	-	-	-	-	-	-	-	-
C10_P9_MED3	52033	7200.00	2.8	0.6	54672	7200.00	4.6	5.7	55002	7200.00	4.2	6.3

For the cases without data, all the solutions are proven infeasible

always have enough vehicles to serve the schedule routes. Thus, it might be interesting option for those decision makers who are less conservative, and prioritize a modelling approach that is able to produce solutions with smaller costs while still mitigating the risks associated with the overall production and routing plans becoming infeasible due to a shortage of available vehicles.

6.4 | Sample Analysis

As a Monte Carlo sampling was made to approximate the real distributions of the random parameters, it is essential to evaluate the stability and validity of the samples used in Section 6.3. We chose two instances with small optimality gaps to perform this assessment, cases that allow us to obtain valuable conclusions (C25_P3_MED3 and C30_P3_MED3), and we performed the analysis on both the [SAP|PBD] and the [SAP-D|PBD] formulations.

Initially, the coefficient of variation (CV) with respect to the values of the objective function is calculated for each instance, considering the corresponding samples used in the computational experiments. The results for the [SAP|PBD] and the [SAP-D|PBD] models are presented respectively on Tables 5 and 6. We present the data in percentage, and stratify them by sample size (N) - 30, 50 and 100 scenarios.

TABLE 5 Coefficient of variation for the [SAP|PBD] reformulation.

		α						
Instance	N	0.50	0.30	0.15	0.10	0.05	0.01	0.00
C25_P3_MED3	30	0.3	0.2	0.2	0.2	0.2	0.2	0.2
	50	0.2	0.1	0.2	0.2	0.1	0.0	0.0
	100	0.2	0.1	0.1	0.1	0.1	0.1	0.0
C30_P3_MED3	30	0.3	0.6	0.6	0.4	0.3	0.5	0.4
	50	0.5	0.3	0.5	0.3	0.4	0.0	0.0
	100	0.3	0.6	0.7	0.5	0.3	0.3	0.0

All the results are given in percentage.

TABLE 6 Coefficient of variation for the [SAP-D|PBD] reformulation.

		α						
Instance	N	0.50	0.30	0.15	0.10	0.05	0.01	0.00
C25_P3_MED3	30	0.1	0.3	0.1	0.1	0.2	0.3	0.3
	50	0.1	0.1	0.1	0.1	0.2	0.1	0.0
	100	0.0	0.1	0.2	0.1	0.1	0.1	0.0
C30_P3_MED3	30	0.3	0.3	0.4	0.6	0.7	0.1	0.1
	50	0.5	0.3	0.4	0.7	0.4	0.4	0.2
	100	0.3	0.3	0.4	0.2	0.3	0.4	0.0

All the results are given in percentage.

It is possible to see that the CV is quite small for all the cases, which demonstrates stability of the results obtained for a given sample size and risk level. For each instance, if we consider the results for a given risk level, we notice that the CV does not change a lot across the different sample sizes. This means that the obtained values of the objective function were reasonably close, and do not depend significantly on the specific scenario set used.

One can also observe that, in general, smaller risk levels are associated with lower CVs. This behaviour is expected, because stricter problems have smaller feasible regions, and, as a consequence, the feasible and optimal solutions are more likely to be closer in value, which leads to smaller CVs.

Furthermore, we evaluated the validity of the samples used, to ensure that with a relatively small sample size one can obtain results that can reflect well the real distribution of probabilities of the random parameter. We sampled 5000 scenarios and verified, for each of the two instances, in how many cases the probabilistic constraint was not enforced, considering the results for each sample previously used and each risk level. The average percentage results for each sample size N and risk level α are presented in Tables 7 and 8, respectively for the [SAP|PBD] and the [SAP-D|PBD] models.

TABLE 7 Percentage of non-respected chance constraints for the [SAP|PBD] reformulation.

		α						
Instance	N	0.50	0.30	0.15	0.10	0.05	0.01	0.00
C25_P3_MED3	30	0.0	1.0	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	1.0	0.0	1.0	0.0	0.0
	100	1.0	0.0	1.0	0.0	0.0	0.0	0.0
C30_P3_MED3	30	0.0	0.0	0.0	1.0	0.0	0.0	0.0
	50	0.0	0.0	1.0	0.0	0.0	0.0	0.0
	100	0.0	1.0	1.0	0.0	0.0	0.0	0.0

All the results are given in percentage.

TABLE 8 Percentage of non-respected chance constraints for the [SAP-D|PBD] reformulation.

		α						
Instance	N	0.50	0.30	0.15	0.10	0.05	0.01	0.00
C25_P3_MED3	30	0.3	0.0	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.0	0.3	0.3	0.7	0.0	0.0
	100	0.0	0.0	0.3	0.3	0.3	0.0	0.0
C30_P3_MED3	30	0.0	0.3	0.0	0.0	0.0	0.0	0.0
	50	0.0	0.3	0.0	0.0	0.0	0.0	0.0
	100	0.3	0.3	0.0	0.0	0.0	0.0	0.0

All the results are given in percentage.

Results show that, on average, the probabilistic constraint is enforced in all the cases, for all the risk levels. This means that the sample sizes are large enough to represent the behaviour of the distributions of probabilities of the random parameters and their effects on the considered problem.

7 | CONCLUSION

In this paper, we proposed a novel stochastic optimization method, that involves the use of chance constraints, to solve the PRP when explicitly considering the uncertainty related to the size of the available fleet of vehicles. Specifically, we propose two variants of the chance constraints (an aggregated and disaggregated version) and solve the resulting models using the SAA approach. Two variants of the BD approach are also developed, i.e., the classical BD and PBD strategies, to solve the scenario-defined models produced by the SAA approach. It was observed that the PBD strategy clearly outperforms the classical variant. Specifically, the computational results showed that the PBD approach was able to produce reasonable optimality gaps for most instances. As expected, as the number of periods and retailers increased, so did the complexity of the instances to solve. However, it was observed that this increase of complexity did not seem to depend significantly on the risk level considered.

The results obtained using the model with disaggregated chance-constraints illustrated how it defined a more flexible modelling approach, when compared with its aggregated counterpart. The use of disaggregated chance-constraints led to more instances being solved to optimality, while also providing higher guarantees in terms of proven feasibility. Additionally, the use of this approach enabled solutions with lower costs to be obtained (especially on the more challenging larger sized instances involving 9 periods). Therefore, these results indicated that the use of disaggregated chance-constraints might be more interesting for those decision makers who are less conservative, i.e., who can benefit from the use of a stochastic optimization approach (to limit risk) while also reducing the costs of the obtained solutions. Lastly, the results obtained also demonstrated stability across different sample sizes, while remaining valid when assessed using much larger samples (providing higher-levels of confidence with the conclusions drawn).

Future work should focus on the development of faster solution methods, or computational enhancements, to solve the studied problem for larger instances. The adoption of heuristics and metaheuristics procedures is strongly advised. The consideration of other complicating aspects such as and multiple products should also be considered.

ACKNOWLEDGMENTS

The computational experiments were enabled by support provided by Calcul Québec (https://www.calculquebec.ca) and and the Digital Research Alliance of Canada (alliancecan.ca).

References

[1] N. Absi, C. Archetti, S. Dauzère-Pérès, and D. Feillet, *A two-phase iterative heuristic approach for the production routing problem*, Transp. Sci. **49** (2015), 784–795.

[2] Y. Adulyasak, J.F. Cordeau, and R. Jans, *Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems*, INFORMS J. Comput. **26** (2014), 103–120.

[3] Y. Adulyasak, J.F. Cordeau, and R. Jans, *Optimization-based adaptive large neighborhood search for the production routing problem*, Transp. Sci. **48** (2014), 20–45.

[4] Y. Adulyasak, J.F. Cordeau, and R. Jans, *Benders Decomposition for Production Routing Under Demand Uncertainty*, Oper. Res. **63** (2015), 851–867.

[5] Y. Adulyasak, J.F. Cordeau, and R. Jans, *The production routing problem: A review of formulations and solution algorithms*, Comput. Oper. Res. **55** (2015), 141–152.

[6] A. Agra, C. Requejo, and F. Rodrigues, *An adjustable sample average approximation algorithm for the stochastic production-inventory-routing problem*, Networks **72** (2018), 5–24.

[7] M. Avci and S.T. Yildiz, *A matheuristic solution approach for the production routing problem with visit spacing policy*, Eur. J. Oper. Res. **279** (2019), 572–588.

[8] J.R. Birge and F. Louveaux, *Introduction to Stochastic Programming*, Springer, New York, 2 edition 2011.

[9] P. Chandra, *A Dynamic Distribution Model with Warehouse and Customer Replenishment Requirements*, J. Oper. Res. Soc. **44** (1993), 681–692.

[10] Z.l. Chen, "*Integrated Production and Distribution Operations: Taxonomy, Models, and Review*," *Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era*, Z. Simchi-Levi, D.; Wu, S.D.; Shen (ed.) 1 edition, Springer US, 2004, pp. 711–745.

[11] G.T. Crainic, M. Hewitt, F. Maggioni, and W. Rei, *Partial Decomposition Strategies for Two-Stage Stochastic Integer Programs*, CIRRELT, Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le tran **37** (2016), 1–38.

[12] T.G. Crainic, M. Hewitt, F. Maggioni, and W. Rei, *Partial benders decomposition: General methodology and application to stochastic network design*, Transp. Sci. **55** (2021), 414–435.

[13] Digital Research Alliance of Canada, 2023. *The Alliance*. Accessed November 04, 2023.

[14] EGC, *Briefing paper on capacity crisis*, Tech. report, Brussels, 2022.

[15] P. Farghadani-Chaharsooghi, P. Kamranfar, M.S. Mirzapour Al-e-Hashem, and Y. Rekik, *A joint production-workforce-delivery stochastic planning problem for perishable items*, Int. J. Production Res. **60** (2022), 6148–6172.

[16] B.S. Fugate, B. Davis-Sramek, and T.J. Goldsby, *Operational collaboration between shippers and carriers in the transportation industry*, Int. J. Logist. Manage. **20** (2009), 425–447.

[17] A.M. Geoffrion, *Elements of large-scale mathematical programming*, Tech. report, The RAND Corporation, 1969.

[18] M.K.K. Habibi, O. Battaïa, V.D. Cung, A. Dolgui, and M.K. Tiwari, *Sample average approximation for multi-vehicle collection–disassembly problem under uncertainty*, Int. J. Production Res. **57** (2019), 2409–2428.

[19] S.F. Ji, X.S. Peng, and R.J. Luo, *An integrated model for the production-inventory-distribution problem in the Physical Internet*, Int. J. Production Res. **57** (2019), 1000–1017.

[20] P. Kent and H. Haralambides, *A perfect storm or an imperfect supply chain? The U.S. supply chain crisis*, Maritime Econ. Logist. **24** (2022), 1–8.

[21] S. Küçükyavuz and R. Jiang, *Chance-constrained optimization under limited distributional information: A review of reformulations based on sampling and distributional robustness*, EURO J. Comput. Optim. **10** (2022), 100030.

[22] J. Luedtke and S. Ahmed, *A sample approximation approach for optimization with probabilistic constraints*, SIAM J. Optim. **19** (2008), 674–699.

[23] J. Lysgaard, A.N. Letchford, and R.W. Eglese, *A new branch-and-cut algorithm for the capacitated vehicle routing problem*, Math. Program. **100** (2004), 423–445.

[24] A. Majidi, P. Farghadani-Chaharsooghi, and S.M.J. Mirzapour Al-E-Hashem, *Sustainable Pricing-Production-Workforce-Routing Problem for Perishable Products by Considering Demand Uncertainty; A Case Study from the Dairy Industry*, Transp. J. **61** (2022), 60–102.

[25] M. Mammarella, V. Mirasierra, M. Lorenzen, T. Alamo, and F. Dabbene, *Chance-constrained sets approximation: A probabilistic scaling approach*, Automatica **137** (2022), 110108.

[26] M.J. Meixell and M. Norbis, *A review of the transportation mode choice and carrier selection literature*, Int. J. Logist. Manage. **19** (2008), 183–211.

[27] F. Neves-Moreira, B. Almada-Lobo, J.F. Cordeau, L. Guimarães, and R. Jans, *Solving a large multi-product production-routing problem with delivery time windows*, Omega (United Kingdom) **86** (2019), 154–172.

[28] X. Peng, S. Ji, R.G. Thompson, and L. Zhang, *Resilience planning for Physical Internet enabled hyperconnected production-inventory-distribution systems*, Comput. Indus. Eng. **158** (2021), 107413.

[29] S. Pratap, S.K. Jauhar, S.K. Paul, and F. Zhou, *Stochastic optimization approach for green routing and planning in perishable food production*, J. Cleaner Production **333** (2022), 130063.

[30] R. Rahmaniani, T.G. Crainic, M. Gendreau, and W. Rei, *The Benders decomposition algorithm: A literature review*, Eur. J. Oper. Res. **259** (2017), 801–817.

[31] Y. Shuang, A. Diabat, and Y. Liao, *A stochastic reverse logistics production routing model with emissions control policy selection*, Int. J. Production Econ. **213** (2019), 201–216.

[32] B. Vahdani, S.T. Niaki, and S. Aslanzade, *Production-inventory-routing coordination with capacity and time window constraints for perishable products: Heuristic and meta-heuristic algorithms*, J. Cleaner Production **161** (2017), 598–618.

[33] M. Wang, L.C. Wood, and B. Wang, *Transportation capacity shortage influence on logistics performance: evidence from the driver shortage*, Heliyon **8** (2022), e09423.

[34] Q. Zhang, A. Sundaramoorthy, I.E. Grossmann, and J.M. Pinto, *Multiscale production routing in multicommodity supply chains with complex production facilities*, Comput. Oper. Res. **79** (2017), 207–222.

[35] Y. Zhang, H. Alshraideh, and A. Diabat, *A stochastic reverse logistics production routing model with environmental considerations*, Ann. Oper. Res. **271** (2018), 1023–1044.

[36] Z. Zhang, Z. Luo, R. Baldacci, and A. Lim, *A benders decomposition approach for the multivehicle production routing problem with order-up-to-level policy*, Transp. Sci. **55** (2021), 160–178.